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# 渐变折射率多模光纤锁模技术的研究现状与展望(特邀)

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**摘　要**：本文综述了基于渐变折射率多模光纤可饱和吸收体的全光纤锁模激光超短脉冲与束缚态孤子产生机理与技术研究现状.采用这种新型全光纤结构的锁模调制器件，光纤激光器可输出传统孤子的单脉冲能量达nJ量级，同时可实现时空锁模运转.渐变折射率多模光纤作为全光纤非线性可饱和吸收体在激光器中具有重要的研究意义和广泛的应用，为更高能量超短脉冲的产生提供了一条重要技术途径.

**关键词**：光纤激光器；锁模；渐变折射率多模光纤；可饱和吸收体；非线性多模干涉
**中图分类号**：O437.5；O437.4　　**文献标识码**：A　　**doi**：10.3788/gzxb20204911.1149003

# Research Status and Development of Graded-index Multi-mode Fiber Mode-locking Technique (Invited)

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**Abstract**: This paper reviews the research status of the mechanism and technology of all-fiber mode-locked laser ultrashort pulses and bound state soliton generation based on graded index multimode fiber saturable absorber. Using this noval type of all-fiber structured mode-locking modulation device, the output the single pulse energy of conventional soliton in fiber laser can up to the order of nJ, meanwhile spatial mode-locking operation can be realized. As the all-fiber nonlinear saturable absorber, the graded index multimode fiber has important research significance and wide application in lasers, and provides an important technical approach for the generation of higher energy ultrashort pulses.

**Key words**: Fiber laser; Mode locking; Graded-index multimode fiber; Saturable absorber; Nonlinear multimode interference
**OCIS Codes**: 320.7090; 320.110; 320.7120; 320.7140; 320.7160

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# 0 Introduction

Temporal soliton pulse generated in fiber lasers has versatile applications in optical sensing, optical amplifier signal source and precision spectroscopy[1-3]. According to the number of transverse modes of a laser, the mode-locking contains phase locking and total mode-locking (called spatiotemporal mode-locking). Until now, passively mode-locked method plays a vital role in the formation of mode-locking operation. Stemming from the interaction between linear and nonlinear processes, various mode-locking pulse behaviors can be observed in the cavity. For example, a conventional soliton pulse for net anomalous cavity dispersion, a dissipative soliton pulse for net normal cavity dispersion, a stretched pulse for zero cavity dispersion. Moreover, the existence of a different pulse mechanism can be observed in a resonant cavity due to the dispersion shift generated by a high nonlinear phase or fiber-based Lyot filter function[4-5]. Among passive mode-locking fiber lasers, an excellent saturable absorber will become the core of fiber laser operation. To date, a number of saturable absorbers have been demonstrated for ultrashort pulse generation in mode-locked fiber lasers, including Semiconductor Saturable Absorber Mirrors (SESAMs)[6-8], Nonlinear Polarization Rotation (NPR)[9-12], Nonlinear Optical Loop Mirrors (NOLMs) [13-14], and nanomaterials saturable absorber such as single-wall Carbon Nanotubes (CNTs)[15-18], graphene[19-20], Transition Molybdenum Disulphide (TMD) [21-22] and so forth. Besides, a novel all-fiber mode-locked modulator based on the nonlinear multimode interference technique stimulates mode-locked fiber lasers research boom, which further promotes the development of the fiber laser. After promoting theoretically by MAFI A[23], numerous mode-locking operations were demonstrated experimentally with multimode fiber saturable absorber in the fiber lasers[24-27]. The realization of this mode locker is achieved by the nonlinear multimode interference in graded-index multimode fiber which leads to the change of the light intensity-dependent self-focusing length when the light transmits in the nonlinear regime. Therefore, it possesses the ability of intensity discrimination, which can make high-intensity signal undergo minimal influence and discriminate against the low-intensity ones. Moreover, the all-fiber saturable absorber has a series of advantages, such as low price, simple fabrication, small insertion loss, wide waveband operation and high optical damage threshold and so on. Obviously, these characteristics are the core of an ideal mode-locker. After the discovery of all-fiber saturable absorber, various types of mode-locking operations are demonstrated from 1 μm to 2 μm wavelength regimes based on nonlinear multimode interference[28-29]. Meanwhile, all kinds of mode-locking operations can be supported, including conventional solitons[27], stretched pulses[30-31], dispersive soliton pulses[28], bound states [32-34], etc. Besides, another mode-locked state, spatiotemporal mode-locking (total mode-locking) has been investigated in nonlinear multimode cavity in the last few years, which was firstly reported by WRIGHT L G, et al. using offset splicing a graded-index multimode fiber to a few-mode active fiber[35]. Compared with other mode-locked operations, the total mode-locking enables very high pulse energy and all kinds of modes. Since discovery, the total mode-locking have attracted considerable attention due to potential applications in various fields.

In this paper, we review the progress of graded-index multimode fiber as nonlinear saturable absorber in the fiber lasers, and point out the existing problems in the mode-locking operation. As a matter of fact, graded-index multimode fiber is explored to scale up the output power of fiber lasers for industrial processing and the generation of supercontinuum light sources, and so on. Comparing with other multimode fiber, the graded-index multimode fiber possesses typical feature, such as equidistant mode wavenumber, which makes periodical self-image when light transmit it. Therefore, it can make a well test bed for the analysis of complex systems.

# 1 Self-imaging based graded-index multimode fiber

In 2008, single-transverse-mode emission is achieved by using an active multimode fiber due to multimode interference from self-imaging in the fiber[36]. In their experiment, a standard passive single-mode fiber is splicing with a short piece of active multimode fiber with core diameter of 25 μm in the cavity. By controlling the length of the multimode fiber, self-imaging occurs at several $Z$ positions which satisfy the following equation[37]

$$(\beta_n - \beta_1) Z = m_n 2\pi \tag{1}$$

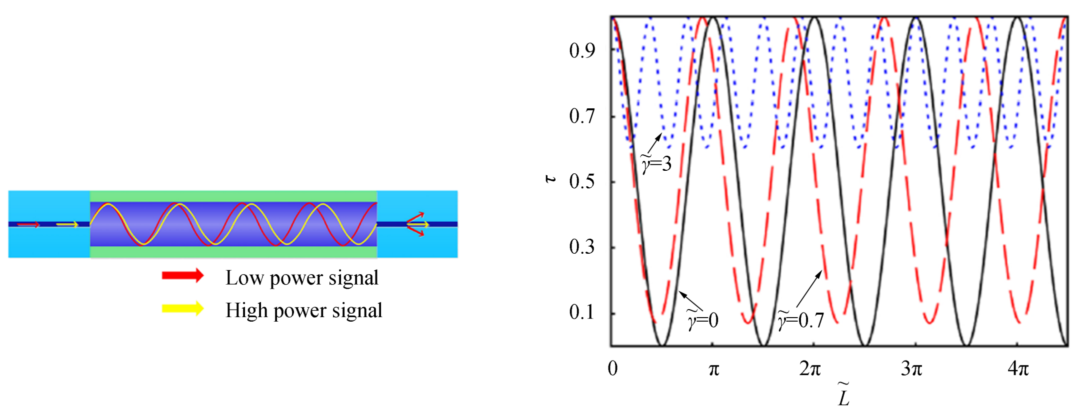
where $\beta_1$ and $\beta_n$ present the propagation constants of the fundamental mode and higher-order modes, respectively. $m_n$ is an integer. When light transmits the multimode fiber, the input field profile is reproduced periodically along the propagation direction. Therefore, the length of the multimode fiber is exactly equal to an integer multiple of the self-imaging length. In order to avoid mode conversion, the length of the active multimode fiber must be limited to a few centimeters, which means that the length of multimode fiber must be controlled precisely. Thus, the multimode fiber with high doping concentration can meet the requirements. In 2011, MAFI A, et al. demonstrated that low-loss coupling could be obtained between two single-mode optical fibers with different mode-field diameters by using a graded-index multimode optical fiber[38]. Later, they studied pulse propagation in a short nonlinear graded-index multimode optical fiber[39]. In 2013, NAZEMOSADAT E and MAFI A proposed to use the differential phase shift among transverse modes to obtain nonlinear multimode interference with a short length of graded-index multimode fiber. Due to the intensity dependent refractive index, the self-imaging period will change accordingly. The relation satisfies the following equation[40]

$$[\beta_n(I)-\beta_1(I)Z]=m_n 2\pi \tag{2}$$

where $I$ presents light intensity. Meanwhile, the self-imaging wavelength which is dependent to intensity can be expressed as[40]

$$\Delta n_{\mathrm{eff},n}(I)L=m_n\lambda_{\mathrm{I}} \tag{3}$$

here, $\Delta n_{\mathrm{eff},n}=(\beta_n-\beta_1)\lambda_0/2\pi$, $L$ is the length of multimode fiber, the wavelength of self-imaging $\lambda_{\mathrm{I}}$ is different from the wavelength $\lambda_0$ which corresponds to minimal cavity loss. In their study, graded-index multimode fiber is spliced between single-mode fibers to form the sandwich structure. In the nonlinear regime, when light propagates along the multimode fiber, which can make high-intensity signal undergo minimal influence and the low-intensity ones are strongly attenuated. Therefore, this structure can act as an effective saturable absorber to obtain mode-locking operation in high energy pulsed fiber lasers. In the structure, nonlinear coupling will occur between the Laguerre-Gauss high-order modes and fundamental mode. They theoretically pointed out that the characteristics of graded-index multimode fiber depend on four parameters, which are the length of multimode fiber, the total number of modes, the intensity of nonlinear effect and the ratio of diameters between high-order modes of multimode fiber and of single mode fiber. Therefore, the transmission of the structure maintains high level at high intensities. In other words, the behavior of the saturable absorber works as Fig.1 shows[23]. Thus, the mode-locked modulator with an all-fiber structure starts to be used in fiber lasers operating in various waveband. Comparing with the traditional saturable absorbers, such as nanomaterials and SESAM, the graded-index multimode fiber saturable absorber has the advantages of high damage threshold, simple structure, low cost, broad spectral waveband, which provides the bed for higher pulse energy lasers. Meanwhile, nonlinear multimode interference effect acts as a long-pass filter to suppress the laser emission below 2 μm, which is required to stabilize the mode-locking operation at 1 μm-waveband fiber laser.



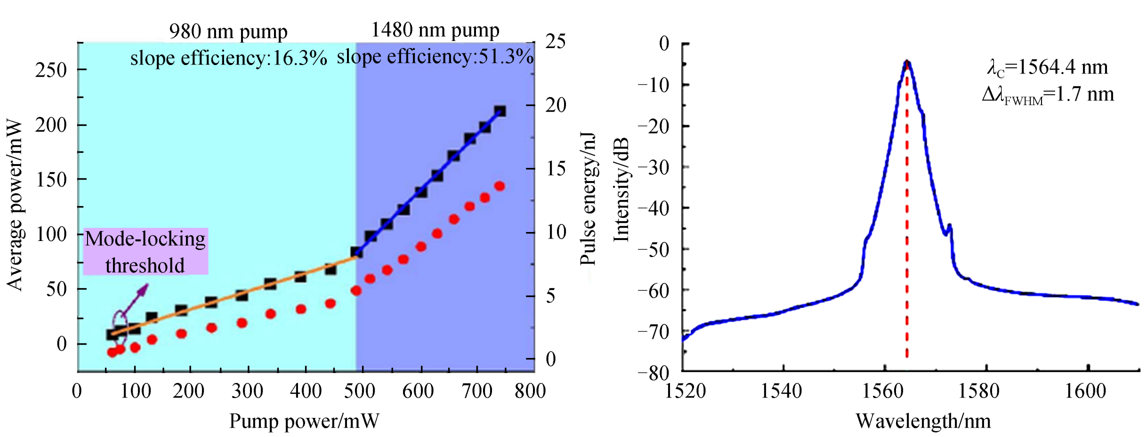

(a) Schematic of multimode fiber saturable absorber

(b) Theoretical relative power transmission from graded-index multimode fiber saturable absorber

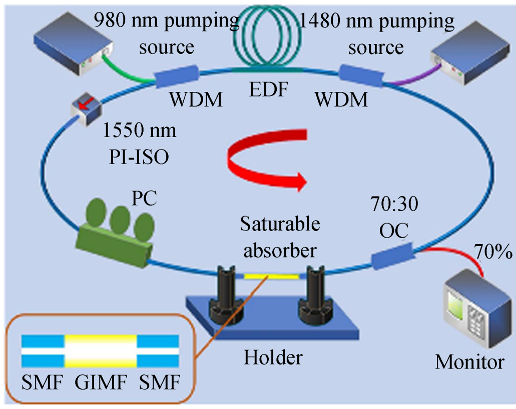
Fig. 1 The behavior of graded-index multimode fiber saturable absorber[23-24]. Reprinted with permission from Ref. [24] Copyright 2013 Optical Society America

## 2 Conventional soliton in anormalous dispersion regime

Based on Kerr-effect induced by changing the self-imaging period with increasing the pump power, a Q-switched fiber laser centered at 1 559.5 nm which includes the nonlinear saturable absorber in the cavity was reported. Due to all-fiber mode-locked modulator, the average output power of 27.6 mW with 0.8 μJ pulse energy was achieved in their experiment[40]. However, the self-imaging period of graded-index multimode fiber is only at the micrometer level. In 2017, WANG Zhao-kun, et al. reported the realization of mode-locked operation by using a modified configuration of graded-index multimode fiber based on nonlinear multimode interference in an Er-doped fiber laser[24]. In their scheme, step-index multimode fiber with an um-level short length is spliced ahead of grade-index multimode fiber to adjust the ratio of diameters between the multimode fibers. By this way, the length restriction of the graded-index multimode fiber can be eliminated. Therefore, the output pulse of 446 fs duration with the corresponding pulse energy of 47 pJ could be generated in the cavity. In the same year, a stable mode-locked state was achieved with the same nonlinear saturable absorber in Tm fiber laser. The output pulse duration is measured to be 1.4 ps at the center wavelength of 1 888 nm[29]. Later, a stable mode-locking operation was obtained in a ring fiber laser by splicing an inner microcavity to graded-index multimode fiber etched with hydrofluoric acid instead of step-index multimode fiber[26]. In 2018, WANG Zhao-kun, et al. exhibited that mode-locking operation could be realized by stretched graded-index multimode fiber as an effective mode-locked modulator[25]. In fact, the nonlinear saturable absorber function can be achieved by adjusting the strength of the stretched fiber in the cavity. In order to elevate the output power, the length of the step-index multimode fiber is required to be shorter than 100 μm. In 2018, conventional soliton pulse with high energy of 2.4 nJ was achieved with the hybrid structure of multimode fiber by optimizing the dispersion and nonlinear parameters[41]. The impact of graded-index multimode fiber core diameter on the characteristics of mode-locking operation was also analyzed. The results showed that a larger diameter can increase the output power in fiber laser. To further increase the output power of the conventional soliton, WANG Zhao-kun, et al. proposed that high energy soliton can be obtained with 70% output coupler by decreasing the length of gain fiber. Therefore, single pulse energy of 13.6 nJ was realized in an Er-doped fiber ring laser, as shown in Fig.2, which is the highest pulse energy obtained in conventional soliton fiber laser so far[42]. Besides, the researchers adopted a no-core fiber instead of the short step-index multimode fiber, stable mode-locking states were obtained by the same method of controlling the excitation of high order modes into graded-index multimode fiber[27]. In 2018, CHEN Tao, et al. proposed that an effective saturable absorber can be formed by splicing a step-index multimode fiber between the single mode fibers[43]. Compared with graded-index multimode fiber, the self-imaging period of step-index multimode fiber with a core diameter of 50 μm can be adjusted from 10 to 43 mm. Therefore, it is easier to control the length of multimode fiber to realize the function of a saturable absorber. By adjusting the length of step-index multimode fiber, the fiber laser can work from Q-switching to soliton mode-locking state.




(a) Average output power (b) Spectrum




(c) Scheme of the experiment

Fig. 2 The scheme and output characteristics of mode-locking operation in anomalous dispersion region[42]

## 3 Dissipative soliton formation in normal dispersion regime

The experimental results mentioned above all relate to fiber laser demonstrated that mode-locking operation in the anomalous dispersion regime. In fact, mode-locking in normal dispersion regime can still be generated with an all-fiber saturable absorber in Yb-doped fiber laser. In 2018, TEGIN U, et al. firstly demonstrated that a stable dissipative soliton with central wavelength of 1 030 nm can be formed based on the nonlinear saturable absorber in 1 μm fiber laser[28]. Fig.3 shows the mode-locking results of an Yb-doped fiber laser. The pulse duration is measured to be 5 ps before compression. As is well known that the nonlinear multimode interference saturable absorber possesses bandpass filtering effect, which is required to sustain dissipative soliton mode locking in Yb-doped fiber laser operating in normal-dispersion regime. However, the output power is relatively lower. How to obtain high energy dispersion soliton pulse will be further discussed in later work. In 2019, the dissipative soliton was also achieved in an Er-doped fiber laser by adding a piece of dispersion compensating fiber[42]. The dissipative soliton with the maximum average power of 72.6 mW and pulse energy of 6.25 nJ could be obtained by adding a section of 10-m long dispersion compensation fiber, as shown in Fig.4.

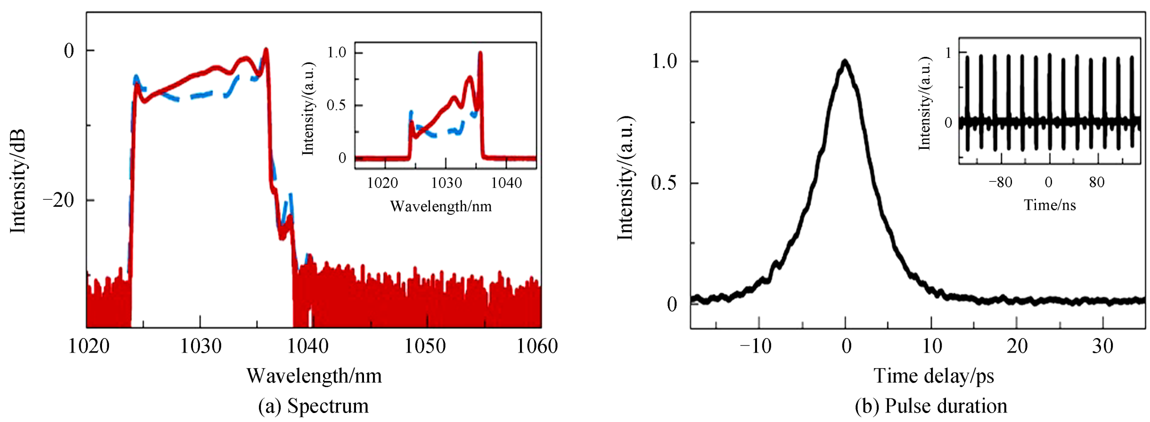
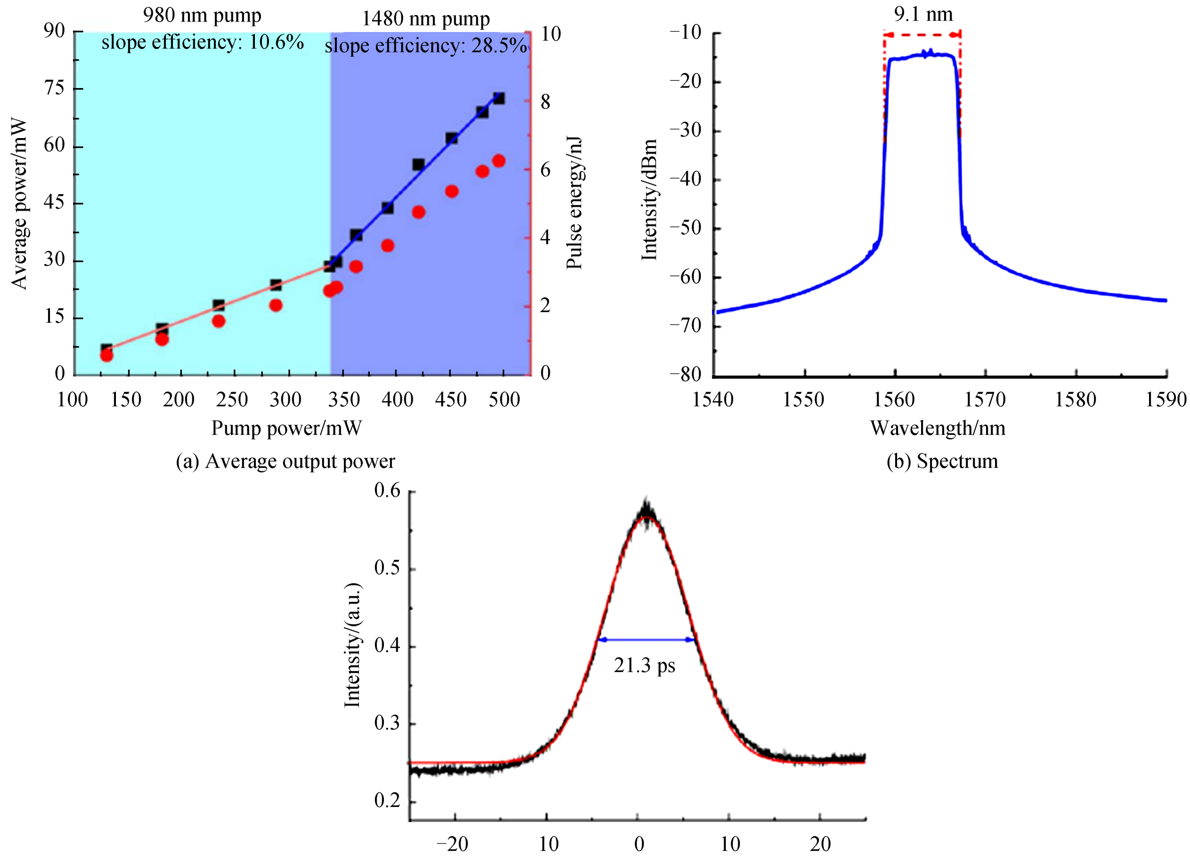



Fig. 3 The characteristics of dissipative soliton in Yb-doped fiber laser[28]. Reprinted with permission from Ref.[28] Copyright 2018 Optical Society America

Besides, another mode-locked state, the Noise-Like Pulse (NLP) operation has also been investigated in the last few years, which was firstly reported by HOROWITZ M[44]. Compared with other mode-locked operations, the NLP mode-locked operation enables very high pulse energy with a penalty of low coherence and longer pulse duration. In 2019, the NLP mode-locking state was demonstrated by LV Zhi-guo, et al. in an Yb-doped fiber laser with hybrid structure of multimode fiber for the first time[45]. In their experiment, 100 μm long step-index multimode fiber with 30 cm long graded-index multimode fiber working as an all-fiber saturable



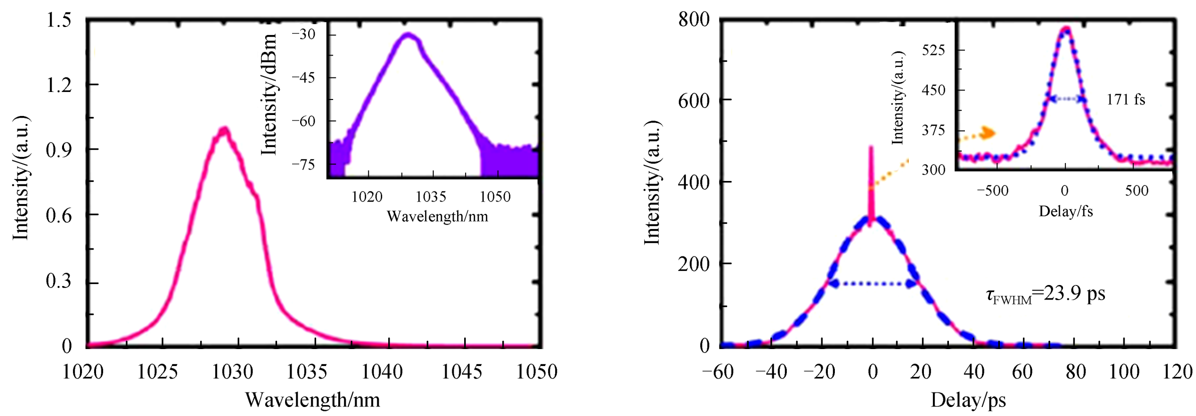

(a) Average output power

(b) Spectrum

(c) Pulse duration

Fig. 4 The characteristics of dissipative soliton in Er-doped fiber laser[42]

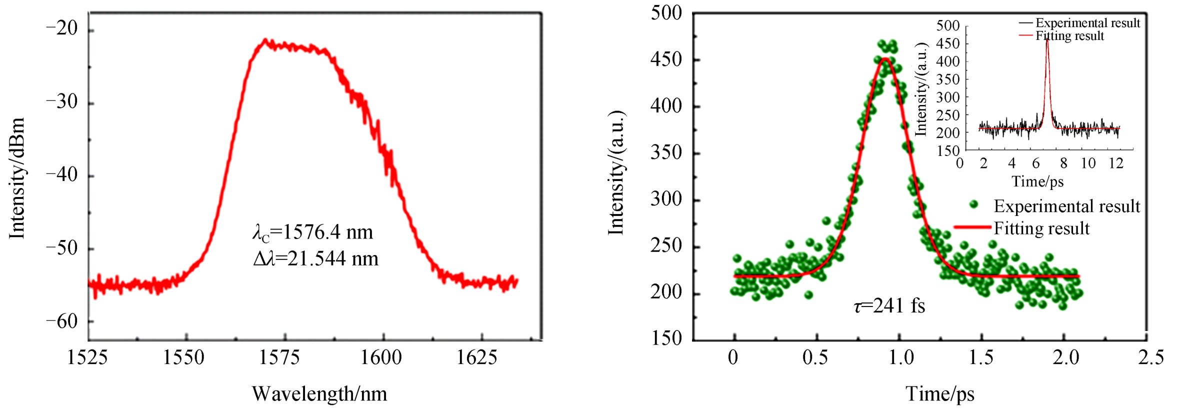
absorber was adopted to realize the generation of NLP at 1 μm waveband, as presented in Fig.5. From Fig.5, we can see a double scale structure with fs-level spike located on the top of ps-level wide pedestal, which is the typical feature of NLP mode-locking operation. The results imply that the formation of NLP due to the complex nonlinear phenomenon of multimode fiber. Besides, we have realized the stable NLP mode-locking state by bending the graded-index multimode fiber in an Er-doped fiber laser, as presented in Fig.6. By analyzing the results obtained, we conclude that the generation of NLP mode-locked state attributes to the soliton collapse and cavity positive feedback.




(a) Output optical spectrum

(b) Autocorrelator trace

Fig. 5 The characteristics of the NLP mode-locking operation with 30-cm long graded index multimode fiber in an Yb-doped fiber laser[45]. Reprinted with permission from Ref.[45] Copyright 2019 The Japan Society of Applied Physics




(a) Output optical spectrum (b) Autocorrelator trace, inset is wide range autocorrelator trace

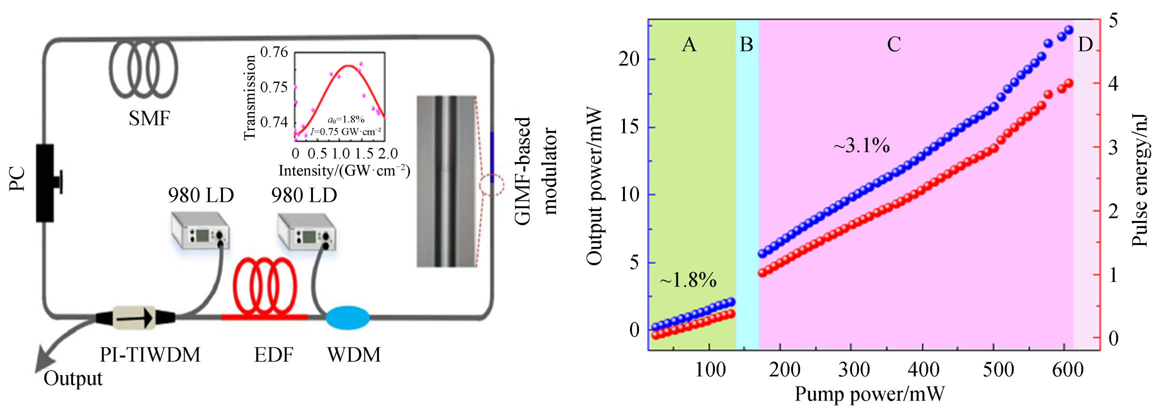
Fig. 6 The characteristics of the NLP mode-locking operation in an Er-doped fiber laser

## 4 Stretched pulse generation in near zero dispersion regime

As we well known that the excellent saturable absorber can be applied to obtain mode-locking operation in different dispersion regimes. Although the high energy conventional soliton can be achieved in anomalous dispersion fiber laser, the single pulse energy is lower relatively. In order to elevate the average output power and realize short pulse laser generation in the cavity, the formation of stretched pulse is a good choice in zero-dispersion cavity. By contrast with conventional soliton, the stretched pulse laser possesses higher pulse energy and shorter pulse width. Therefore, the researchers have demonstrated the formation of stretched pulse in fiber lasers with graded index multimode fiber in the cavity.

### 4.1 Experimental results

In view of this, we used the hybrid structure multimode fiber saturable absorber with a short step-index multimode fiber of 100 μm in zero dispersion Er-doped fiber ring laser. The stable stretched pulse mode-locking state could be easily achieved with the length of graded-index multimode fiber varying from 20 to 60 cm. Taking 20 cm long graded index multimode fiber for an example, the center wavelength and the corresponding 3 dB spectral bandwidth of the mode-locked pulse are 1 608 nm and 14.2 nm respectively[30]. 310 fs is the shortest pulse duration obtained based on the nonlinear all-fiber saturable absorber until now. Under an incident pump power of 700 mW, the single pulse energy is recorded to be 0.85 nJ. Moreover, conventional soliton was also observed by changing the input pump power in the same cavity. Similar observations were also reported by CHEN Guang-wei, et al[31]. In their experiment, the maximum pulse energy reaches 4 nJ with the stretched graded-index multimode fiber in a near zero-dispersion fiber laser. Fig. 7 exhibited their experimental configuration and the variation of the average output power.




(a) The schematic of the stretched pulse mode-locking operation (b) Average output power

Fig. 7 The experiment of the stretched pulse mode locked fiber laser[31]

### 4.2 Theoretical analysis

Why conventional soliton and stretched pulse can coexist in the same cavity? We explain this coexistence as the following reason:

We can infer that the transition between conventional soliton and the stretched pulse mainly depends on the changing dispersion of the graded index multimode fiber in the cavity. In lower-coupling regime (corresponding to lower pump power), the Difference Group Delay (DGD) is proportional to the square root of the length, as expressed by the following equation[46]

$$\Delta\tau=\sqrt{\left(\frac{\partial\Delta\beta}{\partial\omega}L\right)^2+\left|\frac{A_2(L)}{\omega}\right|^2}=L\sqrt{\left(\frac{\partial\Delta\beta}{\partial\omega}\right)^2+\left(\frac{n_0k_0w\kappa}{2\omega}\right)^2} \tag{4}$$

where, $\Delta\beta=\beta_2-\beta_1$ refers to the difference between the propagation constants of the two coupled modes considered. $L$ is the multimode fiber length, $A_2$ is envelope of the second mode, $\kappa$ is the curvature of the bending multimode fiber. $n_0$ is the nominal core refractive index of the fiber. $k_0$ is a constant for multimode fiber. $w$ is the mode radius, which is given by Eq. (5)[46].

$$w^2=\frac{\sqrt{2}\,a}{k_0n_0\sqrt{\Delta}} \tag{5}$$

where the parameters $\Delta$ and $a$ are the index difference between the core and cladding of the graded-index multimode fiber and the core radius. Thus, the DGD changes linearly with the length of the multimode fiber. Meanwhile, the curve also induces a certain DGD. As the increase of the pump power, mode coupling will be enhanced. Eq. (4) gives the DGD in low coupling regime which could not be used to solve the DGD in high-coupling regime (corresponding to high pump power). For high pump power or in high coupling regime, the characteristics of the DGD can be studied by solving the coupling stochastic differential equations[45]. Thus, the DGD in high-coupling regime relays on the following equation[46]

$$\lim_{hz\to\infty}E\left[\Delta\tau^2(z)\right]\Big|_{z=L}=\left(\frac{\partial\Delta\beta}{\partial\omega}\right)^2\frac{L}{h} \tag{6}$$

where $E[\Delta\tau^2]$ is the mean-square of DGD. $h$ is the ensemble-average rate where power is transferred between modes. Evidently, in high-coupling regime, the DGD is proportional to the square of the length $L$ and inverse proportional to $h$. Certainly, $h$ relates to the curve of multimode fiber. Therefore, the analysis mentioned above magnifies that the coexistence of mode-locking states mainly attributes to the varying DGD of the graded-index multimode fiber in the cavity. In other words, the dispersion of the graded-index multimode fiber will be changed under different pump power which corresponds to different coupling regimes.

## 5 Total mode-locking and soliton molecules formation

In 2017, WRIGHT L G, et al. observed total mode-locking in multimode fiber laser, for the first time. In their experiment, a segment of graded-index multimode fiber was spliced to a few-mode Yb-doped fiber by the method of offset[35]. Meanwhile, a combination of spectral filtering and intracavity NPR technique was used to achieve mode-locking operation in the normal dispersion fiber laser. Similar observations were reported by using a section of multimode active fiber in a fiber laser. Although the output of the multimode laser has higher pulse energy, spatial beam quality is relatively poor in fiber lasers. Hence, in order to obtain high-beam quality and high pulse energy simultaneously, different experimental methods need to be designed in practice.

In 2018, QIN Hua-qiang, et al. reported the total mode-locking by using the similar laser cavity to that proposed by WRIGHT L G, soliton molecules in the experiment was also observed[47]. By changing the incident pump power or rotating the intracavity waveplates, pulse pairs, triplets, and quadruplets were generated in the cavity, as exhibited in Fig.8. From then on, various bound states have been observed based on multimode fiber by the researchers. In 2018, bound states were observed based on hybrid structure multimode fiber saturable absorber in an Er-doped fiber laser for the first time[33]. By comparing the ratio of the single pulse duration to the pulse-pulse interval, the formation of bound states depends on the interaction between solitons rather than interaction with the radiated dispersive wave. By adjusting the dispersion and nonlinear parameters of the cavity, triplets, and quadruplets were formed through bending the longer graded-index multimode fiber instead of

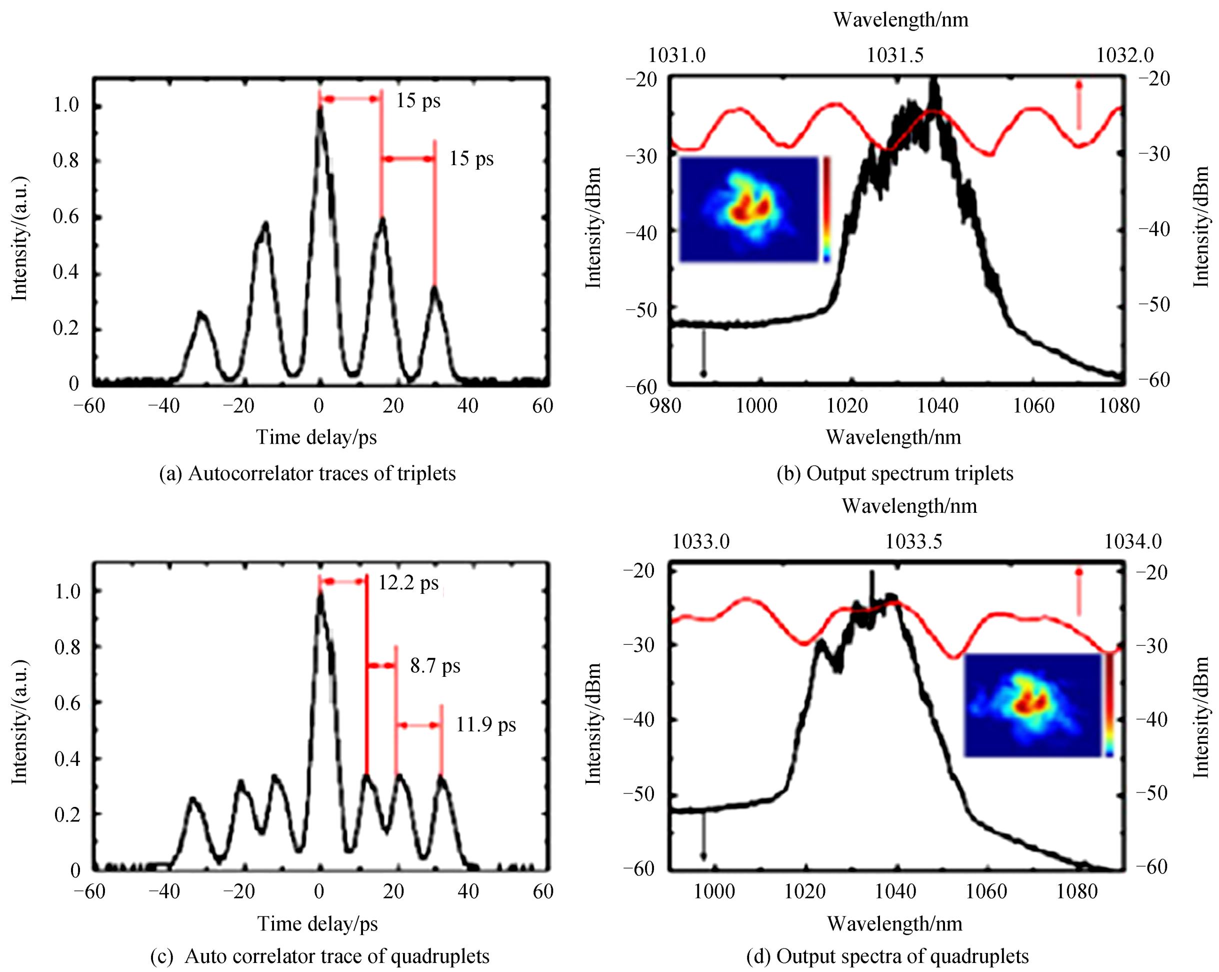



Fig. 8 Soliton molecules formation with triplets and quadruplets in an Yb-doped laser[47]. Reprinted with permission from Ref. [47] Copyright 2018 Optical Society of America

hybrid structure multimode fiber saturable absorber[34]. Meanwhile, vibration soliton pairs composed of conventional soliton was also observed in the same fiber laser. In 2019, LV Zhi-guo, et al. observed bound states by adopting all-fiber saturable absorber in an Yb-doped fiber laser[48]. Therefore, novel nonlinear phenomena were realized with an all-fiber mode-locked modulator in the fiber lasers, which evidently accelerates the development of fiber laser.

## 6 Conclusions and perspectives

In summary, a full new range of spatiotemporal dynamics, including transverse multimode locking, as well as the formation of spatiotemporal soliton molecules could be studied in the multimode fiber cavities. Moreover, passive multimode fiber cavities could be a topic for further investigations. In the ultrafast laser cavity, a stable dissipative soliton in the normal dispersion regime has been obtained. However, the single pulse energy is relatively low. The multi-stage amplification technique is a good choice to realize high energy dissipative soliton. Although stretched pulse mode-locking operations are obtained in near zero-dispersion regime, the single pulse energy is also relatively low. Therefore, how to boost the pulse energy while maintaining the ultrashort pulse duration will be further studied in future works.

Although the mode-locking operation was observed at different waveband from 1 μm to 2μm, the midinfrared mode-locking could not be obtained based on the graded-index multimode fiber yet. Unlike the silica based multimode fiber, the midinfrared multimode fiber with a controlled refractive profile is difficult to access. Hence, it is important to form nonlinear saturable absorber based on the midinfrared graded-index multimode fiber for high energy midinfrared fiber lasers.

Spatiotemporal soliton molecules are fascinating topics. Moreover, soliton molecules were observed in different dispersion regimes. Meanwhile, three-dimensional optical soliton molecular complexes and supramolecular structures could be formed and interactions can occur among themselves[49-50]. Therefore, the study of optical soliton molecule dynamics becomes a research focus. The moving soliton molecules, incoherent dissipative solitons, optical rogue waves may exist in multimode fiber cavities[50], which will be explored in future works.

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