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三能级单量子态控制双向量子隐形传态

廖聪,张炜,柳华勃,王思萌
(西安邮电大学 理学院,西安 710072)

摘 要:采用张量表示和广义三维贝尔基测量的方法,提出了实现三能级单量子态控制双向量子隐形传态的协议.协议中,控制者 Carol 的量子态为任意广义三维贝尔基.选择六粒子纠缠态作为量子通道,并给出了判断任意六粒子纠缠态能否作为量子通道的必要条件.基于该条件,借助 SO(3)群元素的幺正性,选择其任意两个元素作为幺正矩阵,给出了构建量子通道的一般方法.列举了两个具体构建量子通道的例子,其中 Alice、Bob、Carol 共同作用,进行相应的广义三维贝尔基测量和对应的幺正变换,最终实现了 Alice 和 Bob 之间量子态的交换,从而验证了所提协议的可行性.

关键词:量子光学;控制双向量子隐传;广义三维贝尔基测量;张量表示;三能级单量子态;通道参数矩阵;量子通信;幺正矩阵

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Bidirectional Quantum Teleportation Controlled by Single-qutrit State

LIAO Cong, ZHANG Wei, LIU Hua-bo, WANG Si-meng
(*School of Science, Xi'an University of Posts and Telecommunications, Xi'an* 710121, *China*)

Abstract: In order to realize the single-qutrit state controlled bidirectional quantum teleportation, a protocol was proposed by using the methods of tensor representation and generalized three-dimensional Bell basis measurement. In this protocol, the quantum state of the controller Carol is arbitrary generalized three-dimensional Bell basis. The six-qutrit entangled state was chosen as the initial quantum channel, and the necessary conditions to judge whether any six-qutrit entangled state could be used as quantum channel were given. Based on the necessary conditions, with the aid of the unitary property of the SO(3) group element, a general method for constructing quantum channels was given by selecting its arbitrary two elements as the unitary matrices. Two specific examples were given, in which Alice, Bob and Carol worked together by performing generalized three-demensional Bell basis measurement and corresponding unitary transformation. At last, the exchange of quantum state between Alice and Bob was achieved, which verified the feasibility of the proposed protocol.

Key words: Quantum optics; Controlled bidirectional quantum teleportation; Generalized three-dimensional Bell basis measurement; Tensor representation; Single-qutrit state; Channel parameter matrix; Quantum communication; Unitary matrix

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0 Introduction

Quantum Teleportation (QT) is an important branch of quantum communication, which is an

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First author: LIAO Cong (1990—), male, M. S. degree candidate, mainly focuses on quantum communication. Email: congliao0202@163.com

Supervisor(Contact author): ZHANG Wei (1974—), male, associate professor, M. S. degree, mainly focuses on condensed state calculation. Email: zhangweio@xiyou.edu.cn

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important field in the research of quantum theory application. QT has drawn many researchers′ attention since it was first proposed by Bennett in 1993[1]. Based on QT, Karlsson et al. proposed the first protocol for Controlled Quantum Teleportation (CQT) in 1998[2]. Since then, many people have been working on the research of CQT.

In 2013, Zha et al. proposed a theoretical scheme for Bidirectional Quantum Controlled Teleportation (BQCT) using the entanglement property of five-qubit cluster state[3]. In Zha′s protocol, two legal users, Alice and Bob, can complete the mutual exchange of quantum information under the cooperation of Charlie. Until now, many BQCT protocols[4-5] have been proposed. In 2014, Fu et al. proposed Bidirectional Quantum Teleportation (BQT) protocol[6] by using four-qubit entangled state as the quantum channel. In Fu′s protocol, two users can simultaneously transmit an unknown quantum state to each other by using the Hadamard operation, Bell basis measurement and unitary transformation. In 2015, Hassanpour et al. proposed another BQCT protocol[7] with the use of Einstein-Podolsky-Rosen states and entanglement swapping. In the next year, Hassanpour et al. also presented the BQT protocol[8] of a pure EPR state by using Greenberger-Horne-Zeilinger states, which is different from the previous one that is teleportation of an unknown quantum state in BQT. It is worth noting that most researchers of quantum communication protocols focus on the research in two-level quantum system, and few of them consider the scenario in a high-level quantum system. But, it is certain that the research done by the predecessors on the QT has a profound inspiration for the future researchers. In fact, the d-level ($d \geqslant 2$) quantum system has drawn many people′s attention[9-10]. In 2001, Zhou et al. proposed a protocol for teleporting an unknown one-particle state of d-level quantum pure states by two-level Einstein-Podolsky-Rosen states[11]. The significance of their teleportation protocol lies not only on the protocol itself, but also on the possibility of further research and application of teleportation. After that Yan et al. have proposed a scheme on probabilistic teleportation of one-particle state of d-level in 2003[12]. Compared with Zhou′s scheme, their scheme is more perfect. In the field of experimental research, Anton Zeilinger and his team have confirmed the correctness of Bennett′s quantum teleportation scheme[13]. Furthermore, Pan et al. have obtained a great achievement in realizing teleportation of multiple degrees of freedom[14], which has special significance for understanding the potential of teleportation.

Compared with two-level quantum system, three-level quantum system has more merits such as increased security in a range of quantum information protocols[15-16], greater capacity of channel[17] for quantum communication. Therefore, the research for the teleportation of three-level quantum system will have significant contribution to quantum communication and quantum information processing area. Other related research of the three-level quantum teleportation can be found in Refs. [18-20]. In the computation and representation of quantum teleportation, Tian et al. proposed tensor representation technology[21-22], which makes the description and calculation of quantum states more concise and convenient.

In this paper, a protocol of single-qutrit state Controlled Bidirectional Quantum Teleportation (CBQT) is proposed by using six-qutrit entangled state as quantum channel based on tensor representation and Generalized Three-dimensional Bell basis Measurement (GTBM). A necessary condition is introduced for determining the availability to be used as a quantum channel for an arbitrary six-qutrit entangled state. The general method is introduced to build quantum channels and two specific examples are used for verifying the rationality of the proposed protocol. The complexity of the quantum channel is determined by the selection of Channel Parameter Matrix (CPM).

1 Necessary conditions for single-qutrit state CBQT using the six-qutrit channel

Suppose Alice wants to teleport an unknown qutrit state $|\varphi\rangle_{\mathrm{a}}$ to Bob, while Bob also wants to teleport an unknown qutrit state $|\varphi\rangle_{\mathrm{b}}$ to Alice. With the aid of tensor representation, the two states can be expressed as

$$|\varphi\rangle_{\mathrm{a}}=x_i|i\rangle,|\varphi\rangle_{\mathrm{b}}=y_j|j\rangle,i,j\in 0,1,2 \tag{1}$$

where the unknown coefficients satisfy the normalization condition $x_i x_i^*=1$ and $y_j y_j^*=1$.

To implement the single-qutrit state CBQT, we take a general six-qutrit entangled state as the initial quantum channel, which can be written as

$$|\varphi\rangle = R_{kltugh}\,|kltugh\rangle \quad k,l,t,u,g,h\in 0,1,2 \tag{2}$$

where $R_{kltugh}R^{*}_{kltugh}=1$. Here, the particles A_1 and A_2 belong to Alice, B_1 and B_2 belong to Bob, C_1 and C_2 belong to Carol, where the states of A_1, A_2, B_1, B_2, C_1 and C_2 are denoted by the indices k, h, l, g, t and u, respectively. The quantum state of C_1C_2 consists of the generalized three-dimensional Bell basis.

Generalized three-dimensional Bell basis can be regarded as

$$\varphi^{\alpha}_{tu} = (\mathrm{e}^{2t\pi ir/3}\,|r,\mathrm{mod}(r+u,3)\rangle)/\sqrt{3} \tag{3}$$

where $r=0,1,2$ and $\alpha=1,2,\cdots\cdots,9$. Let $\boldsymbol{T}$ denote a transformation matrix between computation basis $|tu\rangle$ and generalized three-dimensional Bell basis φ^{α}_{tu}, that is

$$\boldsymbol{T}=\begin{bmatrix}
T^1_{00} & T^1_{10} & T^1_{20} & T^1_{01} & T^1_{11} & T^1_{21} & T^1_{02} & T^1_{12} & T^1_{22}\\
T^2_{00} & T^2_{10} & T^2_{20} & T^2_{01} & T^2_{11} & T^2_{21} & T^2_{02} & T^2_{12} & T^2_{22}\\
T^3_{00} & T^3_{10} & T^3_{20} & T^3_{01} & T^3_{11} & T^3_{21} & T^3_{02} & T^3_{12} & T^3_{22}\\
T^4_{00} & T^4_{10} & T^4_{20} & T^4_{01} & T^4_{11} & T^4_{21} & T^4_{02} & T^4_{12} & T^4_{22}\\
T^5_{00} & T^5_{10} & T^5_{20} & T^5_{01} & T^5_{11} & T^5_{21} & T^5_{02} & T^5_{12} & T^5_{22}\\
T^6_{00} & T^6_{10} & T^6_{20} & T^6_{01} & T^6_{11} & T^6_{21} & T^6_{02} & T^6_{12} & T^6_{22}\\
T^7_{00} & T^7_{10} & T^7_{20} & T^7_{01} & T^7_{11} & T^7_{21} & T^7_{02} & T^7_{12} & T^7_{22}\\
T^8_{00} & T^8_{10} & T^8_{20} & T^8_{01} & T^8_{11} & T^8_{21} & T^8_{02} & T^8_{12} & T^8_{22}\\
T^9_{00} & T^9_{10} & T^9_{20} & T^9_{01} & T^9_{11} & T^9_{21} & T^9_{02} & T^9_{12} & T^9_{22}
\end{bmatrix} \tag{4}$$

The total state of the system has the following form as

$$|\varphi\rangle_{\mathrm{T}} = |\varphi\rangle_{\mathrm{a}}\otimes|\varphi\rangle_{\mathrm{b}}\otimes|\varphi\rangle = x_i y_j R_{kltugh}\,|i\rangle\,|j\rangle\,|kltugh\rangle \tag{5}$$

and the entangled quantum channel $|\varphi\rangle$ can be illustrated in Fig. 1. Curve① and curve② denote the GTBM and its direction, respectively, while curve③ and curve④ curve denote the quantum channel and the direction of the collapsed state after the GTBM, respectively.

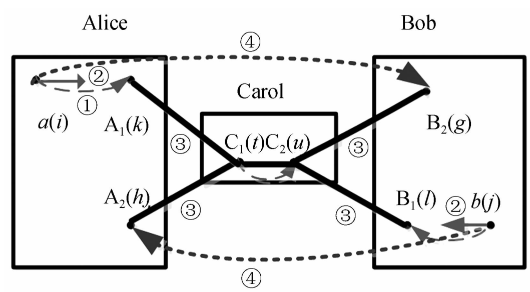

Fig. 1 Initial entangled quantum channel of the six qutrit in the single-qutrit state CBQT

As the first step of the single-qutrit CBQT, Carol decomposes C_1C_2 in Bell basis φ^{α}_{tu} via the transformation matrix $\boldsymbol{T}$, which determines the transformation between the computation basis $|tu\rangle$ and the Bell basis φ^{α}_{tu}, then the quantum state of channel can be regarded as $|\varphi\rangle = R_{kltugh}T^{\alpha}_{tu}\,|kl\alpha gh\rangle$.

Secondly, Carol performs the GTBM on his particles C_1C_2, namely, $\langle\varphi^{\alpha}_{tu}|\varphi\rangle = R_{klgh}\sigma^{\alpha}_{tu}\,|klgh\rangle$.

Finally, Carol performs an unitary transformation on (C_1, C_2), with the transmission $|\varphi'\rangle = (\sigma^{\alpha}_{tu})^{-1}_{C_1C_2}\langle\varphi^{\alpha}_{tu}|\varphi\rangle = R_{klgh}\,|klgh\rangle$.

So, the total state of system can be written as $|\boldsymbol{\Psi}\rangle_{\mathrm{T}} = |\varphi\rangle_a\otimes|\varphi\rangle_b\otimes|\varphi'\rangle = x_i y_j R_{klgh}\,|i\rangle\,|j\rangle\,|klgh\rangle$, and the transformed entangled quantum channel can be illustrated in Fig. 2.

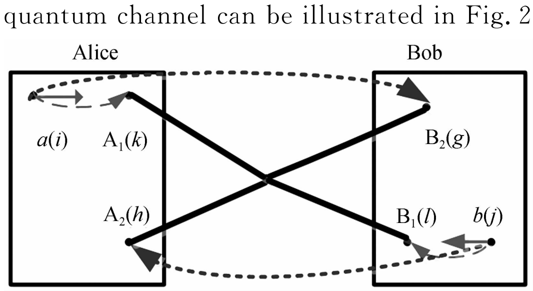

Fig. 2 Transformed entangled quantum channel in the single-qutrit state CBQT

The matrix $\boldsymbol{R}$ is CPM as

$$\boldsymbol{R}=\begin{bmatrix} R_{0000} & R_{0100} & R_{0200} & R_{1000} & R_{1100} & R_{1200} & R_{2000} & R_{2100} & R_{2200} \\ R_{0001} & R_{0101} & R_{0201} & R_{1001} & R_{1101} & R_{1201} & R_{2001} & R_{2101} & R_{2201} \\ R_{0002} & R_{0102} & R_{0202} & R_{1002} & R_{1102} & R_{1202} & R_{2002} & R_{2102} & R_{2202} \\ R_{0010} & R_{0110} & R_{0210} & R_{1010} & R_{1110} & R_{1210} & R_{2010} & R_{2110} & R_{2210} \\ R_{0011} & R_{0111} & R_{0211} & R_{1011} & R_{1111} & R_{1211} & R_{2011} & R_{2111} & R_{2211} \\ R_{0012} & R_{0112} & R_{0212} & R_{1012} & R_{1112} & R_{1212} & R_{2012} & R_{2112} & R_{2212} \\ R_{0020} & R_{0120} & R_{0220} & R_{1020} & R_{1120} & R_{1220} & R_{2020} & R_{2120} & R_{2220} \\ R_{0021} & R_{0121} & R_{0221} & R_{1021} & R_{1121} & R_{1221} & R_{2021} & R_{2121} & R_{2221} \\ R_{0022} & R_{0122} & R_{0222} & R_{1022} & R_{1122} & R_{1222} & R_{2022} & R_{2122} & R_{2222} \end{bmatrix} \tag{6}$$

$$\boldsymbol{T}^{\beta}=\begin{bmatrix} T^{\beta}_{00} & T^{\beta}_{10} & T^{\beta}_{20} \\ T^{\beta}_{01} & T^{\beta}_{11} & T^{\beta}_{21} \\ T^{\beta}_{02} & T^{\beta}_{12} & T^{\beta}_{22} \end{bmatrix} \boldsymbol{T}^{\gamma}=\begin{bmatrix} T^{\gamma}_{00} & T^{\gamma}_{10} & T^{\gamma}_{20} \\ T^{\gamma}_{01} & T^{\gamma}_{11} & T^{\gamma}_{21} \\ T^{\gamma}_{02} & T^{\gamma}_{12} & T^{\gamma}_{22} \end{bmatrix} \tag{7}$$

where $\boldsymbol{T}^{\beta}$ and $\boldsymbol{T}^{\gamma}$ are measurement matrices, $\sigma^{\beta\gamma}_{ikjl}$ is an element of the transmission matrix $\boldsymbol{\sigma}^{\beta\gamma}$ and $\boldsymbol{\sigma}^{\beta\gamma}_{ikjl}=\boldsymbol{R}_{klgh}\boldsymbol{T}^{\beta}_{ik}\boldsymbol{T}^{\gamma}_{jl}$.

In order to realize their quantum state interchanged, the CPM $\boldsymbol{R}$ must be reduced into a direct product form of two unitary matrices $\boldsymbol{R}^1$ and $\boldsymbol{R}^2$, namely, $\boldsymbol{R}=\boldsymbol{R}^1\otimes\boldsymbol{R}^2$, that is to say

$$|\varphi'\rangle=|\varphi\rangle_{\mathrm{A_1B_2}}\otimes|\varphi\rangle_{\mathrm{B_1A_2}}=R^1_{kg}R^2_{lh}|kg\rangle_{\mathrm{A_1B_2}}|lh\rangle_{\mathrm{B_1A_2}} \tag{8}$$

The total state also can be writen as

$$|\boldsymbol{\Psi}\rangle_{\mathrm{T}}=x_iy_jR^1_{kg}R^2_{lh}|i\rangle|j\rangle|kg\rangle|lh\rangle \tag{9}$$

with $R_{klgh}=R^1_{kg}R^2_{lh}$. Here, the CPM $\boldsymbol{R}$ is 9×9 complex matrix, $\boldsymbol{R}^1$ and $\boldsymbol{R}^2$ are 3×3 complex matrices.

Alice and Bob decompose $|\boldsymbol{\Psi}\rangle_{\mathrm{T}}$ with the generalized three-dimensional Bell basis

$$|\boldsymbol{\Psi}\rangle_{\mathrm{T}}=x_iy_jR^1_{kg}R^2_{lh}T^{\beta}_{ik}T^{\gamma}_{jl}|\beta\rangle|\gamma\rangle|g\rangle|h\rangle=x_iy_j\sigma_i^{(\beta)g}\sigma_j^{(\gamma)h}|\beta\gamma gh\rangle \tag{10}$$

After Alice and Bob finishing the GTBM on their particles $(a,\mathrm{A_1})$ and $(b,\mathrm{B_1})$, respectively, the remanent state of the system has the next form as

$$|\Phi\rangle_{\mathrm{T}}=\langle\varphi^{\beta}_{ik}|\varphi^{\gamma}_{jl}|\Psi\rangle_{\mathrm{T}}=x_iy_j\sigma^{\beta}\sigma^{\gamma}|gh\rangle_{\mathrm{B_2A_2}} \tag{11}$$

with $\sigma^{\beta\gamma}=(\boldsymbol{R}^1\otimes\boldsymbol{R}^2)(\boldsymbol{T}^{\beta}\otimes\boldsymbol{T}^{\gamma})=(\boldsymbol{R}^1\boldsymbol{T}^{\beta})\otimes(\boldsymbol{R}^2\boldsymbol{T}^{\gamma})=\sigma^{\beta}\otimes\sigma^{\gamma}$.

Alice and Bob individually perform an unitary transformation on particles $\mathrm{A_2}$ and $\mathrm{B_2}$ to swap their quantum state, and the unitary transformation can be represented as follow

$$(\sigma^{\gamma})^{-1}_{\mathrm{A_2}}(\sigma^{\beta})^{-1}_{\mathrm{B_2}}|\boldsymbol{\Phi}\rangle_{\mathrm{T}}=x_g|g\rangle_{\mathrm{B_2}}y_h|h\rangle_{\mathrm{A_2}} \tag{12}$$

hence we can easily find that the single-qutrit state CBQT is successfully achieved.

Based on the result of the above analysis, one can determine the necessary condition which can be used to judge whether a six-qutrit state can be regarded as quantum channel for the single-qutrit state CBQT: 1) The initial quantum channel is a single channel which is composed of a genuine quantum entangled state; 2) Carol must perform a GTBM on his particles $(\mathrm{C_1}, \mathrm{C_2})$ and then performs an unitary transformation; 3) The transformed quantum CPM can be written as a direct product of two unitary matrices. If we want to realize a probabilistic CBQT, the CPM needs to be reduced into direct product form of two invertible matrices.

2 General method of the constructing channel for CBQT

As is known to all, the nature representation element of SO(3) group is all unitary, so it will be a suit candidate of CPM $\boldsymbol{R}$ for perfect teleportation. In order to realize perfect teleportation in qutrit system, for instance, we can select universal rotational matrix as the matrix $\boldsymbol{R}^1$ and $\boldsymbol{R}^2$. We can define $\boldsymbol{R}^1$ as

$$\boldsymbol{R}^1=\begin{bmatrix} \mathrm{e}^{-\mathrm{i}(\alpha_1+\gamma_1)}\cos^2(\beta_1/2) & -\sqrt{2}\,\mathrm{e}^{-\mathrm{i}\alpha_1}\cos(\beta_1/2)\sin(\beta_1/2) & \mathrm{e}^{-\mathrm{i}(\alpha_1-\gamma_1)}\sin^2(\beta_1/2) \\ \sqrt{2}\,\mathrm{e}^{-\mathrm{i}\gamma_1}\cos(\beta_1/2)\sin(\beta_1/2) & \cos^2(\beta_1/2)\sin^2(\beta_1/2) & -\sqrt{2}\,\mathrm{e}^{\mathrm{i}\gamma_1}\cos(\beta_1/2)\sin(\beta_1/2) \\ \mathrm{e}^{\mathrm{i}(\alpha_1-\gamma_1)}\sin^2(\beta_1/2) & \sqrt{2}\,\mathrm{e}^{\mathrm{i}\alpha_1}\cos(\beta_1/2)\sin(\beta_1/2) & \mathrm{e}^{\mathrm{i}(\alpha_1+\gamma_1)}\cos^2(\beta_1/2) \end{bmatrix} \tag{13}$$

where α_1, β_1, γ_1 are Euler angle, $0\leqslant\alpha_1\leqslant 2\pi$, $0\leqslant\beta_1\leqslant\pi$, $0\leqslant\gamma_1\leqslant 2\pi$. Similarly, we can define $\boldsymbol{R}^2$ as

$$\boldsymbol{R}^2=\begin{bmatrix} \mathrm{e}^{-\mathrm{i}(\alpha_2+\gamma_2)}\cos^2(\beta_2/2) & -\sqrt{2}\,\mathrm{e}^{-\mathrm{i}\alpha_2}\cos(\beta_2/2)\sin(\beta_2/2) & \mathrm{e}^{-\mathrm{i}(\alpha_2-\gamma_2)}\sin^2(\beta_2/2) \\ \sqrt{2}\,\mathrm{e}^{-\mathrm{i}\gamma_2}\cos(\beta_2/2)\sin(\beta_2/2) & \cos^2(\beta_2/2)\sin^2(\beta_2/2) & -\sqrt{2}\,\mathrm{e}^{\mathrm{i}\gamma_2}\cos(\beta_2/2)\sin(\beta_2/2) \\ \mathrm{e}^{\mathrm{i}(\alpha_2-\gamma_2)}\sin^2(\beta_2/2) & \sqrt{2}\,\mathrm{e}^{\mathrm{i}\alpha_2}\cos\left(\dfrac{\beta_2}{2}\right)\sin(\beta_2/2) & \mathrm{e}^{\mathrm{i}(\alpha_2+\gamma_2)}\cos^2\left(\dfrac{\beta_2}{2}\right) \end{bmatrix} \tag{14}$$

where α_2, β_2, γ_2 are Euler angle, $0\leqslant\alpha_2\leqslant2\pi$, $0\leqslant\beta_2\leqslant\pi$, $0\leqslant\gamma_2\leqslant2\pi$.

The general quantum channel $|\varphi\rangle$ is

$$|\varphi\rangle=|\varphi\rangle_{A_1B_2}\otimes|\varphi\rangle_{B_1A_2}\otimes|\varphi\rangle_{C_1C_2}=R_{kltugh}|kltugh\rangle \tag{15}$$

where $|\varphi\rangle_{A_1B_2}=R^1_{kg}|kg\rangle$, its expansion is

$$|\varphi\rangle_{A_1B_2}=\frac{1}{\sqrt{3}}[\mathrm{e}^{-\mathrm{i}(\alpha_1+\gamma_1)}\cos^2(\beta_1/2)|00\rangle+\cdots+\mathrm{e}^{\mathrm{i}(\alpha_1+\gamma_1)}\cos^2(\beta_1/2)|22\rangle] \tag{16}$$

$|\varphi\rangle_{B_1A_2}=R^2_{lh}|lh\rangle$, its expansion is

$$|\varphi\rangle_{B_1A_2}=\frac{1}{\sqrt{3}}[\mathrm{e}^{-\mathrm{i}(\alpha_2+\gamma_2)}\cos^2(\beta_2/2)|00\rangle+\cdots+\mathrm{e}^{\mathrm{i}(\alpha_2+\gamma_2)}\cos^2(\beta_2/2)|22\rangle] \tag{17}$$

$|\varphi\rangle_{C_1C_2}=\varphi^{\alpha}_{tu}$, its quantum state is represented by arbitrary generalized three-dimensional Bell basis.

3 Specific examples

3.1 Example one

Selecting $\boldsymbol{R}^1$ and $\boldsymbol{R}^2$ as

$$\boldsymbol{R}^1=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta_1 & -\sin\theta_1\\ 0 & \sin\theta_1 & \cos\theta_1\end{bmatrix} \tag{18}$$

$$\boldsymbol{R}^2=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta_2 & -\sin\theta_2\\ 0 & \sin\theta_2 & \cos\theta_2\end{bmatrix} \tag{19}$$

where $0\leqslant\theta_1\leqslant2\pi$ and $0\leqslant\theta_2\leqslant2\pi$. It is obvious that $\boldsymbol{R}^1$ and $\boldsymbol{R}^2$ are unitary matrices. Let $\theta_1=\theta_2=\pi/4$, then $\boldsymbol{R}^1=\boldsymbol{R}^2$. Because coefficients of a state must be normalized, the channel parameter will be suitably adjusted and the CPM $\boldsymbol{R}$ can be written as $\boldsymbol{R}=\boldsymbol{R}^1\otimes\boldsymbol{R}^2/3$. The initial channel state is given by

$$\left|\varphi\rangle=\frac{1}{3}(\right|00\alpha00\rangle+\frac{\sqrt{2}}{2}|01\alpha01\rangle+\frac{\sqrt{2}}{2}|01\alpha02\rangle-\frac{\sqrt{2}}{2}\left|02\alpha01\rangle+\frac{\sqrt{2}}{2}|02\alpha02\rangle+\cdots+\frac{1}{2}|21\alpha22\rangle+\right. \\ \frac{1}{2}\left|22\alpha11\rangle-\frac{1}{2}|22\alpha12\rangle-\frac{1}{2}\right|22\alpha21\rangle+\frac{1}{2}|22\alpha22\rangle) \tag{20}$$

where α is equivalent to a generalized three-dimensional Bell basis φ^{α}_{tu} which represents the quantum state of C_1C_2. Carol performs a GTBM on his particles (C_1, C_2). If he obtains the result φ^{α}_{tu}, the channel state will be marked as

$$|\varphi'\rangle=\langle\varphi^{\alpha}_{tu}|\varphi\rangle=\frac{1}{3}(\left|0000\rangle+\frac{\sqrt{2}}{2}\right|0101\rangle+\frac{\sqrt{2}}{2}|0102\rangle-\frac{\sqrt{2}}{2}\left|0201\rangle+\frac{\sqrt{2}}{2}|0202\rangle+\cdots+\right. \\ \frac{1}{2}|2122\rangle+\frac{1}{2}\left|2211\rangle-\frac{1}{2}|2212\rangle-\frac{1}{2}\right|2221\rangle+\frac{1}{2}|2222\rangle) \tag{21}$$

The system state of the six particles is given by $|\Psi\rangle_{\mathrm{T}}=|\varphi\rangle_{\mathrm{a}}\otimes|\varphi\rangle_{\mathrm{b}}\otimes|\varphi'\rangle$. When Alice and Bob perform the GTBM on their particles (a, A_1) and (b, B_1), respectively, and obtain the results $\varphi^2_{\mathrm{a}A_1}$ and $\varphi^3_{\mathrm{b}B_1}$, the system state will collapse to the state $|\Psi^{23}\rangle_{\mathrm{T}}=\langle\varphi^3_{\mathrm{b}B_1}|\varphi^2_{\mathrm{a}A_1}|\Psi\rangle_{\mathrm{T}}$, with $\sigma^2=\boldsymbol{R}^1(\sqrt{3}\boldsymbol{T}^2)$, and $\sigma^3=\boldsymbol{R}^2(\sqrt{3}\boldsymbol{T}^3)$, $\sigma^{23}=\sigma^2\otimes\sigma^3$ being the transformation matrix.

Alice and Bob perform an appropriate unitary transformation on their particles A_2 and B_2, respectively, to swap their quantum state. The final state is $(\sigma^3)^{-1}_{A_2}(\sigma^2)^{-1}_{B_2}|\Psi^{23}\rangle_{\mathrm{T}}$, which is marked as

$$|\Phi\rangle_{swap}=\frac{1}{9}[x_0|0\rangle+x_1|1\rangle+x_2|2\rangle]_{B_2}\otimes[y_0|0\rangle+y_1|1\rangle+y_2|2\rangle]_{A_2} \tag{22}$$

therefore the single-qutrit state CBQT is successfully achieved.

For the other 80 kinds of Bell basis $\varphi^{\beta}_{\mathrm{a}A_1}\varphi^{\gamma}_{\mathrm{b}B_1}$ measurement results, Alice and Bob operate a suitable unitary transformation on their particles to interchange their quantum states.

3.2 Example two

Let $\theta_1=\theta_2=\frac{\pi}{2}$, then the CPM $\boldsymbol{R}$ can be represented as $\boldsymbol{R}=\boldsymbol{R}^1\otimes\boldsymbol{R}^2/3$. In the same way with Section 2, the initial channel state can be remarked as

$$\left|\varphi\right\rangle=\frac{1}{3}[|00\alpha00\rangle+|01\alpha02\rangle-|02\alpha01\rangle+|10\alpha20\rangle+|11\alpha22\rangle-|12\alpha21\rangle-|20\alpha10\rangle-|21\alpha12\rangle+|22\alpha11\rangle] \tag{23}$$

After Carol performs a GTBM on his particles (C_1, C_2), and obtains the result φ_{tu}^{α}, the channel state will be marked as

$$|\varphi'\rangle=\langle\varphi_{tu}^{\alpha}|\varphi\rangle=\frac{1}{3}[|0000\rangle+|0102\rangle-|0201\rangle+|1020\rangle+|1122\rangle-|1221\rangle-|2010\rangle-|2112\rangle+|2211\rangle] \tag{24}$$

The system state of six particles is $|\Psi\rangle_{\mathrm{T}}=|\varphi\rangle_{\mathrm{a}}\otimes|\varphi\rangle_{\mathrm{b}}\otimes|\varphi'\rangle$. When Alice and Bob perform a GTBM on their particles (a, A_1) and (b, B_1), respectively, and Alice's and Bob's respective measurement result are $\varphi_{\mathrm{aA}_1}^{1}$ and $\varphi_{\mathrm{bB}_1}^{1}$, the system state will collapse to the state $|\Psi^{11}\rangle_{\mathrm{T}}=\langle\varphi_{\mathrm{bB}_1}^{1}|\varphi_{\mathrm{aA}_1}^{1}|\Psi\rangle_{\mathrm{T}}$, with $\sigma^1=\boldsymbol{R}^1(\sqrt{3}T^1)$, $\sigma^{11}=\sigma^1\otimes\sigma^1$.

Alice and Bob perform an appropriate unitary transformation on their particles A_2 and B_2, respectively, to swap their quantum state. The final state is $(\sigma^1)_{\mathrm{A}_2}^{-1}(\sigma^1)_{\mathrm{B}_2}^{-1}|\Psi^{11}\rangle_{\mathrm{T}}$, which is marked as

$$\left|\Phi\right\rangle_{swap}=\frac{1}{9}[x_0|0\rangle+x_1|1\rangle+x_2|2\rangle]_{\mathrm{B}_2}\otimes[y_0|0\rangle+y_1|1\rangle+y_2|2\rangle]_{\mathrm{A}_2} \tag{25}$$

As a consequence, the single-qutrit state CBQT is successfully achieved.

For the other 80 kinds of Bell basis $\varphi_{\mathrm{aA}_1}^{\beta}\varphi_{\mathrm{bB}_1}^{\gamma}$ measurement results, Alice and Bob operate a suitable unitary transformation on their particles to swap their quantum states.

4 Conclusion

The quantum state of controller Carol is selected as Bell basis φ_{tu}^{α}. Due to the orthogonality of Bell basis, the Bell basis φ_{tu}^{α} is chosen to measure the controller. After the initial quantum channel is measured by the Bell basis, the transformed quantum channel parameter matrix will be able to be represented as the direct product of two unitary matrices. For legal user Alice sand Bob, having performed the corresponding Bell basis measurement and unitary transformation respectively ensures their exchange of the quantum information between each other theoretically. However, the complexity of the quantum channel is still determined by the selection of R. In order to deal with the problem flexibly, we discussed two specific examples of making R.

In summary, the protocol of CBQT for the single-qutrit state is proposed by using six-qutrit entangled state as the quantum channel. Besides, a necessary condition is introduced for determining the availability to be used as a quantum channel for an arbitrary six-qutrit entangled state. In addition, based on necessary condition proposed, the general approach is introduced to build quantum channels and two specific examples are given.

In fact, the proposed protocol is an ideal CBQT, with a transmission success probability of 100%. However, assuring the success rate of probabilistic CBQT requires Rbeing a reversible matrix instead of unitary matrix. Besides, Alice and Bob will need to perform the corresponding reverse transformation as well.

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