doi:10.3788/gzxb20184709.0927002

二粒子态控制双向量子隐形传态

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摘　要:为实现二粒子态的控制双向量子隐形传态,基于张量分析、贝尔基测量和冯·诺依曼测量等方法,提出二粒子态控制双向量子隐形传态的通道准则,准则中,选择九粒子纠缠态为量子通道,并给出任意九粒子纠缠态能否作为量子通道的必要条件.基于该条件,控制者完成冯·诺依曼测量后,通道被分为单通道、双通道、三通道和四通道.以双通道为例进行计算分析,在 Alice、Bob 和 Controller 的共同作用下,经冯·诺依曼测量后进行贝尔基测量和对应的幺正变换,最终实现 Alice 和 Bob 之间量子态的交换,通过双通道的隐形传输,验证了所提准则的可行性并给出量子通道选择的一般方法.

关键词:量子光学;双向控制量子隐形传态;贝尔基测量;张量分析;冯·诺依曼测量;通道参数矩阵;量子通信

中图分类号:O413.1; O431.2　　**文献标识码**:A　　**文章编号**:1004-4213(2018)09-0927002-7

Controlled Bidirectional Two-qubit Quantum Teleportation

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Abstract: In order to achieve controlled bidirectional quantum teleportation of two qubits, based on tensor analysis, von Neumann and Bell basis measurements, we propose criteria for assessing whether an arbitrary nine-qubit entangled state can be used as quantum channel for controlled bidirectional quantum teleportation. Using such a channel, the controller performs a von Neumann measurement, dividing the initial quantum channel into single, dual, triple and quadruple channel. In order to achieve CBQT of two qubits using the proposed criteria, taking the dual channel as an example, Alice, Bob, and the controller work together, using von Neumann measurement, Bell basis measurements, and finally unitary transformations. we verify the feasibility of the proposed standard and set out a general quantum channel selection method.

Key words: Quantum optics; Controlled bidirectional quantum teleportation; Tensor analysis; Bell basis measurement; Von Neumann measurement; Channel parameter matrix; Quantum communication

OCIS Codes: 270.0270; 270.5565; 270.5585; 000.3860; 200.3050

0 Introduction

Quantum Teleportation (QT) is a communication protocol that transmits a system's complete state information from one place to another via a quantum channel without transmitting the system itself. The essential property underlying this type of information transfer is the existence of quantum correlations, which can be characterized in several different ways[1]. Since the initial work of BENNETT C, et al. in

Foundation item: The National Natural Science Foundation of China (No. 51402235)

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Received: Nov.15, 2018; **Accepted**: May 21, 2018

1993, QT has been a focus of many researchers[2]. In particular, Pan, et al. made a breakthrough by achieving QT via photon entanglement[3-4]. Recently, ground-to-satellite QT of independent single-photon qubits has been achieved, which is an essential step toward a global-scale quantum internet[5].

Significant progress toward QT has been achieved in several different areas. In 1998, Karlsson and Bourennane proposed a controlled teleportation protocol that uses Greenberger - Horne - Zeilinger (GHZ) states as a quantum channel[6], while Tian, et al. suggested using a tensor representation method to construct quantum channels for teleportation[7].

Since the quantum correlations in channel are vital for teleportation from a practical point of view, the connection between entanglement, nonlocality, and teleportation fidelity has also been investigated extensively[8-9]. In 2013, Zha, et al. proposed a theoretical scheme for Controlled Bidirectional QT (CBQT)[10-11]. In Zha's approach, Alice and Bob exchanged quantum information using entangled five-qubit cluster states. In 2014, Fu, et al. proposed a QT (BQT) protocol that uses four-qubit entangled states as the quantum channel. Fu's protocol uses tensor analysis and Bell basis measurements to realize the transmission of unknown quantum states[12]. In 2015, Hassanpour, et al. proposed a novel CBQT protocol that teleports an unknown single-qubit state using an approach which a controller controls the communication in only one direction[13]. The following year, Hassanpour, et al. proposed a novel BQT scheme that uses an entanglement swapping technique where two users simultaneously transmit pure Einstein-Podolsky-Rosen (EPR) states to each other[14]. Significantly, most communication protocol studies have focused on transmitting single-qubit states, and little work has been done on using multi-qubit states.

Although teleporting a single degree of freedom has attracted the attention of many researchers and important progress has been made, a single degree of freedom can carry much less information than multiple degrees of freedom. Since any quantum system inevitably interacts with its surroundings, the problem of achieving QT in the face of environmental decoherence has also been studied by many researchers[15-16]. Aside from single-qubit QT, the majority of the remaining work has been devoted to studying the teleportation of two-qubit states, including directly teleporting an EPR pair and an arbitrary two-qubit state[17], probabilistically teleporting a two-qubit state[18], and teleporting a two-qubit state in a decohering environment[19]. In addition, Pan, et al. have realized the teleportation of multiple degrees of freedom, a significant achievement that has a special importance for understanding the potential of teleportation[4].

In this paper, we therefore use nine-qubit entangled states for our quantum channel. We characterize these states using the tensor representation and show that there is an intrinsic link between the Bell basis measurements T and the Channel Parameter Matrix (CPM) R, which makes the quantum state representation simpler and clearer. Clearly, selecting an appropriate quantum channel is important for two-qubit CBQT. Here, we present a general method for choosing a channel type, based on analyzing the CPM R for both the von Neumann and Bell basis measurements, and give necessary criteria that can be used to assess whether a given nine-qubit entangled state can be used as a quantum channel for two-qubit CBQT in terms of whether the resulting entangled states are separable in these two bases. There are four quantum channel types: single, dual, triple, and quadruple channels. Our general approach provides a theoretical basis for channel selection and flexibility for two-qubit CBQT experiments.

1 Tensor representation for two-qubit CBQT

Before constructing quantum channels for teleporting two-qubit states, we first need to introduce some preliminaries related to characterizing unknown two-qubit states. Suppose Alice wants to teleport an unknown state $|\varphi\rangle_{A_1A_2}$ to Bob, and Bob wants to simultaneously teleport an unknown state $|\varphi\rangle_{B_1B_2}$ to Alice. With the help of the tensor representation, $|\varphi\rangle_{A_1A_2}$ and $|\varphi\rangle_{B_1B_2}$ can be expressed as

$$\begin{cases}|\varphi\rangle_{A_1A_2}=X^{hi}|hi\rangle\\|\varphi\rangle_{B_1B_2}=Y^{jk}|jk\rangle\end{cases}\tag{1}$$

where X^{hi} and Y^{jk} are unknown coefficients that satisfy $X^{hi}X^{*}_{hi}=1$ and $Y^{jk}Y^{*}_{jk}=1$, with $h,i,j,k\in\{0,1\}$. To implement CBQT, we take the general nine-qubit entangled state $|\varphi\rangle_{a_1a_2b_3b_4Cb_1b_2a_3a_4}$ as the quantum channel, which can be expressed as

$$|\varphi\rangle_{a_1a_2b_3b_4Cb_1b_2a_3a_4} = R^{lmpqtnors}|lmpqtnors\rangle \tag{2}$$

where $l, m, p, q, t, n, o, r, s \in \{0,1\}$ and the CPM $\boldsymbol{R}$ satisfies $R^{lmpqtnors}R^*_{lmpqtnors}=1$. Qubits a_1, a_2, a_3 and a_4 belong to Alice, b_1, b_2, b_3 and b_4 belong to Bob, and C belongs to the controller. The total system state has the following form

$$|\varphi\rangle_{\mathrm{T}} = |\varphi\rangle_{A_1A_2}\otimes|\varphi\rangle_{B_1B_2}\otimes|\varphi\rangle_{a_1a_2b_3b_4Cb_1b_2a_3a_4} = X^{hi}Y^{jk}R^{lmpqtnors}|hi\rangle|jk\rangle|lmpqtnors\rangle \tag{3}$$

The entangled quantum channel is illustrated in Fig.1, where Alice wants to teleport $|\varphi\rangle_{A_2A_2}$ to Bob, and Bob wants to teleport $|\varphi\rangle_{B_1B_2}$ to Alice, and both of them rely on the CPM and the controller.

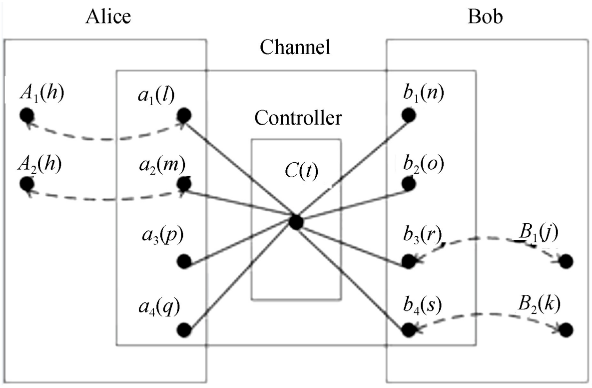


Fig.1 Entangled nine-qubit quantum channel for two-qubit CBQT

CBQT can be implemented by using the following two-step.

Step 1: By performing a Hadamard operation on the qubit C, we change the total state $|\varphi\rangle_{\mathrm{T}}$ to

$$\begin{aligned} H|\varphi\rangle_{\mathrm{T}} &= X^{hi}Y^{jk}R^{lmpqtnors}|hi\rangle|jk\rangle|lmpqnors\rangle H|t\rangle_c = \\ &\frac{1}{\sqrt{2}}X^{hi}Y^{jk}R^{lmpqnors}|hi\rangle|jk\rangle|lmpqnors\rangle[|0\rangle+(-1)^t|1\rangle]_c \end{aligned} \tag{4}$$

The controller then performs a von Neumann measurement on C in the basis $\{|0\rangle, |1\rangle\}$. If the outcome is $|0\rangle$, the original state $|\varphi\rangle_{\mathrm{T}}$ becomes $|\boldsymbol{\Phi}_1\rangle_{\mathrm{T}} = \frac{1}{\sqrt{2}}X^{hi}Y^{jk}R^{lmpqnors}|hi\rangle|jk\rangle|lmpqnors\rangle$, and likewise (but with a factor of $(-1)^t$) for the outcome $|1\rangle$.

Step 2: To transmit the two-qubit states, we need to perform Bell basis measurements based on (A_1, a_1), (A_2, a_2), (B_1, b_3), and (B_2, b_4). The Bell basis is $|\phi^\alpha\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\phi^\beta\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, where $\alpha=1,2$ and $\beta=3,4$. With the measurement matrices $\boldsymbol{T}$ given in Ref. [12], which we define as $\boldsymbol{T}^{\beta_1}, \boldsymbol{T}^{\beta_2}, \boldsymbol{T}^{\gamma_1}$, and $\boldsymbol{T}^{\gamma_2}$ in this case, the quantum channel becomes $|\varphi\rangle = R^{nopq\beta_1\beta_2\gamma_1\gamma_2}T^{\beta_1}T^{\beta_2}T^{\gamma_1}T^{\gamma_2}|nopq\beta_1\beta_2\gamma_1\gamma_2\rangle$ and the four Bell basis measurement matrices $\boldsymbol{T}$ can be represented as $\boldsymbol{T}^1 = I/\sqrt{2}$, $\boldsymbol{T}^2 = \sigma_z/\sqrt{2}$, $\boldsymbol{T}^3 = \sigma_x/\sqrt{2}$, $\boldsymbol{T}^4 = -\mathrm{i}\,\sigma_y/\sqrt{2}$, where $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{\sigma}_x$, $\boldsymbol{\sigma}_y$, and $\boldsymbol{\sigma}_z$ are the Pauli matrices. Alice and Bob now perform Bell basis measurements, yielding $\langle\phi^{\gamma_2}|\phi^{\gamma_1}|\phi^{\beta_2}|\phi^{\beta_1}|\varphi\rangle = R^{nopq}\sigma^{\beta_1\beta_2\gamma_1\gamma_2}|nopq\beta_1\beta_2\gamma_1\gamma_2\rangle$.

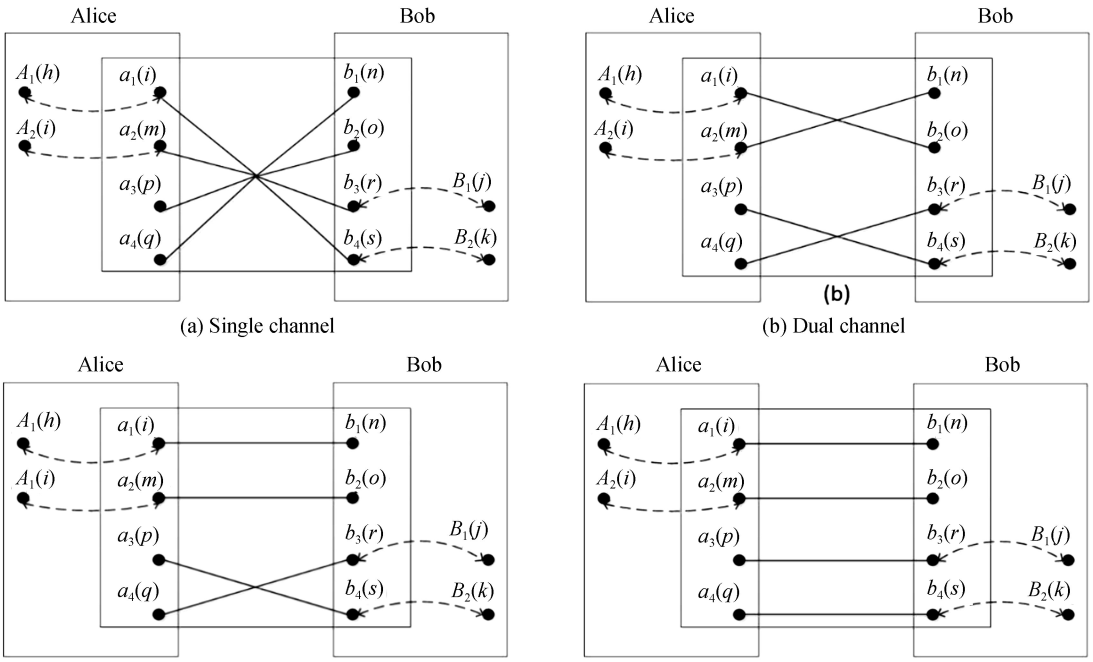
Finally, they perform unitary transformations, giving $(\sigma^{\beta_1\beta_2\gamma_1\gamma_2})^{-1}\langle\phi^{\gamma_2}|\phi^{\gamma_1}|\phi^{\beta_2}|\phi^{\beta_1}|\varphi\rangle$.

After the measurements and transformations above, the resulting CPM $\boldsymbol{R}$ can be one of four different types, giving a single, dual, triple and quadruple channel, as illustrated in Fig.2.

If the CPM $\boldsymbol{R}$ can not be written as a direct product of two or more unitary matrices, we call the channel a single channel, as shown in Fig.2(a). In other words, it is genuinely entangled, and can not be used to realize bidirectional teleportation between Alice and Bob.

If the CPM $\boldsymbol{R}$ can be represented as $\boldsymbol{R}^{lmpqnors} = \boldsymbol{R}_1^{lmno}\boldsymbol{R}_2^{pqrs}$, we call the channel a dual channel, as shown in Fig.2(b). Here, $\boldsymbol{R}$ is a 16×16 complex matrix, while $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are 4×4 complex matrices. Alice and Bob can decompose $|\boldsymbol{\Phi}_1\rangle_{\mathrm{T}}$ using Bell basis measurements, transforming the state $|\boldsymbol{\Phi}_1\rangle_{\mathrm{T}}$ into

$$
|\boldsymbol{\Phi}_1\rangle_{\mathrm{T}}=\frac{1}{\sqrt{2}}X^{hi}Y^{jk}R_1^{lmno}R_2^{pqrs}T^{\beta_1}_{A_1a_1}T^{\beta_2}_{A_2a_2}T^{\gamma_1}_{B_1b_3}T^{\gamma_2}_{B_2b_4}|nopq\beta_1\beta_2\gamma_1\gamma_2\rangle=
\frac{1}{\sqrt{2}}X^{hi}Y^{jk}\sigma_{hi}^{(\beta_1\beta_2)no}\sigma_{jk}^{(\gamma_1\gamma_2)pq}|nopq\beta_1\beta_2\gamma_1\gamma_2\rangle \tag{5}
$$



(a) Single channel (b) Dual channel (c) Triple channel (d) Quadruple channel

Fig.2 The four channel types in two-qubit CBQT

After Alice and Bob have completed their Bell basis measurements on their qubits (A_1, a_1), (A_2, a_2), and (B_1, b_3), (B_2, b_4), respectively, the resulting state becomes $\langle\phi^{\gamma_2}_{ks}|\phi^{\gamma_1}_{jr}|\phi^{\beta_2}_{im}|\phi^{\beta_1}_{hl}|\Phi_1\rangle_{\mathrm{T}}=\frac{1}{\sqrt{2}}X^{hi}Y^{jk}\sigma^{\beta_1\beta_2\gamma_1\gamma_2}|nopq\rangle_{b_1b_2a_3a_4}=|\Phi_2\rangle_{\mathrm{T}}$, where $\sigma^{\beta_1\beta_2\gamma_1\gamma_2}$ is the transmission matrix

$$
\sigma^{\beta_1\beta_2\gamma_1\gamma_2}=(\boldsymbol{R}_1\otimes\boldsymbol{R}_2)(\boldsymbol{T}^{\beta_1}\otimes\boldsymbol{T}^{\beta_2}\otimes\boldsymbol{T}^{\gamma_1}\otimes\boldsymbol{T}^{\gamma_2})=[\boldsymbol{R}_1(\boldsymbol{T}^{\beta_1}\otimes\boldsymbol{T}^{\beta_2})]\otimes[\boldsymbol{R}_2(\boldsymbol{T}^{\gamma_1}\otimes\boldsymbol{T}^{\gamma_2})]=\sigma^{\beta_1\beta_2}\otimes\sigma^{\gamma_1\gamma_2} \tag{6}
$$

Here, $\sigma^{\beta_1\beta_2\gamma_1\gamma_2}$ is a 16×16 matrix. In order to realize the quantum state exchange perfectly, the matrix $\sigma^{\beta_1\beta_2\gamma_1\gamma_2}$ must be unitary, and Alice and Bob have to individually perform unitary transformations on (A_1, a_3), (A_2, a_4) and (B_1, b_1), (B_2, b_2) to perfectly exchange their quantum states. The two matrices $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are unitary, and we can realize two-qubit CBQT if Alice (Bob) performs an appropriate unitary transformation $(\sigma^{\gamma_1\gamma_2})^{-1}{}_{a_3a_4}((\sigma^{\beta_1\beta_2})^{-1}{}_{b_1b_2})$ on the state $|\boldsymbol{\Phi}_2\rangle_{\mathrm{T}}$. This transformation can be expressed as

$$
(\sigma^{\beta_1\beta_2}_{b_1b_2})^{-1}(\sigma^{\gamma_1\gamma_2}_{a_3a_4})^{-1}|\boldsymbol{\Phi}_2\rangle_{\mathrm{T}}=X^{no}|no\rangle_{b_1b_2}Y^{pq}|pq\rangle_{a_3a_4} \tag{7}
$$

If the CPM $\boldsymbol{R}$ can be written as $\boldsymbol{R}^{lmpqnors}=\boldsymbol{R}_1^{ln}\boldsymbol{R}_2^{mo}\boldsymbol{R}_3^{pqrs}$, we call the channel a triple channel, as shown in Fig.2(c). Now, $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are 2×2 complex matrices, while $\boldsymbol{R}_3$ is a 4×4 complex matrix. Here, Alice and Bob decompose $|\boldsymbol{\Phi}_1\rangle_{\mathrm{T}}$ using the Bell basis via the measurement matrix $\boldsymbol{T}$, so $|\boldsymbol{\Phi}_1\rangle_{\mathrm{T}}$ becomes

$$
|\boldsymbol{\Phi}_1\rangle_{\mathrm{T}}=\frac{1}{\sqrt{2}}X^{hi}Y^{jk}R_1^{ln}T^{\beta_1}_{A_1a_1}R_2^{mo}T^{\beta_2}_{A_2a_2}R_3^{pqrs}T^{\gamma_1}_{B_1b_3}T^{\gamma_2}_{B_2b_4}|nopq\beta_1\beta_2\gamma_1\gamma_2\rangle=
\frac{1}{\sqrt{2}}X^{hi}Y^{jk}\sigma_h^{(\beta_1)}{}_n\sigma_i^{(\beta_2)}{}_o\sigma_{jk}^{(\gamma_1\gamma_2)pq}|nopq\beta_1\beta_2\gamma_1\gamma_2\rangle \tag{8}
$$

After Alice and Bob have finished this measurement on their qubits (A_1, a_1), (A_2, a_2) and (B_1, b_3), (B_2, b_4), respectively, the collapsed state takes the form $\langle\varphi^{\gamma_2}_{ks}|\varphi^{\gamma_1}_{jr}|\varphi^{\beta_2}_{im}|\varphi^{\beta_1}_{hl}|\Phi_1\rangle_{\mathrm{T}}=\frac{1}{\sqrt{2}}X^{hi}Y^{jk}\sigma^{\beta_1}\sigma^{\beta_2}\sigma^{\gamma_1\gamma_2}|nopq\rangle_{b_1b_2a_3a_4}=|\Phi_2\rangle_{\mathrm{T}}$ where the transmission matrix $\sigma^{\beta_1\beta_2\gamma_1\gamma_2}$ is

$$
\sigma^{\beta_1\beta_2\gamma_1\gamma_2}=\sigma^{\beta_1}\sigma^{\beta_2}\sigma^{\gamma_1\gamma_2} \tag{9}
$$

To implement CBQT, Alice and Bob each perform his or her own unitary transformations on $(A_1$,

a_3),(A_2,a_4) and (B_1,b_1),(B_2,b_2), respectively, to perfectly exchange their quantum states. Again, the matrices $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, and $\boldsymbol{R}_3$ are unitary, and we can achieve two-qubit CBQT if Alice (Bob) performs the appropriate unitary transformation $(\sigma^{\gamma_1})^{-1}{}_{a_3}(\sigma^{\gamma_2})^{-1}{}_{a_4}((\sigma^{B_1B_2})^{-1}{}_{b_3b_4})$ on the state $|\boldsymbol{\Phi}_2\rangle_{\mathrm{T}}$, which yields

$$(\sigma^{\gamma_1})^{-1}{}_{a_3}(\sigma^{\gamma_2})^{-1}{}_{a_4}(\sigma^{\beta_1\beta_2})^{-1}{}_{b_1b_2}|\Phi_2\rangle_{\mathrm{T}}=X^{no}|no\rangle_{b_1b_2}Y^{pq}|pq\rangle_{a_3a_4} \tag{10}$$

If the CPM $\boldsymbol{R}$ can be represented as $R^{lmpqnors}=R_1^{ln}R_2^{mo}R_3^{pr}R_4^{qs}$, we call the channel a quadruple channel, as shown in Fig. 2(d). The four matrices $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, and $\boldsymbol{R}_4$ are 2×2 complex matrices. This time, Alice and Bob decompose $|\boldsymbol{\Phi}_1\rangle_{\mathrm{T}}$ in the Bell basis using the measurement matrix $\boldsymbol{T}$, obtaining

$$\begin{aligned}|\boldsymbol{\Phi}_1\rangle_{\mathrm{T}}=&\frac{1}{\sqrt{2}}X^{hi}Y^{jk}R_1^{ln}T^{\beta_1}_{A_1a_1}R_2^{mo}T^{\beta_2}_{A_2a_2}R_3^{pr}T^{\gamma_1}_{B_1b_3}R_4^{qs}T^{\gamma_2}_{B_2b_4}|nopq\beta_1\beta_2\gamma_1\gamma_2\rangle=\\&\frac{1}{\sqrt{2}}X^{hi}Y^{jk}\sigma_h^{(\beta_1)n}\sigma_i^{(\beta_2)o}\sigma_j^{(\gamma_1)p}\sigma_k^{(\gamma_2)q}|nopq\beta_1\beta_2\gamma_1\gamma_2\rangle\end{aligned} \tag{11}$$

After Alice and Bob have finished this measurement on their qubits, (A_1,a_1),(A_2,a_2) and (B_1,b_3),(B_2, b_4), respectively, the resulting state takes the form

$$\langle\phi^{\gamma_2}_{ks}|\phi^{\gamma_1}_{jr}|\phi^{\beta_2}_{im}|\phi^{\beta_1}_{hl}|\Phi_1\rangle_{\mathrm{T}}=\frac{1}{\sqrt{2}}X^{hi}Y^{jk}\sigma^{\beta_1}\sigma^{\beta_2}\sigma^{\gamma_1}\sigma^{\gamma_2}|nopq\rangle_{b_1b_2a_3a_4}=|\Phi_2\rangle_{\mathrm{T}}$$

where the transmission matrix $\sigma^{\beta_1\beta_2\gamma_1\gamma_2}$ is

$$\sigma^{\beta_1\beta_2\gamma_1\gamma_2}=\sigma^{\beta_1}\sigma^{\beta_2}\sigma^{\gamma_1}\sigma^{\gamma_2} \tag{12}$$

The situation is the same as before: in order to realize perfect quantum state exchange, Alice and Bob each have to perform a unitary transformation on (A_1,a_3),(A_2,a_4) and (B_1,b_1),(B_2,b_2), respectively. Again, the matrices $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, and $\boldsymbol{R}_4$ are unitary, and we can achieve two-qubit CBQT if Alice (Bob) performs an appropriate unitary transformation $(\sigma^{\gamma_1})^{-1}{}_{a_3}(\sigma^{\gamma_2})^{-1}{}_{a_4}((\sigma^{\beta_1})^{-1}{}_{b_1}(\sigma^{\beta_2})^{-1}{}_{b_2})$ on the state $|\boldsymbol{\Phi}_2\rangle_{\mathrm{T}}$, yielding

$$(\sigma^{\gamma_1})^{-1}{}_{b_1}(\sigma^{\gamma_2})^{-1}{}_{b_2}(\sigma^{\beta_1})^{-1}{}_{a_3}(\sigma^{\beta_2})^{-1}{}_{a_4}|\Phi_2\rangle_{\mathrm{T}}=X^{no}|no\rangle_{b_1b_2}Y^{pq}|pq\rangle_{a_3a_4} \tag{13}$$

Based on the analysis, we obtain the following necessary conditions for the channel to serve as a quantum channel for two-qubit CBQT:

1)The initial quantum channel must be a single channel and in a genuinely entangled state.

2)The controller must perform a von Neumann measurement on its own qubit C, and then reduce the quantum channel from a nine-qubit to an eight-qubit entangled state.

3)After performing the von Neumann and Bell basis measurements, the resulting CPM can be written as a direct product of unitary matrices. This can produce four different channel types, and true CBQT is only possible when the channel can be divided, i.e., if it is not a single channel.

2 Selecting nine-qubit channels for two-qubit CBQT

Given the above channel requirements for achieving two-qubit CBQT, the channel must have no fewer than nine qubits. Here, we choose a nine-qubit, genuinely entangled state as the quantum channel, and focus on the dual channel case, where the CPM is composed of two general unitary matrices, $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, as follows

$$\boldsymbol{R}_1=\begin{pmatrix}-\sin\theta_1 & 0 & 0 & \cos\theta_1\\ 0 & \cos\theta_1 & \sin\theta_1 & 0\\ 0 & -\sin\theta_1 & \cos\theta_1 & 0\\ \cos\theta_1 & 0 & 0 & \sin\theta_1\end{pmatrix}\quad 0\leqslant\theta_1\leqslant\pi \tag{14}$$

$$\boldsymbol{R}_2=\begin{pmatrix}-\sin\theta_2 & 0 & 0 & \cos\theta_2\\ 0 & \cos\theta_2 & \sin\theta_2 & 0\\ 0 & -\sin\theta_2 & \cos\theta_2 & 0\\ \cos\theta_2 & 0 & 0 & \sin\theta_2\end{pmatrix}\quad 0\leqslant\theta_2\leqslant\pi \tag{15}$$

For simplicity, we set $\theta_1=0$ and $\theta_2=\pi/6$ in the following discussion. The normalization condition ensures the channel parameters are suitably adjusted, and the CPM $\boldsymbol{R}$ can then be represented as $\boldsymbol{R}=\boldsymbol{R}_1\otimes\boldsymbol{R}_2$, while the channel's quantum state becomes

$$|\varphi\rangle_{a_1a_2b_3b_4Cb_1b_2a_3a_4}=\frac{1}{4\sqrt{2}}\Big[-\frac{1}{2}(|0000C1100\rangle+|0001C1110\rangle+\cdots)+\cdots$$

$$+\frac{\sqrt{3}}{2}\left(|0000C1111\rangle+|0001C1100\rangle+\cdots\right)\Bigg] \tag{16}$$

where $C=0,1$. If Alice wants to teleport $|\varphi\rangle_{A_1A_2}$ to Bob, and Bob wants to teleport $|\varphi\rangle_{B_1B_2}$ to Alice, with

$$|\varphi\rangle_{A_1A_2}=X^{00}|00\rangle+X^{01}|01\rangle+X^{10}|10\rangle+X^{11}|11\rangle \tag{17}$$

$$|\varphi\rangle_{B_1B_2}=Y^{00}|00\rangle+Y^{01}|01\rangle+Y^{10}|10\rangle+Y^{11}|11\rangle \tag{18}$$

then the state of the whole 13-qubit system is given by

$$\begin{aligned}|\varphi\rangle_{\mathrm{T}}=&\ |\varphi\rangle_{A_1A_2}\otimes|\varphi\rangle_{B_1B_2}\otimes|\varphi\rangle_{a_1a_2b_3b_4Cb_1b_2a_3a_4}=\\ &\frac{1}{4\sqrt{2}}\Bigg[-\frac{1}{2}X^{00}Y^{00}\left(|00000000C1100\rangle+|00000001C1100\rangle+\cdots\right)+\cdots\\ &+\frac{\sqrt{3}}{2}X^{11}Y^{11}\left(|11110000C1111\rangle+|11110001C1100\rangle+\cdots\right)\Bigg]\end{aligned} \tag{19}$$

After the von Neumann measurement, the state of the remaining 12-qubit system collapses to

$$\begin{aligned}|\varphi\rangle_{\mathrm{T}}=&\ |\varphi\rangle_{A_1A_2}\otimes|\varphi\rangle_{B_1B_2}\otimes|\varphi\rangle_{a_1a_2b_3b_4Cb_1b_2a_3a_4}\\ &=\frac{1}{4\sqrt{2}}\Bigg[-\frac{1}{2}X^{00}Y^{00}\left(|00000000C1100\rangle+|00000001C1100\rangle+\cdots\right)+\\ &+\frac{\sqrt{3}}{2}X^{11}Y^{11}\left(|11110000C1111\rangle+|11110001C1100\rangle+\cdots\right)\Bigg]\end{aligned} \tag{20}$$

If Alice and Bob perform standard Bell basis measurements on (A_1,a_1), (A_2,a_2) and (B_1,b_3), (B_2,b_4), respectively, then, assuming their measurement outcomes are ϕ^1,ϕ^2 and ϕ^3,ϕ^4, the whole system will collapse into the following state

$$\begin{aligned}|\Phi_2\rangle_{\mathrm{T}}=&\langle\phi^4_{B_2b_4}|\phi^3_{B_1b_3}|\phi^2_{A_2a_2}|\phi^1_{A_1a_1}|\Phi_1\rangle_{\mathrm{T}}=\\ &\frac{1}{4\sqrt{2}}\Bigg[\left(-\frac{\sqrt{3}}{2}X^{11}Y^{00}-\frac{1}{2}X^{11}Y^{11}\right)|0000\rangle+\left(\frac{1}{2}X^{11}Y^{01}-\frac{\sqrt{3}}{2}X^{11}Y^{10}\right)|0001\rangle+\cdots\\ &+\left(-\frac{\sqrt{3}}{2}X^{00}Y^{01}-\frac{1}{2}X^{00}Y^{10}\right)|1110\rangle+\left(\frac{1}{2}X^{00}Y^{00}-\frac{\sqrt{3}}{2}X^{00}Y^{11}\right)|1111\rangle\Bigg]\end{aligned} \tag{21}$$

where $\sigma^{12}=R_1(\sqrt{2}\,T^1\otimes\sqrt{2}\,T^2)$ and $\sigma^{34}=R_2(\sqrt{2}\,T^3\otimes\sqrt{2}\,T^4)$.

To exchange their quantum states, Alice and Bob then perform appropriate unitary transformations on b_1, b_2, and a_3,a_4, respectively. This gives the final state $(\sigma^{34})^{-1}{}_{b_1b_2}(\sigma^{12})^{-1}{}_{a_3a_4}|\Phi_2\rangle_{\mathrm{T}}$, or, more explicitly

$$\begin{aligned}|\Phi\rangle_{\mathrm{swap}}=&\frac{1}{\sqrt{2}}\left(X^{00}Y^{00}|0000\rangle+X^{00}Y^{01}|0001\rangle+\cdots+X^{11}Y^{11}|1111\rangle\right)_{b_1b_2a_3a_4}=\frac{1}{\sqrt{2}}(X^{00}|00\rangle+X^{01}|01\rangle+\\ &X^{10}|10\rangle+X^{11}|11\rangle)_{b_1b_2}\otimes(Y^{00}|00\rangle+Y^{01}|01\rangle+Y^{10}|10\rangle+Y^{11}|11\rangle)_{a_3a_4}\end{aligned} \tag{22}$$

In other words, Alice has sent $|\varphi\rangle_{A_1A_2}$ to Bob, and Bob has simultaneously transmitted $|\varphi\rangle_{B_1B_2}$ to Alice, successfully achieving two-qubit CBQT. This is also true for the other 255 types of Bell basis measurement $\phi^{\alpha}_{B_2b_4}\phi^{\beta}_{B_1b_3}\phi^{\gamma}_{A_2b_2}\phi^{\delta}_{A_1a_1}$, where $\alpha,\beta,\gamma,\delta=1,2,3,4$.

3 Conclusion

In this paper, we have presented a two-qubit CBQT protocol with potential theoretical applications, and set out necessary conditions for selecting a quantum channel that uses nine-qubit entangled states to achieve two-qubit CBQT. We then gave a general expression for the channel's quantum state, taking the specific example of a dual channel. We have also proposed a general channel selection method for the four different channel types and verified its feasibility theoretically. This method describes how to choose an appropriate and simple quantum channel to realize two-qubit CBQT, and provides flexibility for quantum experiments. Clearly, there are many ways to implement the proposed protocol theoretically, with a wide range of applications, leading to unique research avenues. We hope that with the improvement of modern optical technology and the progress of QT, this criterion can be verified experimentally. In the future, we plan to investigate ways for Alice and Bob to exchange something complex more than just a two-qubit state with the help of the controller.

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引用格式：WANG Si-meng, ZHANG Wei, LIAO Cong. Controlled Bidirectional Two-qubit Quantum Teleportation[J]. *Acta Photonica Sinica*, 2018, **47**(9): 0927002

王思萌,张炜,廖聪.二粒子态控制双向量子隐形传态[J].光子学报,2018,**47**(9):0927002