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光子纠缠态在海洋湍流中纠缠退化的研究

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摘　要：基于量子光学和Kolmogorov海洋湍流谱理论，建立了双量子比特路径纠缠态在海洋湍流中传输的数学模型，得到了其纠缠退化的解析表达式。利用共生纠缠度度量方法，进一步数值分析了双量子比特路径纠缠态在海洋湍流中的纠缠退化问题。结果表明：双量子比特路径纠缠态在海洋湍流中传输时，其纠缠退化程度与制备该纠缠态的实验装置密切相关。空间距离分布较小的双量子比特路径纠缠态在海洋湍流中传输时纠缠保真度较高。通过数值计算进一步定量分析了多种海洋湍流因素及传输距离对纠缠态退相干问题，得出了双量子比特路径纠缠态在均方温度耗散率较低且动能耗散率较大的海洋湍流中传输时纠缠保真度较高。研究结果对基于光子纠缠态的远距离水下量子通信具有参考意义。

关键词：量子光学；海洋光学；Kolmogorov 海洋湍流；双量子比特路径纠缠态；共生纠缠度
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Entanglement Degradation of Photons Entangled States in Oceanic Turbulence

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Abstract：Based on the theory of Kolmogorov oceanic turbulence spectrum and quantum optics，the theoretical model that the spatial two-qubit photons entangled states prepared by parametric down-converted propagate through the Kolmogorov oceanic turbulence is constructed. The theoretical expressions for entanglement degradation of the spatial two-qubit photons entangled states in oceanic turbulence are obtained. Then，using the Wootters's concurrence，the influence of Kolmogorov oceanic turbulence on the spatial two-qubit photons entangled states with numerical simulation is analyzed. The results show that the parameters of the laboratory device which are prepared the spatial two-qubit entangled states will make a great impact on the entanglement. And the smaller separation of two signal (idler) apertures or the separation between the two signal and the idler apertures is，the higher fidelity of the spatial two-qubit photon entangled states is. The entanglement of spatial two-qubit states can well maintain in the salinity-induced oceanic turbulence when the rate of dissipation of mean-square temperature is small and the rate of dissipation of turbulent kinetic energy per unit mass of fluid is big with numerical calculation. These results have important significance for

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long distance underwater quantum communication via quantum entangled channel.

Key words: Quantum optics; Ocean optics; Kolmogorov oceanic turbulence; Spatial two-qubit entangled states; Concurrence

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0 Introduction

It is well known that Spontaneous Parametric Down-Conversion (SPDC) is an important way to produce entangled photons with various degrees of freedom, such as polarization, spatial, time and Orbital Angular Momentum (OAM) and so on[1-6]. As a main resource in quantum information processing, entangled photons have been applied in quantum teleportation[7], quantum dense coding[8], Quantum Imaging (GI)[3, 9] and quantum metrology[10-11]. Two-qubit states based on the position correlations of entangled photons are referred to as spatial two-qubit states[12-16], which can be collected by position correlated detectors in linear optical system. This spatial two-qubit entangled states can also be used in quantum teleportation[17], quantum secure direct communication[18], quantum key distribution[19] and the verification of quantum nonlocality[20-21].

In all application of quantum information technology, quantum communication may be one of the earliest commercial application. Recently satellite-ground quantum communication through free-space air and inter-city quantum communication network through optical fibers have been made significant progress. At the same time, ocean as a significant natural resource takes up three quarters of the earth, so the long-distance large capacity secure underwater communication is an urgent task, because it is of importance both for the development of communication and quantum technology, and for the application in the fields of ocean, military affairs etc. In order to obtain high band width underwater channel, underwater wireless optical communication has been widely investigated[22-26]. Meanwhile, to achieve unconditionally secure underwater communication, underwater quantum communication has attracted a great deal of attentions[27-31].

Entangled photons, as the carrier of quantum communication, are inevitably affected by the environment in the transmission process, which is also the case in underwater quantum communication. So the influence of various underwater environmental factors on the evolution of entanglement becomes more and more important for the practical application of long-distance quantum communication, such as scattering, absorption and turbulence. Turbulence is a flow state of fluid, which motion has randomness in time and space. The theoretical model Kolmogorov turbulence can describe the inertial subrange which fluid is homogeneous and isotropic turbulence in Kolmogorov scale without energy loss. Atmospheric turbulence is mainly caused by atmospheric refractive index[32]. Since the quantum teleportation have been distributed over 144 km[33], the propagation of entangled photons through the atmospheric turbulence has been studied extensively[34-36]. Using the Wootters's concurrence, the effects of atmospheric turbulence on the entanglement of spatial two-qubit states was investigated[34]. Besides, the polarization-entangled of states propagate in the turbulent atmosphere was also studied[37]. These theoretical researches have provided some guidance for the free-space quantum communication. Oceanic turbulence is a perturbed state which the ocean has different velocity at different disposition ocean. The turbulence flow contains eddies of various size, and the energy is transferred from larger eddies until it is drained out by viscous dissipation. Kolmogorov's asserts that for large Reynolds numbers the small scale structure of turbulence is statistically steady, isotropic and locally homogeneous, and independent of the detailed structure of the large scale components of turbulence. The oceanic turbulence which keeps away from coast and sea level can be regarded as homogeneous and isotropic turbulence. The oceanic turbulence theoretical model which is considered temperature and salinity can be well described the homogeneous and isotropic weak oceanic turbulence[38]. Oceanic turbulence is a key factor in underwater wireless optical communication[38-39]. Using the unified theory of coherence and polarization, the changing of polarization and coherence propagating through oceanic turbulence are widely studied[40-41]. Meanwhile, in order to obtain large capacity underwater quantum communication, several theoretical models have also been established to analyze the effect of oceanic turbulence aberrations on photon OAM[42-45]. The results show that OAM has large channel capacity, but it is easily effected by oceanic turbulence, causing high bit error rate. So the other quantum state may be chosen for underwater quantum communication, such as spatial entangled state and polarized entangled state.

With the development of underwater quantum communication, the underwater transport mechanism of quantum state should be studied. Oceanic turbulence as a key factor affects the underwater quantum communication bit error ratio. And the spatial two-qubit entangled states is a basic quantum resources, which are widely used in quantum communication. To the best of our knowledge, the properties of spatial two-qubit entangled states through oceanic turbulence have never been investigated so far. Due to the randomness of oceanic turbulence, the modern statistical theory and the Rytov approximation perturbation theory have become a typical method to deal with laser beam propagating through weak oceanic turbulence[46]. In this paper, we wish to discover how the oceanic turbulence affects the spatial two-qubit entangled states, which can help for long-distance underwater quantum communication.

1 Theoretical analysis

Based on the theory of partial coherence, the spatial coherence properties of spatial two-qubit entangled states have been precisely analyzed[46]. In addition, the effect of Kolmogorov atmospheric turbulence on the entanglement of spatial two qubit states was analyzed in detail[33]. A typical method called Wootter's concurrence, is utilized to quantify the entanglement of two-qubit states[16, 47]. With the development of underwater quantum communication, the evolution of entangled two-qubit states in underwater channel is more and more important. Based on the theory of partial coherence and Wootter's concurrence, theoretical model that spatial two-qubit entangled states prepared by parametric down-converted propagate through the Kolmogorov oceanic turbulence is constructed. Fig. 1 depicts that the spatial two-qubit states are prepared by SPDC and detected by detectors D_s and D_i located at the position $\boldsymbol{r}_s$ and $\boldsymbol{r}_i$, respectively, after passing through the oceanic turbulence. The transverse position of the apertures define the qubit spaces. Thus the two-dimentional orthonormal bases for the signal photon and idler photon are formed by $\left\{\left|s_1\right\rangle,\left|s_2\right\rangle\right\}$ and $\left\{\left|i_1\right\rangle,\left|i_2\right\rangle\right\}$, respectively, where $\left|i_1\right\rangle$ represents the state of the idler photon passing through the aperture located at position $\boldsymbol{r}_i\equiv\left(\boldsymbol{\rho}_i, z\right)$, etc. Meanwhile, the four-dimentional basis set for the two-qubit state can be represented by $\left\{\left|s_1\right\rangle\left|i_1\right\rangle,\left|s_1\right\rangle\left|i_2\right\rangle,\left|s_2\right\rangle\left|i_1\right\rangle,\left|s_2\right\rangle\left|i_2\right\rangle\right\}$, where $\left|s_2\right\rangle\left|i_2\right\rangle$ represents the joint state of the signal and idler photons when the signal photon passes through the hole located at $\boldsymbol{\rho}_{s_2}$ and the idler photon passes through the hole located at $\boldsymbol{\rho}_{i_2}$, etc.

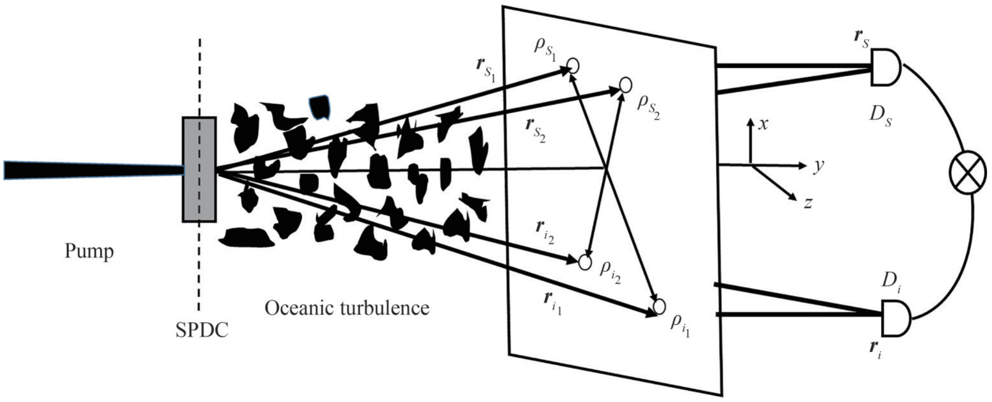


Fig.1 A generic scheme used to investigate the effect of oceanic turbulence on the spatial two-qubit entangled states prepared by SPDC

The sum of the signal and idler photons arriving at detectors D_s and D_i are equal to the positive-frequency part of electric fields $\hat{E}_s^{(+)}\left(\boldsymbol{r}_s, t\right)$ and $\hat{E}_i^{(+)}\left(\boldsymbol{r}_i, t\right)$ at detectors D_s and D_i. The positive-frequency part of the signal field $\hat{E}_s^{(+)}\left(\boldsymbol{r}_s, t\right)$ at position $\boldsymbol{r}_s$ and the positive-frequency part of the idler field $\hat{E}_i^{(+)}\left(\boldsymbol{r}_i, t\right)$ at position $\boldsymbol{r}_i$ are written as

$$\hat{E}_s^{(+)}\left(\boldsymbol{r}_s, t\right)=k_{s_1}\hat{E}_{s_1}^{(+)}\left(\boldsymbol{r}_{s_1}\right)\exp\left[-\mathrm{i}\omega_s\left(t-t_{s_1}\right)\right]+k_{s_2}\hat{E}_{s_2}^{(+)}\left(\boldsymbol{r}_{s_2}\right)\exp\left[-\mathrm{i}\omega_s\left(t-t_{s_2}\right)\right] \tag{1}$$

$$\hat{E}_i^{(+)}\left(\boldsymbol{r}_i, t\right)=k_{i_1}\hat{E}_{i_1}^{(+)}\left(\boldsymbol{r}_{i_1}\right)\exp\left[-\mathrm{i}\omega_i\left(t-t_{i_1}\right)\right]+k_{i_2}\hat{E}_{i_2}^{(+)}\left(\boldsymbol{r}_{i_2}\right)\exp\left[-\mathrm{i}\omega_i\left(t-t_{i_2}\right)\right] \tag{2}$$

The constant factor k_{s_1} depends on the size of the aperture at $\boldsymbol{r}_{s_1}$ and the geometry of the arrangement. Assuming that the quantum efficiencies of detectors D_s and D_i are $\alpha_s$and α_i respectively, the coincidence count rate $R_{si}\left(\boldsymbol{r}_s, \boldsymbol{r}_i\right)$, which a signal photon is detected at position $\boldsymbol{r}_s$ at time t and a idler photon is detected at position $\boldsymbol{r}_i$at time $t+\tau$, can be written as

$$\begin{aligned} R_{si}\left(\boldsymbol{r}_s, \boldsymbol{r}_i\right) = & k_1^2 S^{(2)}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, z\right) + k_2^2 S^{(2)}\left(\boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}, z\right) + k_1 k_2 W^{(2)}\left(\boldsymbol{r}_{\mathrm{s}_1}, \boldsymbol{r}_{i_1}, \boldsymbol{r}_{\mathrm{s}_2}, \boldsymbol{r}_{i_2}, z\right) \times \\ & \exp\left\{\mathrm{i}\left[\omega_s\left(t_{s_1} - t_{s2}\right) + \omega_i\left(t_{i_1} - t_{i_2}\right)\right]\right\} + \mathrm{c.c} \end{aligned} \tag{3}$$

where $W^{(2)}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, \boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}, z\right)$ is the two photons cross-spectral density function. It can quantify the coherence between the two-photon field at the two pairs of positions $\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}\right)$ and $\left(\boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}\right)$. $S^{(2)}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, z\right) = W^{(2)}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, \boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, z\right)$ is the two photons cross-spectral density function at the position $\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}\right)$. The constant factors $k_1 = \sqrt{\alpha_s \alpha_i}\, k_{s_1} k_{i_1}$ and $k_2 = \sqrt{\alpha_s \alpha_i}\, k_{s_2} k_{i_2}$ depend on the size of the aperture, the geometry of the arrangement and quantum efficiencies of detectors.

In turbulent media, the concurrence of this spatial two-qubit states is given by[48]

$$C = \frac{2k_1 k_2 \left| W^{(2)}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, \boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}, z\right) \right|}{k_1^2 S^{(2)}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, z\right) + k_2^2 S^{(2)}\left(\boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}, z\right)} \tag{4}$$

For a spatial two-qubit state, the concurrence is proportional to the magnitude of the two photon cross-spectral density at the two pairs of transverse position that define the two-qubit state. Using the theory of partial coherence, the two photons cross-spectral density function in the oceanic turbulence can be written as

$$W^{(2)}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, \boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}\right) = \left\langle \operatorname{tr}\left\{\boldsymbol{\rho}_{\mathrm{tp}} \hat{\hat{E}}_{s_1}^{(-)}\left(\boldsymbol{r}_{s_1}\right) \hat{\hat{E}}_{i_1}^{(-)}\left(\boldsymbol{r}_{i_1}\right) \hat{\hat{E}}_{s_2}^{(+)}\left(\boldsymbol{r}_{s_2}\right) \hat{\hat{E}}_{i_2}^{(+)}\left(\boldsymbol{r}_{i_2}\right)\right\} \right\rangle_{\mathrm{ocean}} \tag{5}$$

where, $\hat{\hat{E}}_{s_2}^{(+)}\left(\boldsymbol{r}_{s_2}\right)$ is the positive frequency parts of the electric field at position $\boldsymbol{r}_{s_2}$. The symbol tr stands for the trace, $\boldsymbol{\rho}_{\mathrm{tp}}$ is the density matrix of the two photons field produced by SPDC. $\langle \cdot \rangle$ denotes the ensemble average effect of isotropic oceanic turbulence.

Restricted to weak fluctuation conditions, the positive frequency field that propagate in the oceanic turbulence with the Rytov approximation can be written as

$$\hat{\hat{E}}_{s_2}^{(+)}\left(\boldsymbol{r}_{s_2}\right) = \hat{E}_{s_2}^{(+)}\left(\boldsymbol{r}_{s_2}\right) \exp\left[\mathrm{i}\psi\left(\boldsymbol{r}_{s_2}\right)\right] \tag{6}$$

where, $\hat{E}_{s_2}^{(+)}\left(\boldsymbol{r}_{s_2}\right)$ represents a deterministic electric field. ψ is the phase perturbation in the isotropic oceanic turbulence. Assuming the fluctuation of oceanic turbulence intensity is sufficiently small compared to the mean turbulence intensity, the two photons cross-spectral density function in the isotropic oceanic turbulence is

$$W^{(2)}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, \boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}\right) = \mu_{\mathrm{turb}}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, \boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}\right) \times \operatorname{tr}\left\{\boldsymbol{\rho}_{tp} \hat{E}_{s_1}^{(-)}\left(\boldsymbol{r}_{s_1}\right) \hat{E}_{i_1}^{(-)}\left(\boldsymbol{r}_{i_1}\right) \hat{E}_{s_2}^{(+)}\left(\boldsymbol{r}_{s_2}\right) \hat{E}_{i_2}^{(+)}\left(\boldsymbol{r}_{i_2}\right)\right\} \tag{7}$$

where

$$\mu_{\mathrm{turb}}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, \boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}\right) = \left\langle \exp\left[-\mathrm{i}\psi\left(\boldsymbol{r}_{s_1}\right) - \mathrm{i}\psi\left(\boldsymbol{r}_{i_1}\right) + \mathrm{i}\psi\left(\boldsymbol{r}_{s_2}\right) + \mathrm{i}\psi\left(\boldsymbol{r}_{i_2}\right)\right] \right\rangle_{\mathrm{ocean}} \tag{8}$$

Assuming the pump laser is typical Gaussian Schell-model beam, the entanglement of spatial two-qubit states affected by the spatial coherence properties of the pump beam have already been studied in detail[12]. In this paper our main aim is to study the effects of oceanic turbulence on the spatial two-qubit states. we take the pump beams to be full coherent Gaussian Schell-model type, considering the special case $k_1^2 S^{(2)}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}\right) = k_2^2 S^{(2)}\left(\boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}\right)$, so the concurrence of the spatial two-qubit entangled states can be written as

$$C_{\mathrm{qubit}} = \mu_{\mathrm{turb}}\left(\boldsymbol{r}_{s_1}, \boldsymbol{r}_{i_1}, \boldsymbol{r}_{s_2}, \boldsymbol{r}_{i_2}\right) = \left\langle \exp\left[-\mathrm{i}\psi\left(\boldsymbol{r}_{s_1}\right) - \mathrm{i}\psi\left(\boldsymbol{r}_{i_1}\right) + \mathrm{i}\psi\left(\boldsymbol{r}_{s_2}\right) + \mathrm{i}\psi\left(\boldsymbol{r}_{i_2}\right)\right] \right\rangle_{\mathrm{ocean}} \tag{9}$$

In the isotropic weak oceanic turbulence, we assume the phase fluctuations to be a Gaussian random variable. Using the quadratic approximation of Rytov phase $\left\langle \exp^{-\mathrm{i}x} \right\rangle = \exp^{-(1/2)\langle x^2 \rangle}$ and the statistical average of phase variations arising from oceanic turbulence, the concurrence can be written as the wave structure function of a spherical wave

$$C_{\mathrm{qubit}}=\exp\left\{-\frac{1}{2}\left[\left\langle\left(\psi\left(\boldsymbol{r}_{s_1}\right)-\psi\left(\boldsymbol{r}_{s_2}\right)\right)^2\right\rangle+\left\langle\left(\psi\left(\boldsymbol{r}_{i_1}\right)-\psi\left(\boldsymbol{r}_{i_2}\right)\right)^2\right\rangle+\left\langle\left(\psi\left(\boldsymbol{r}_{s_1}\right)-\psi\left(\boldsymbol{r}_{i2}\right)\right)\right\rangle^2+\right.\right.$$
$$\left.\left.\left\langle\left(\psi\left(\boldsymbol{r}_{s_2}\right)-\psi\left(\boldsymbol{r}_{i_1}\right)\right)^2\right\rangle-\left\langle\left(\psi\left(\boldsymbol{r}_{s_1}\right)-\psi\left(\boldsymbol{r}_{i_1}\right)\right)^2\right\rangle-\left\langle\left(\psi\left(\boldsymbol{r}_{s_2}\right)-\psi\left(\boldsymbol{r}_{i_2}\right)\right)^2\right\rangle\right]\right\} \tag{10}$$

where phase structure functions of a spherical wave can be written as

$$\left\langle\left(\psi\left(\boldsymbol{r}_{s_1}\right)-\psi\left(\boldsymbol{r}_{s_2}\right)\right)^2\right\rangle=2\left(\frac{\boldsymbol{\rho}_{s_1}-\boldsymbol{\rho}_{s_2}}{\rho_{\mathrm{osp}}}\right)^{\frac{5}{3}} \tag{11}$$

ρ_{osp} is the spatial coherence length of spherical wave in the isotropic oceanic turbulence, which have the following form[49]

$$\rho_{\mathrm{osp}}=\left[\frac{\pi^2 k^2 z_i}{3}\int_0^{\infty}\kappa^3\phi_n(\kappa)\,\mathrm{d}\kappa\right]^{-\frac{1}{2}}=\left[\left(3.603\times 10^{-7}k^2 z\varepsilon^{-1/3}\chi_T/2\omega^2\right)\times\left(0.419\omega^2-0.838\omega+0.419\right)\right]^{-\frac{3}{5}} \tag{12}$$

here, $\phi_n(\kappa)$ donates the spatial power spectrum of the refractive-index fluctuations for homogeneous and isotropic oceanic water, which has the form

$$\phi_n(\kappa)=0.388\times 10^{-8}\varepsilon^{-\frac{1}{3}}\kappa^{-\frac{11}{3}}\left[1+2.35(\kappa\eta)^{\frac{2}{3}}\right]\frac{\chi_T}{w^2}\left[w^2\exp\left(-A_T\delta\right)+\exp\left(-A_s\delta\right)-2w\exp\left(-A_{Ts}\delta\right)\right] \tag{13}$$

where $A_T=1.863\times 10^{-2}$, $A_s=1.9\times 10^{-4}$, $A_{Ts}=9.41\times 10^{-3}$, $\delta=8.248(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2$, and ε is the rate of dissipation of turbulent kinetic energy per unit mass of fluid that ranges $10^{-1}\,\mathrm{m}^2/\mathrm{s}^3$ from $10^{-10}\,\mathrm{m}^2/\mathrm{s}^3$. And χ_T is the rate of dissipation of mean-square temperature varying from $10^{-4}\,\mathrm{K}^2\mathrm{s}^{-1}$ to $10^{-10}\,\mathrm{K}^2\mathrm{s}^{-1}$. $\eta=10^{-3}\,\mathrm{m}$ is the inner scale of the Kolmogorov microscale. The relative strength of temperature and salinity fluctuation w varies from -5 to 0, the minus sign of the parameter w denotes that there is a reduction in temperature and an increase in salinity with depth. 0 corresponding to the case when temperature-driven turbulence dominates, -5 corresponding to the situation when salinity-driven turbulence prevails. So the concurrence of the spatial two-qubit states in the isotropic oceanic turbulence has the following form

$$C_{\mathrm{qubit}}=\exp\left\{-\rho_{\mathrm{osp}}^{-\frac{5}{3}}\left[\left|\boldsymbol{\rho}_{s_1}-\boldsymbol{\rho}_{s_2}\right|^{\frac{5}{3}}+\left|\boldsymbol{\rho}_{i_1}-\boldsymbol{\rho}_{i_2}\right|^{\frac{5}{3}}+\left|\boldsymbol{\rho}_{s_1}-\boldsymbol{\rho}_{i_2}\right|^{\frac{5}{3}}+\left|\boldsymbol{\rho}_{s_2}-\boldsymbol{\rho}_{i_1}\right|^{\frac{5}{3}}-\left|\boldsymbol{\rho}_{s_1}-\boldsymbol{\rho}_{i_1}\right|^{\frac{5}{3}}-\left|\boldsymbol{\rho}_{s_2}-\boldsymbol{\rho}_{i_2}\right|^{\frac{5}{3}}\right]\right\} \tag{14}$$

In order to simplify the conception, we assume the two pairs of signal and idler apertures in Symmetry position, as $\boldsymbol{\rho}_{s_1}=-\boldsymbol{\rho}_{i_1}$ and $\boldsymbol{\rho}_{s_2}=-\boldsymbol{\rho}_{i_2}$. Meanwhile, to investigate the displacement parameters in terms of the transverse position vectors of the signal and idler photons, the displacement parameters are defined as $\Delta\boldsymbol{\rho}=\boldsymbol{\rho}_{s_1}-\boldsymbol{\rho}_{s_2}$, $\Delta\boldsymbol{\rho}'=\boldsymbol{\rho}_{s_1}-\boldsymbol{\rho}_{i_2}$, so the Eq. (10) has the form

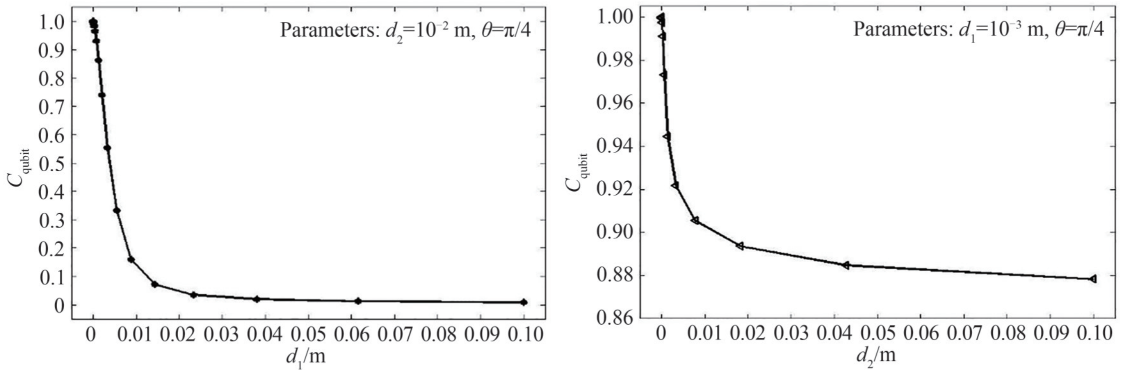
$$C_{\mathrm{qubit}}=\exp\left\{-\left[3.603\times 10^{-7}k^2 z\varepsilon^{-1/3}\frac{\chi_T}{2\omega^2}\times\left(0.419\omega^2-0.838\omega+0.419\right)\right]\times\right.$$
$$\left.\left[2d_1^{\frac{5}{3}}+2d_2^{\frac{5}{3}}-\left(d_1^2+d_2^2+2d_1 d_2\cos\theta\right)^{\frac{5}{6}}-\left(d_1^2+d_2^2-2d_1 d_2\cos\theta\right)^{\frac{5}{6}}\right]\right\} \tag{15}$$

where $d_1=\left|\Delta\boldsymbol{\rho}\right|$ can be taken as a measurement of the effective physical size of the two-qubit states. $d_2=\left|\Delta\boldsymbol{\rho}'\right|$ can be taken as a measure of the separation between the two signal and the idler apertures. θ is the angle between $\Delta\boldsymbol{\rho}$ and $\Delta\boldsymbol{\rho}'$.

2 Numerical calculations and discussion

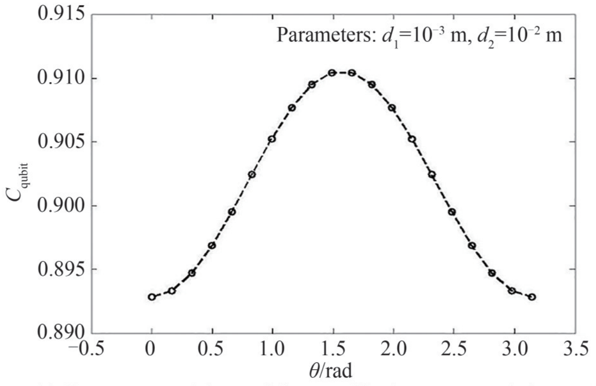
According to Eq. (15), the concurrence is discussed by using numerical analysis, to investigate the effects of oceanic turbulence on the spatial two-qubit entangled states. Different spatial two-qubit states and different oceanic turbulence environment are analyzed in detail, so the mechanism of spatial two-qubit states propagating through oceanic turbulence is clearly presented.

Fig. 2 reveals that the laboratory setup which produce the spatial two-qubit states evidently affect the concurrence in the oceanic turbulence. Fig. 2(a) shows that the concurrence approximates 1 when the effective physical size close to 0. But the concurrence is degraded dramatically when the effective physical size exceeds 0.000 5 m. The concurrence close to 0 when the effective physical size exceeds 0.07 m. It is depicted in Fig. 2(b) that the concurrence decreases with the increasing of the separation between the two signal and the idler apertures. The rate of decline changes, which the concurrence falls dramatically at the start and begins to be stable when d_2 exceeds 0.018 3 m. In Fig. 2(c), the concurrence is Gaussian distribution with the angle θ. The concurrence can obtain max value when $\theta = \pi/2$. The angle θ also represents the spatial distance of path qubit. The phase perturbation in the isotropic oceanic turbulence will degrade the path coherent information of path qubit. In order to maintain good entanglement of the spatial two-qubit states in the ocean, we can set the effective physical size and the separation between the two signal and the idler apertures.



(a) Concurrence of the spatial two-qubit photons entangled states in oceanic turbulence with different d_1

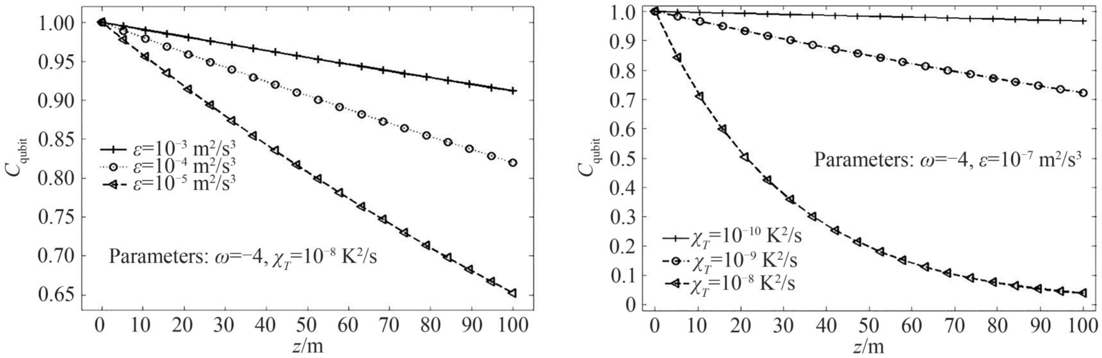
(b) Concurrence of the spatial two-qubit photons entangled states in oceanic turbulence with different d_2



(c) Concurrence of the spatial two-qubit photons entangled states in oceanic turbulence with different θ

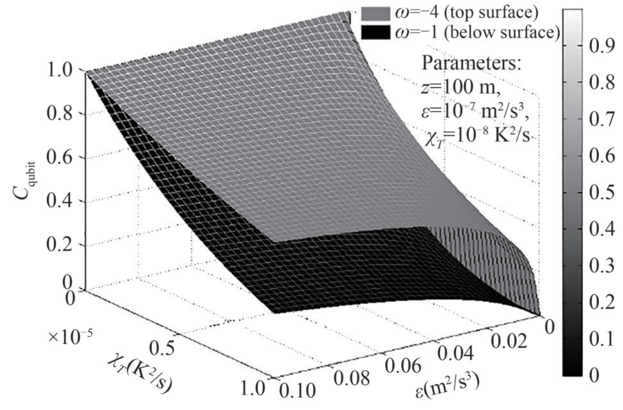
Fig. 2 The change of concurrence with the laboratory setup in the oceanic turbulence ($\lambda = 532\,\text{nm}$, $z = 100\,\text{m}$, $\omega = -4$, $\chi_T = 10^{-7}\,\text{K}^2/\text{s}$, $\varepsilon = 10^{-5}\,\text{m}^2/\text{s}^3$)

From Fig. 3, it is observed that the concurrence is reduced gradually with the distance increase. However, increasing the rate of dissipation of mean-square temperature χ_T results in low concurrence and increasing the rate of dissipation of turbulent kinetic energy per unit mass of fluid ε results in high concurrence. The concurrence approaches to 0.987 8 when the value of ε is $10^{-1}\,\text{m}^2/\text{s}^3$ and the value of χ_T is $10^{-10}\,\text{K}^2/\text{s}$. The concurrence approaches to zero when the value of ε is $10^{-10}\,\text{m}^2/\text{s}^3$ and the value of χ_T is $10^{-5}\,\text{K}^2/\text{s}$. The concurrence is 0.328 5 and 0.647 3 corresponding to the corresponding value ω of -1 and -4, respectively. Clearly, the smaller value of the relative strength of temperature and salinity fluctuation ω is, the smaller concurrence is. So the entanglement is more affected by the temperature-induced oceanic turbulence than the salinity-induced oceanic turbulence. These three parameters in Kolmogorov ocean turbulence affect the coherent length of light field when the spatial two-qubit photons entangled states propagate through the Kolmogorov oceanic turbulence, and then the concurrence is reduced gradually with the distance increase.



(a) Concurrence of the spatial two-qubit photons entangled states for different oceanic turbulence ε

(b) Concurrence of the spatial two-qubit photons entangled states for different oceanic turbulence χ_T



(c) Concurrence of the spatial two-qubit photons entangled states for different oceanic turbulence ω

Fig. 3 The effects of oceanic turbulence on the concurrence ($d_1=10^{-3}$ m, $d_2=10^{-2}$ m, $\theta=\pi/4$, $\lambda=532$ nm)

3 Conclusion

In this paper, the theoretical model that the spatial two-qubit entangled states propagate through the Kolmogorov oceanic turbulence is constructed and the effect of oceanic turbulence on entanglement of the spatial two-qubit states is investigated based on the theory of Kolmogorov oceanic turbulence spectrum and quantum optics. The results show that the parameters of the laboratory devices which produce the spatial two photons can obviously affect the entanglement. The smaller physical size or the separation between the two signal and the idler apertures is, the higher entanglement maintains. And the entanglement of spatial two-qubit states can well maintain in the salinity-induced oceanic turbulence when the rate of dissipation of mean-square temperature is small and the rate of dissipation of turbulent kinetic energy per unit mass of fluid is big. These theoretical model are helpful for the underwater quantum communication in natural oceanic environments. Next, the error rate of concurrence degradation in specific underwater quantum communication scheme will be analyzed. Meanwhile, the relevant underwater quantum communication experiment will also be carried out.

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