doi:10.3788/gzxb20184709.0914005

注入锁定半导体锁模激光器的时序抖动特性

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摘 要:采用简化的孤子微扰模型方程组,研究了注入锁定时被动锁模半导体激光器的时序抖动噪声特性.研究发现,当注入锁定的耦合系数为 10^{-3} ps^{-1}时,在 100 kHz 到 10 GHz 的频率范围内,从锁模激光器的时序抖动噪声可从自由运转情况下的皮秒量级(3.83 ps)下降至几十飞秒.还讨论了主从锁模激光器的稳态相位差、注入耦合系数、线宽增强因子等参数对时序抖动噪声特性的影响.计算结果表明,时序抖动对稳态相位差不敏感,而耦合系数的变化对其则有显著影响;此外,线宽增强因子越小,时序抖动噪声越小.

关键词:时序抖动噪声;注入锁定;被动锁模;半导体激光器
中图分类号:TN248.4; TN365 **文献标识码**:A **文章编号**:1004-4213(2018)09-0914005-8

Timing Jitter in Pulsed Injection Locking of a Passively Mode-locked Semiconductor Laser

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Abstract: The effects of injection locking on the timing-jitter noise properties of a passively mode locked semiconductor laser are investigated theoretically via a set of simplified soliton perturbation equations. We find that, with injection coupling coefficient of 10^{-3} ps^{-1}, the timing-jitter noise of the slave mode-locked laser can be reduced from a few picoseconds (3.83 ps) to a few femtoseconds in the frequency range from 100 kHz to 10 GHz. We also discuss the impacts of the steady state phase difference between the injection and slave pulses, coupling coefficient, and linewidth enhancement factor on the timing-jitter noise. Calculations show that timing jitter is insensitive to the phase difference while coupling coefficient has a strong effect on the timing jitter, and a smaller linewidth enhancement factor leads to better timing jitter performance.

Key words: Timing-jitter noise; Injection-locking; Passively mode-locking; Semiconductor lasers
OCIS Codes: 140.4050; 140.4050; 140.5960; 250.5960; 270.2500

0 Introduction

Mode-locked semiconductor lasers can provide compact optical sources of stable pulse trains with high repetition rate[1] for optical communications and signal processing. Examples of applications include high

Foundation item: The Major Project of Science and Technology of Fujian Province (No.2017HZ0004-1) and the National Program on Key Basic Research Project (No. 2016YFB0402300)

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Received: Apr.17, 2018; **Accepted**: Jun.6, 2018

speed optical sampling[2], analog to digital conversion[3] and optical time division multiplexed transmission[4]. Among all these applications, high quality mode-locking lasers with low timing-jitter are required, because timing jitter is related to the phase noise in the optical frequency components of the pulse train, which could induce the deviation of the temporal pulse positions from those in a fully periodic train. In recent years, the timing jitter in passively Mode-Locked Laser Diodes (MLLDs) has attracted lots of attention and reports on a few picoseconds (ps) or hundreds of femtoseconds (fs) magnitude jitter have been published[5-6].

Jiang[7] noticed that the carrier dynamics have a great effect on timing jitter of mode-locked semiconductor lasers and derived the analytical expression for the amplitude, frequency, timing and carrier phase noise of MLLDs. In experiment, he demonstrated a timing jitter of 1.7 ps in a monolithic semiconductor with hybrid mode-locking scheme, while 4.7 ps with passive mode-locking. Another efficient and inexpensive approach to reduce timing jitter is based on all-optical feedback from an external cavity, which can reduce the average pulse-to-pulse Root-Mean-Squared (RMS) timing jitter from 295 fs to 32 fs per cycle[6]. Recently, there are articles proposing that the timing-jitter of a passively mode-locked femtosecond fiber laser can be further improved through the optical injection-locking technique[8]. The theoretical explanation of this technique was given by Margilit[9] using soliton perturbation model. However, so far, there is lacking of similar analysis on using injection-locking to reduce the noise in passively mode-locked semiconductor lasers.

In this paper, we focus on thetiming jitter of pulsed injection locked passively MLLDs. In section 1, we introduce the evolution of electric-field inside the laser cavity and deduce the soliton perturbation model with pulse injection terms. Through a linear stochastic analysis, we derive the analytical expression of the noise contributions from the fluctuations of energy, carrier phase, frequency, timing and carrier population. In section 2, a numerical analysis of the noise spectra and the pulse-to-clock root-mean-squared timing jitter of the injection locked passively MLLDs is given. We also discuss the impacts of the steady state phase difference, coupling coefficient and linewidth enhancement factor on the noise spectrum of timing jitter.

1 Model

The timing jitter of a mode-locked laser is limited by quantum noise, but in most cases it is dominated by vibrations and drifts of thelaser resonator. So we begin our analysis with the evolution of electric-field of the circulating pulse in the cavity. The electric field after the intracavity transmission can be expressed as

$$A_{n+1}(t)=\mathrm{e}^{\mathrm{EL}(t)}A_n(t)+S_n(t) \tag{1}$$

in which, n is the number of round-trips the pulse propagated in the cavity, t is used to express the temporal laser dynamics in the short time periods, $A_n(t)$ is the n^{th} electric-fleld in the laser cavity, $\mathrm{EL}(t)$ represents all the interactions between the cavity medium and the optical electric field, including loss, gain, optical bandpass filtering, saturable absorption, group velocity dispersion and Kerr effect, the quantity $S_n(t)$ is the noise of the electric-fleld. Subtracting $A_n(t)$ from both sides of Eq. (1) yields

$$A_{n+1}(t)-A_n(t)=(\mathrm{e}^{\mathrm{EL}(t)}-1)A_n(t)+S_n(t) \tag{2}$$

Supposing that T describes the long term variable of the laser dynamics during the round-trips, T_{R} is the round trip time of the cavity, then $n=T/T_{\mathrm{R}}$. By making the substitutions $A_n(t)\rightarrow A(t,T)$ and $S_n(t)\rightarrow S(t,T)$, we can convert the discrete equation of motion into a continuous time equation. Utilizing the approximation $\mathrm{e}^{\mathrm{EL}(t)}\approx 1+\mathrm{EL}(t)$, Eq. (2) becomes

$$T_{\mathrm{R}}\frac{\mathrm{d}A(t,T)}{\mathrm{d}T}=\mathrm{EL}(t)A(t,T)+T_{\mathrm{R}}S(t,T) \tag{3}$$

where $A(t,T)$ is the normalized electric field amplitude . To agree with the notation in Haus and Mecozzi ′ s paper[10], we rescale the noise term $S(t,T)$ by $T_{\mathrm{R}}S(t,T)$. The concrete expression $\mathrm{EL}(t)$ has been addressed by Huas and Mecozzi. Using their formula, the differential equation for a mode-locked diode laser becomes

$$T_{\mathrm{R}}\frac{\mathrm{d}A(t,T)}{\mathrm{d}T}=\left[-l+g+\frac{1}{\Omega_{\mathrm{f}}^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}+\mathrm{j}D\frac{\mathrm{d}^2}{\mathrm{d}t^2}+(\gamma-\mathrm{j}\delta)\left|A(t,T)\right|^2\right]A(t,T)+T_{\mathrm{R}}S(t,T) \tag{4}$$

in which, l and g are the total cavity loss and gain per pass, Ω_f is optical filter bandwidth which induce wavelength-dependent phase shifts due to dielectric coatings, D is the group velocity dispersion coefficient, γ is the saturable absorber coefficient which can be created by a reversed biased diode laser, δ is the self-phase modulation coefficient due to Kerr effect which will produce a phase shift in the pulse.

An additional term $kI(t,T)$ is used to replace the term $S(t,T)$ in the master equation to study the dynamics of the pulse injection locking passively mode-locked semiconductor laser. The injection sheme brings another two elements t_0 and p_0, which are the initial differences in round trip time and center optical frequency between the injected signal and the free running passive mode locking laser respectively. According to Margalit's[9-11] analysis, under the two following assumptions: 1) the difference between the harmonics of the repetition rates of the free running laser and the injected pulses train is much smaller than either of the above two repetition rates; 2) the maximum difference between interacting injected frequency component and cavity modes is no more than half of the cavity mode spacing, the injection locking characteristics of a passively MLLD can be described as

$$T_R\frac{dA(t,T)}{dT}=\left[j(w_0-w_i)T_R-l+g+\frac{1}{\Omega_f^2}\frac{d^2}{dt^2}+jD\frac{d^2}{dt^2}+(\gamma-j\delta)|A|^2+t_0\frac{d}{dt}+j\frac{2P_0}{\Omega_f^2}\frac{d}{dt}\right]\cdot A(t,T)+T_RkI(t,T) \tag{5}$$

where w_0 is the center frequency of the gain spectrum, w_i is the injected pulse carrier frequency. When the frequency discrepancy (w_0-w_i) is offset by the injection term, the phase difference between the injected and slave pulse remains constant, locking is expected. k is the normalized coupling coefficient of the injected signal. If the injected signal is normalized so that $|I(t,T)|^2$ is equal to power, then it was shown[12] that the coupling coefficient k is in a unit of s^{-1}.

With a linear analysis, the external noise, compared to the intrinsic noise of the slave pulse itself, comes from a random variation of the injected pulse parameters, which can be describe as

$$I(t,T)=I_0(t,T)+\Delta\omega_s I_\omega(t,T)+\Delta\theta_s I_s(t,T)+\Delta p_s I_p(t,T)+\Delta t_s I_t(t,T) \tag{6}$$

where $I_0(t,T)$ is a determined injected pulse, $I_x(x=\omega,\theta,p,t)$, is the expansion function which describes the projection of the injected pulse noise parameters[9] and the four parameters $\Delta\omega_s$, $\Delta\theta_s$, Δp_s and Δt_s are small variations of the corresponding pulse parameters, which represent energy, phase, center frequency, and temporal differences respectively. The projection function for 0^{th} and 1^{st} expansion is evaluated by

$$I_x(\Delta\omega,\Delta\theta,\Delta p,\Delta t)=I_x(\Delta\omega,\Delta\theta,\Delta p,\Delta t)|_{I_0}+y\frac{\partial}{\partial y}\{I(t-\Delta t_s,T)e^{j(\Delta p_s t+\Delta\theta_s)}\}|_{I_0} \tag{7}$$

$$y=\Delta\omega_s,\Delta\theta_s,\Delta p_s,\Delta t_s$$

To simplify the analysis, we assume that the free running passively MLLD and the injected signal has the same pulse shape and pulse width, and pulse injection power is low enough that the energy projection of the injected signal can be neglected. We further assume that the deviations from the steady state of the laser are small and the initial frequency detuning p_0 is zero.

Using the above assumptions and (7), we can obtain the detail expansion termof the injected signal $I(t,T)$

$$\begin{cases} I_p=I_p|_{I_0}+I_{pp_s}\Delta p_s+I_{pt_s}\Delta t_s=\cos(\Delta\theta_0)(\Delta p+\Delta p_s)+\dfrac{2}{3\tau^2}\sin(\Delta\theta_0)(\Delta t+\Delta t_s) \\ I_\theta=I_\theta|_{I_0}+I_{\theta\theta}\Delta\theta_s=\cos(\Delta\theta_0)(\Delta\theta+\Delta\theta_s) \\ I_t=I_t|_{I_0}+I_{tt_s}\Delta t_s+I_{tp_s}\Delta p_s=\cos(\Delta\theta_0)(\Delta t+\Delta t_s)-\dfrac{\pi^2\tau^2}{6}\sin(\Delta\theta_0)(\Delta p+\Delta p_s) \end{cases} \tag{8}$$

In which, $\Delta\theta_0$ is the steady state phase difference between the injection and slave pulses, τ is the pulsewidth. The other expansion functions equal to zero because they are anti-symmetric.

Without injection, the steady state solution to (5) is an unchirped soliton[10]. The noise driven by Amplified Spontaneous Emission (ASE) can be treated as a perturbation to the soliton solution. The perturbation analysis leads to four orthogonal fluctuations: energy ($\Delta\omega$), carrier phase ($\Delta\theta$), frequency (Δp), timing (Δt). Since the carrier dynamics are important in describing the noise in semiconductor mode-locked lasers, a fifth equation for carrier population (ΔN) which is related to the fluctuations of gain and group refractive index[13] is added. Putting the projections of the injected signal to the motion equations, we get the following five equations to describe the evolution of the perturbations for pulsed injection locked passively MLLDs

$$\frac{\mathrm{d}}{\mathrm{d}T}\Delta\omega = A_{\omega\omega}\Delta\omega + A_{\omega N}\Delta N + S_{\omega} \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}T}\Delta\theta = A_{\theta\omega}\Delta\omega + A_{\theta\theta s}(\Delta\theta + \Delta\theta_s) + A_{\theta N}\Delta N + S_{\theta} \tag{10}$$

$$\frac{\mathrm{d}}{\mathrm{d}T}\Delta p = A_{pp}\Delta p + A_{pps}(\Delta p + \Delta p_s) + A_{pts}(\Delta t + \Delta t_s) + A_{pN}\Delta N + S_p \tag{11}$$

$$\frac{\mathrm{d}}{\mathrm{d}T}\Delta t = A_{t\omega}\Delta\omega + A_{tp}\Delta p + A_{tps}(\Delta p + \Delta p_s) + A_{tts}(\Delta t + \Delta t_s) + A_{tN}\Delta N + S_t \tag{12}$$

$$\frac{\mathrm{d}}{\mathrm{d}T}\Delta N = A_{N\omega}\Delta\omega + A_{NN}\Delta N + S_N \tag{13}$$

with $A_{\omega\omega} = \frac{-2g}{T_{\mathrm{R}}} - \frac{2}{3\Omega_{\mathrm{f}}^2\tau^2 T_{\mathrm{R}}} + \frac{\omega_{0Y}}{\tau T_{\mathrm{R}}}$, $A_{\omega N} = \sigma N_{p0} h v$, $A_{\theta\omega} = \frac{-\delta}{2\tau T_{\mathrm{R}}}$, $A_{\theta\theta s} = k\cos(\Delta\theta_0)$, $A_{\theta N} = \frac{-\alpha\sigma}{2}$, $A_{pp} = \frac{-4}{3\Omega_{\mathrm{f}}^2\tau^2 T_{\mathrm{R}}}$, $A_{pps} = k\cos(\Delta\theta_0)$, $A_{pts} = \frac{2k}{3\tau^2}\sin(\Delta\theta_0)$, $A_{pN} = \frac{-\alpha\sigma}{2T_{\mathrm{R}}}$, $A_{t\omega} = \frac{C_g}{T_{\mathrm{R}}} - \frac{1}{\omega_0\Omega_{\mathrm{f}} T_{\mathrm{R}}}$, $A_{tp} = \frac{-2|D|}{T_{\mathrm{R}}}$, $A_{tps} = \frac{-k\pi^2\tau^2}{6}\sin(\Delta\theta_0)$, $A_{tts} = k\cos(\Delta\theta_0)$, $A_{tN} = \frac{-\alpha\sigma}{4\pi v}$, $A_{N\omega} = \frac{-N_0\sigma}{hv}$, $A_{NN} = -\left(\sigma N_{p0} + \frac{1}{\tau_N}\right)$.

In which ω_0 is the average pulse energy, $N_{p0} = \omega_0/hv$ is the steady-state photon population, N_0 is the steady-state carrier population, σ is the carrier-photon coupling coefficient, τ_N is the carrier lifetime, α is referred to the linewidth enhancement factor, C_g is energy-timing coupling coefficient. $S_i(i=\omega,\theta,p,t,N)$ are the internal noise sources of the passively MLLD, and can be given by[7, 13]

$$\langle S_i(T)S_i(T')\rangle = D_i\delta(T-T') \quad i=\omega,\theta,p,t,N \tag{14}$$

where D_i is diffusion constants.

2 Calculation and discussion

Weanalyze the above model given by Eq.(9)～(13) with all the parameters presented in Table 1[7].

Table 1 Parameter list

Parameter	Symbol	Value
Pulse width	τ	1.3 ps
Carrier lifetime	τ_N	2 ns
Optical filter bandwidth	Ω_{f}	1.5×10^{13} rad/s
Carrier frequency	$v = p_0/(2\pi)$	2×10^{14} Hz
Carrier photon coupling coefficient	σ	$4.22\times10^4\ \mathrm{s^{-1}}$
Net group delay dispersion	D	$1\times10^{-26}\ \mathrm{s^2}$
Energy-timing coupling coefficient	C_g	8×10^3
Gain per pass	g	11 $\mathrm{m^{-1}}$
Round-trip time	T_{R}	22 ps
Carrier population	N_0	3×10^8
Photon population	$N_{p0} = \omega_0/(hv)$	4.5×10^5
Energy diffusion coeffciment	D_ω	$9.48\times10^{-20}\ \mathrm{J^2/m/s}$
Frequency diffusion coeffciment	D_p	$5.26\times10^{30}\ \mathrm{s^{-3}/m}$
Timing diffusion coeffciment	D_t	1.18×10^{-17} s/m
Carrier diffusion coeffciment	D_N	$3\times10^{17}\ \mathrm{s^{-1}}$

The noise spectra is obtained by takingthe Fourier transform of above Eqs (9) ～ (13) with $\frac{\mathrm{d}}{\mathrm{d}T}\rightarrow \mathrm{j}\Omega$. The timing noise spectra density for pulsed injection locking of passively MLLD is

$$\langle|\Delta t(\Omega)|^2\rangle = \frac{D_t[\Omega^2 + (A_{pp}+A_{pps})^2] + D_p(A_{tps}+A_{tp})^2 + \frac{D_N\langle|\mathrm{DET2}(\Omega)|^2\rangle + D_\omega\langle|\mathrm{DET3}(\Omega)|^2\rangle}{\langle|\mathrm{DET1}(\Omega)|^2\rangle}}{\langle|\mathrm{DET}(\Omega)|^2\rangle} \tag{15}$$

where

$\mathrm{DET}(\Omega)=(\mathrm{j}\Omega-A_{tts})(\mathrm{j}\Omega-A_{pp}-A_{pps})-(A_{tps}+A_{tp})A_{pts}$, $\mathrm{DET1}(\Omega)=(\mathrm{j}\Omega-A_{NN})(\mathrm{j}\Omega-A_{\omega\omega})-A_{\omega N}$, $\mathrm{DET2}(\Omega)=-A_{tN}(\mathrm{j}\Omega-A_{\omega\omega})(\mathrm{j}\Omega-A_{pp}-A_{pps})-A_{\omega N}A_{t\omega}(\mathrm{j}\Omega-A_{pp}-A_{pps})-(A_{tps}+A_{tp})A_{pN}(\mathrm{j}\Omega-A_{\omega\omega})$, $\mathrm{DET3}(\Omega)=A_{t\omega}(\mathrm{j}\Omega-A_{NN})(\mathrm{j}\Omega-A_{pp}-A_{pps})+A_{N\omega}A_{tN}(\mathrm{j}\Omega-A_{pp}-A_{pps})+(A_{tps}+A_{tp})A_{pN}A_{N\omega}$.

Eq.(15) shows that the timing jitter spectrum has a complex expression which includes four terms: timing noise contribution (D_t), frequency noise contribution (D_p), energy noise contribution (D_ω), and carrier dynamics contribution (D_N).

2.1 Noise spectrum without injection

To better understand the effect of pulsed injection, we first illustrate the noise spectrum without injection. It′s a free-running passively mode locked laser when the coupling coefficient k is set to zero. To discuss the impact of each contribution term, the noise spectrum of the timing jitter for a fundamentally passive mode locked semiconductor laser is plotted in Fig.1. The vertical axis shows $10\log|\Delta t(\Omega)|^2$ in dB, rather than directly as $|\Delta t(\Omega)|^2$ in s^2. At low frequencies (under 30 MHz), it′s obvious that the frequency noise source makes the dominant contribution to the total timing jitter noise spectrum density through strong dispersion , and the second major influence comes from the carrier dynamics term. Compared to the above two noise sources, the timing fluctuation affects only a small part of timing jitter. In general, the gain dynamics contribution is therefore insignificant in this case. At high frequencies (above 30 MHz), the frequency noise contribution decreases due to the limit of optical filter bandwidth Ω_f. Because energy fluctuation has minimal impact on timing jitter, we ignore this term in following simulation.

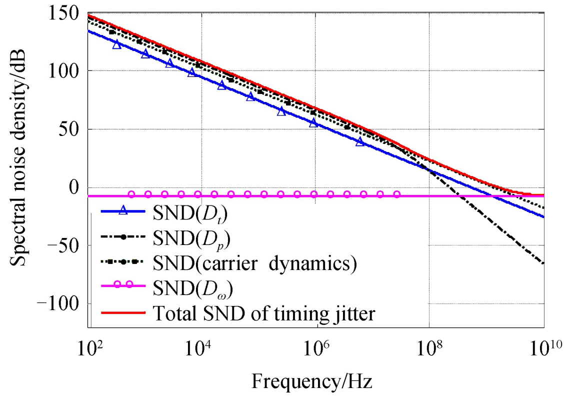

Fig.1 Noise spectral density of the timing jitter for a free running passively MLLD

2.2 Stability analysis under pulsed injection

The noise of an injection locked passively MLLD is usually attributed to two uncorrelated sources, the internal noise and the noise accompanying with the injected signal. But the stability of the injection locked passively MLLD only depends on the internal parameters. Setting Eqs. (9) ~(13) to zero, the steady-state solution is obtained as

$$A_{\theta\theta s}(A_{NN}A_{\omega\omega}-A_{\omega N}A_{N\omega})[A_{tts}(A_{pp}+A_{pps})-A_{pts}(A_{tp}+A_{tps})]>0 \tag{16}$$

The stability diagram as a function of the coupling constant k and steady-state phase difference $\Delta\theta_0$ is shown in Fig.2. The stable locking regime (gray area) is asymmetry and the locking range shrinks with the increasing of the coupling constant. The asymmetry is attributed to that the timing parameter decouples from the frequency variations when $\Delta\theta_0$ is an integer multiple of 2π, while the frequency parameter may affect the frequency term through the dispersion even lacking of injection. This simulation result is consistent with our assumption that low injected power is needed to maintain the steady-state condition.

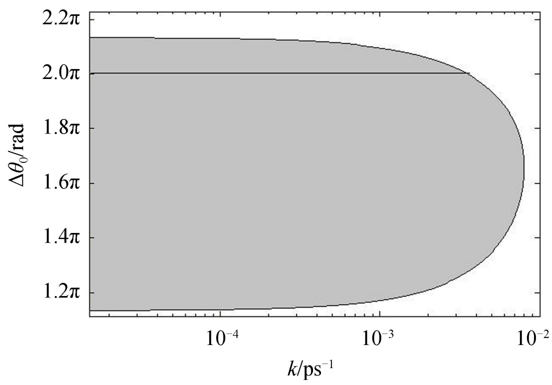

Fig.2 Stability diagram for an injection locked passively MLLD

2.3 Noise analysis

Assuming that the injected signal has low noise content, the dominant noise of the injection locked passively MLLD is depending on the internal parameters itself within the stable locking regime. According to Fig.2, the stable range relies on the coupling coefficient k and steady state phase difference $\Delta\theta_0$, thus we emphasized on analysing the influences of these two factors.

As shown in Fig.3, we show the spectral noise density of the timing jitter, when the steady state phase difference $\Delta\theta_0$ operates at different radians, while the linewidth enhancement factor is 5 and the injection coefficient is 10^{-3}. The timing jitter noise spectral density has a different response shape under different phase, while the value difference at the identical frequency is normally less than 10 dB which is much smaller compared to the huge difference of 70dB between the free running laser and the slave laser. The same result is obtained for different values of α and k. All of these calculation results reveal that the timing jitter spectrum is insensitive to steady-state phase difference. This result will have a great guiding significance for the experiment.

Then we make a further discussion about the effect of coupling coefficient on the timing jitter spectral noise density within the stable range. From Fig.4, we find that timing jitter decreases significantly with increasing k. At zero coupling coefficient, the free running passively MLLD shows a characteristic slope of f^{-4}, which mainly contributed by frequency noise term and has been illustrated in Fig.1. At the maximum allowable coupling coefficient 8×10^{-3} ps^{-1}, the noise density has a reduction of 90 dB at 100 kHz.

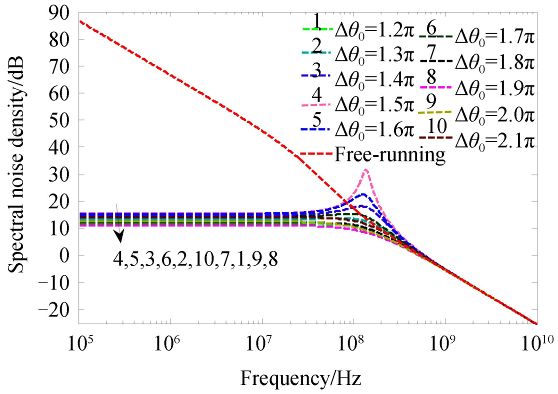

Fig.3 Spectral noise density of the timing parameter Δt for different $\Delta\theta_0$ and $\alpha=5$, $k=10^{-3}$ ps^{-1}

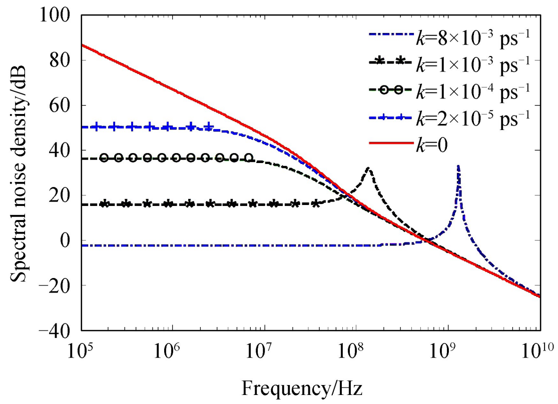

Fig.4 The effect of injecting coupling coefficient k on the noise spectral density of timing jitter for an injection locking passively MLLDs with $\Delta\theta_0=1.5\pi$ and $\alpha=5$

During the simulation, we also find that three contribution terms have the different weight on the timing jitter spectrum under different $\Delta\theta_0$ and k, which is distinct from the case without injection (Fig.1). In some cases, the spectrum mainly depends on the frequency noise sources, while in other cases, timing noise sources play a leading role. Compared to the other two ASE noise, the carrier population fluctuation which is driven by shot noise doesn't dominate the timing jitter spectrum in all cases.

To evaluate the mean-squared timing jitter from the noise spectrum, the pulse-to-clock rms timing fluctuations in the frequency range from f_{low} to f_{high} can be easily obtained by using the relationship [14]

$$\sigma_{t,pc}=\left[\frac{1}{\pi}\int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}}\langle|\Delta t(\Omega)|^2\rangle\mathrm{d}\Omega\right]^{1/2} \tag{17}$$

The value of timing jitter in the frequency range 100 kHz～10 GHz under different $\Delta\theta_0$ and k is listed in Table 2. For free running passively mode locked laser, the pulse-to-clock rms timing jitter is 3.83 ps, which reduce to a few fs or hundreds of fs magnitude under injection. The minimum value of timing jitter we calculated within the stable range is 13.9 fs with coupling coefficient equaling to 4.7×10^{-3} ps^{-1} and phase difference operating at 1.96π. We show that the timing jitter is insensitive to steady state phase difference as given in Table 2. However, the timing jitter decreases rapidly with increasing coupling coefficient. These are consistent with the analysis of spectral noise density given in previous sections.

Table 2 The pulse-to-clock rms timing jitter in the frequency of interest from 100 kHz to 10 GHz

$\sigma_{t,pc}$/fs; $\Delta\theta_0$/radian; k/ps^{-1}	1.2π	1.4π	1.6π	1.8π	2π	2.1π
0	3 830	3 830	3 830	3 830	3 830	3 830
$2*10^{-5}$	1 150	521	446	513	898	2 510
$1*10^{-4}$	627	187	148	189	609	1 510
$1*10^{-3}$	34.2	59.7	77.7	35.3	33.9	40.6

2.4 Influence of linewidth enhancement factor on timing jitter

The analysis given above is under the condition that the linewidth enhancement factor α equals to 5. Practically, the linewidth enhancement factor α has a strong effect on the timing jitter through the carrier population contribution terms which are related to the fluctuations of gain and group refractive index and hence the changes of the pulse timing. It′s necessary to further explore the influence of α. Thus we compare the effect of linewidth enhancement factor in Fig. 5. It reveals that smaller linewidth enhancement factor leads to lower timing jitter spectral noise density. In Quantum Dot (QD) semiconductors, α is small and approximately equals to zero[15], while in other quantum materials, α is typically between 2 and 6[16]. Therefore, QD lasers generally have a better performance owing to their low linewidth enhancement factor.

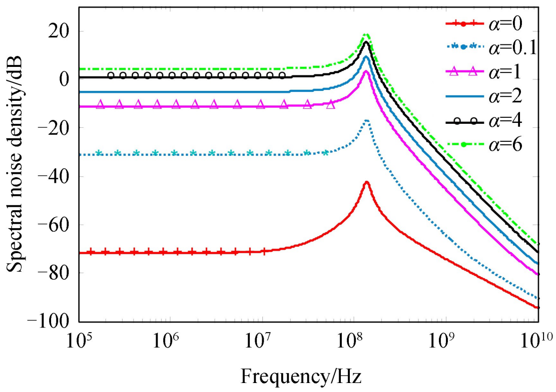

Fig.5 The effect of α on the spectral density of the timing jitter for an injected locking passively MLLD with $\Delta\theta_0=1.5\pi$ and $k=10^{-3}$ ps^{-1}

3 Conclusion

Basing on a simplified soliton perturbation equation model, the noise properties of injection-locked passively MLLD have been investigated taking into account the influences of the carrier population fluctuations. The stability condition is analyzed for an injection locking passively MLLD which is strongly dependent on the value of the steady state phase difference and coupling coefficient. Calculations show that timing jitter is insensitive to the phase difference while the coupling coefficient has a strong effect on the timing jitter. Compare to the free-running passively MLLD, calculation results point out that the noise spectral density of the timing jitter decreases 90 dB under pulsed injection at frequencies lower than 100 kHz. It is demonstrated that the pulse-to-clock root-mean-squared timing fluctuations in the frequency range from 100 kHz to 10 GHz can be reduced from 3.83 ps to a few fs, when the passively mode locked laser is subject to injection locking. We also find that the timing jitter decreases with decreasing linewidth enhancement factor, implying QD lasers could have performance advantages.

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引用格式：XU Yu-lan, LIN Zhong-xi, LIN Qi, *et al*. Timing Jitter in Pulsed Injection Locking of a Passively Mode-locked Semiconductor Laser[J]. *Acta Photonica Sinica*, 2018, **47**(9): 0914005

徐玉兰，林中晞，林琦，等.注入锁定半导体锁模激光器的时序抖动特性[J].光子学报，2018，**47**(9)：0914005