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基于压缩光的量子激光雷达技术

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摘　要:利用Wigner函数对真空态、单光子态、压缩态在相空间的噪声分布进行仿真,并系统分析了基于压缩光的量子相干激光雷达和压缩光注入式量子激光雷达.研究表明,相比经典激光雷达,较高压缩度有利于量子相干激光雷达探测信噪比的提升,理论上8dB的压缩度可以使信噪比提高6.25倍;而压缩光注入式量子激光雷达系统的空间分辨率主要取决于真空压缩光的压缩度和无噪声相敏放大系统的增益.由于压缩光对探测信噪比的提升作用,量子激光雷达在微弱信号探测和高分辨率成像领域具有显著优势.

关键词:压缩光;量子激光雷达;相干探测;压缩真空注入;激光雷达
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Quantum Lidar Based on Squeezed Sates of Light

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Abstract: The simulations of the noise distribution of vacuum state, single photon state and squeezed state in phase space were presented by Wigner function, respectively and the technology of the quantum lidar based on the squeezed field was systematically analyzed, including quantum coherent lidar and quantum lidar with squeezed light injection. Compared to the classical lidar, a higher squeezing degree benefits improvement of the detection Signal-To-Noise Ratio (SNR) for quantum coherent lidar, and a squeezing degree of 8dB would afford a 6.25-fold improvement in SNR theoretically, while the quantum lidar with squeezed vacuum injection has higher spatial resolution, which depends on the squeezing level of the squeezed vacuum and the gain of the phase sensitive amplifier. The results show that the quantum lidar has obvious advantages in the fields of weak signal detection and high-resolution imaging as the improvements of the signal-to-noise by using squeezed light.

Key words: Squeezed light; Quantum lidar; Coherent detection; Squeezed vacuum injection; Lidar
OCIS Codes: 270.6570; 280.3640; 270.1670

0 Introduction

Compared to the microwave radar, the laser radar (lidar) has higher spatial resolution as the shorter wavelength, which is essential for target detection and recognition. In real applications the detection ability of the lidar should be improved to meet modern military requirements, especially for appearance of stealth target and stealth platform. According to the classical lidar theory, the spatial resolution of the classical lidar is limited by the Rayleigh resolution and the Signal-To-Noise Ratio (SNR)[1]. In real experiments, the only possible improvement of the spatial resolution is to alter the aperture of the receivers at certain wavelength, while progress in the development of practical receivers with large aperture has been slow because of technical difficulties. In order to break through the detection limit of the classical lidar system,

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researchers mostly focus on studies of the quantum lidar and the quantum technology is essentially expected to improve the performance of the classical lidar[2-6].

It is known that the squeezed states of light has the potential for use in optical precision measurement and gravitational wave detection, as the quantum property of being below the quantum noise limit[7,8]. In reality the quantum resources are very difficult to survive as the fact that these states are very fragile and every loss would transform the squeezed field into a vacuum field. As a result, based on the performance of the squeezed states of light, quantum lidar using receivers enhanced with quantum technology seems feasible in the near future and the classical lidar would benefit most from an improvement in SNR and image resolution by use of squeezed states of light. Also it is easier for this type of quantum lidar to become a useful tool for engineering applications[1-4].

In this paper, we show the quantum properties of the squeezed light by Wigner function and then present an analysis of squeezed-based quantum lidars systematically, including quantum coherent lidar and quantum lidar with squeezed light injection. To the best of our knowledge, there has been an increasing interest in engineering applications since the development of the quantum lidar and the systematic analysis of the quantum lidar using receivers with quantum technology would be beneficial for future applications in high SNR detection and imaging.

1 Quantum properties of the squeezed light

1.1 Squeezed light

According to relevant theory, at certain fixed frequency, the electric field of an electromagnetic wave in an isotropic, insulating medium can be described by[9]

$$E(r,t)=E_0\left[a(r)\mathrm{e}^{-\mathrm{i}\omega t}-a(r)^*\mathrm{e}^{+\mathrm{i}\omega t}\right]$$

The operator for the electric field also can be written as

$$E(r,t)=E_0\left[X_1\cos(\omega t)-X_2\sin(\omega t)\right]$$

where $X_1(r)$ and $X_2(r)$ indicate the position and momentum operators , respectively and can be expressed as

$$X_1(r)=a^*(r)+a(r)$$

$$X_2(r)=\mathrm{i}\left[a^*(r)-a(r)\right]$$

Considering the quantization of the electromagnetic field, the quadrature operators are

$$\hat{X}_1=\hat{a}^\dagger+\hat{a}$$

$$\hat{X}_2=\mathrm{i}(\hat{a}^\dagger-\hat{a})$$

where $\hat{X}_1$ and $\hat{X}_2$ correspond to amplitude quadrature operator and phase quadrature operator, respectively. Then the quadrature operator at angle θ can be defined as

$$\hat{X}_\theta=\hat{X}_1\cos\theta+\hat{X}_2\sin\theta$$

From the commutation relation of the quadrature operators

$$[\hat{X}_1,\hat{X}_2]=2\mathrm{i}$$

and taking into account the module of the commutation relation

$$|\langle[\hat{X}_1,\hat{X}_2]\rangle|=2$$

then the uncertainty relation of the quadrature operators can be described by the Heisenberg's uncertainty relation

$$\Delta\hat{X}_1\Delta\hat{X}_2\geqslant 1 \tag{1}$$

where $\Delta\hat{X}_1$ and $\Delta\hat{X}_2$ are the standard deviation of measurements of the amplitude quadrature operator and phase quadrature operator, respectively. A state of minimum uncertainty is given by the equal sign in Eq. (1), which means the noise is equally distributed among the two field quadratures. This will always be the special case for coherent state and vacuum state.

Eq. (1) puts a limit to the product of the variance of the two orthogonal quadratures, while the uncertainty in one of the quadratures is not restricted and the squeezed light is defined in the case of $\Delta\hat{X}_1<1$

or $\Delta\hat{X}_2 < 1$. Consequently, squeezed light is a special state in which the uncertainty of one quadrature can be smaller than vacuum noise and this noise feature shows essential differences between the squeezed field and the vacuum field. Fig. 1 shows an intuitive picture of the noise properties of the three light field[9-11]: the vacuum state, the coherent state and the squeezed state. Evidently, for the vacuum state and coherent state, equal noise is seen in the amplitude and phase quadrature and a circle marks the variance of this distribution. In the case of the squeezed states of light, the quantum noise of one quadrature is reduced at the expense of the other and described by a ellipse in Fig. 1.

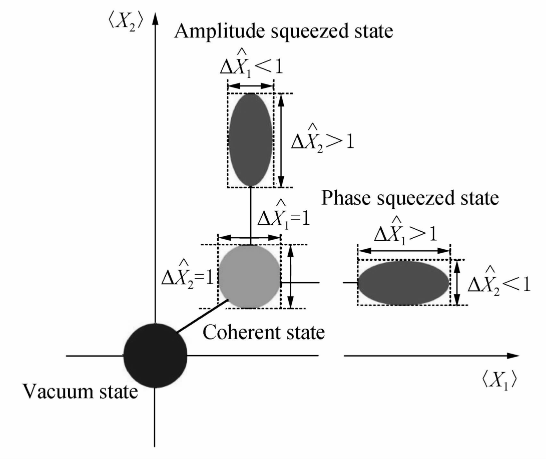


Fig. 1 Noise properties of the three light field

1.2 Phase-space picture of squeezed light

A classical electromagnetic oscillation can be represented by probability distribution function $P(q,p)$ in classical physics, which allows to determine the probability of the position q and momentum p simultaneously[10-11]. Furthermore, such a function has to be normalized and positive and a negative probability has no physical significance.

Following the quantum properties of the squeezed light described above, i. e., both non-commuting variables cannot be measured simultaneously with arbitrarily precision, which is limited by the Heisenberg′s uncertainty relation. To describe the quantum states completely, just as the probability distribution function in classical physics, a function was introduced by Wigner in 1932, which is a quasi-probability distribution function and has a powerful effect in quantum physics[12-13]. Compared with other methods, expectation values of an arbitrary operator could be calculated and it is more easier and intuitive to describe the quantum states by the Wigner function. As demonstrated in Refs. [12-13], the Wigner function can be expressed as

$$W(q,p)=\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}\mathrm{d}x\exp\left(\frac{\mathrm{i}px}{\hbar}\right)\langle q-\frac{1}{2}x|\hat{\rho}|\rangle$$

where $\hat{\rho}$ stands for an arbitrary density operator and is given by

$$\hat{\rho}=\sum_i p_i|\Psi_i\rangle\langle\Psi_i|$$

Considering Hermitian operators $\hat{\rho}$, the Wigner function is real and normalized

$$W^*(q,p)=W(q,p)$$

$$\int_{-\infty}^{\infty}\mathrm{d}p\int_{-\infty}^{\infty}\mathrm{d}qW(q,p)=1$$

It is convenient for us to describe quantum states by Wigner function in phase space. Taking the vacuum state for example, whose Wigner function is calculated as follows[10,14]

$$W_0(q,p)=\frac{1}{\pi}\exp(-q^2-p^2)$$

Similarly, the Wigner function of the coherent state can be obtained by displacing the vacuum state′s Wigner function, as any coherent state could be derived by applying the displacement operator to the vacuum state, resulting in

$$W_0(q,p)=\frac{1}{\pi}\exp(-(q-q_0)^2-(p-p_0)^2)$$

As shown in Fig. 2 (a), the quantum noise of the vacuum state follows Gaussian distribution, meaning that the variances of the two quadratures are equal. It is actually the same case for the coherent state. In comparison the Wigner function of the single photon is given by[10,14]

$$W_n(q,p)=\frac{2q^2+2p^2-1}{\pi}\exp(-q^2-p^2)$$

Fig. 2 (b) shows the quantum noise distribution of the single photon. It is obviously that the fluctuations of the two quadratures are founded to be non-Gaussian and the probabilities have negative values near the origin, representing the quantum property of the single photon. For this reason, the Wigner function is called the quasi-probability distribution function.

Note that theoretically the conjugate observables of the squeezed light have different quantum noise, considering a squeezed vacuum state, the Wigner function is[10,14-15]

$$W_s(q,p)=\frac{1}{\pi}\exp\left(-e^{2\zeta}q^2-e^{-2\zeta}p^2\right)$$

where ζ is a parameter described by the squeezed parameter r and squeezed angle θ, i. e. ,

$$\zeta=re^{i\theta}$$

Fig. 2(c) displays the two quadrature variables noise distribution of the squeezed vacuum state in the phase space. Compared to the noise distribution of the vacuum state and the single photon state, the quantum noise of the squeezed vacuum state are redistributed between the two quadratures and noise from the variable q relocated into the non-commuting variable p, which consistent with the definition of the squeezed state described in section 2. 1.

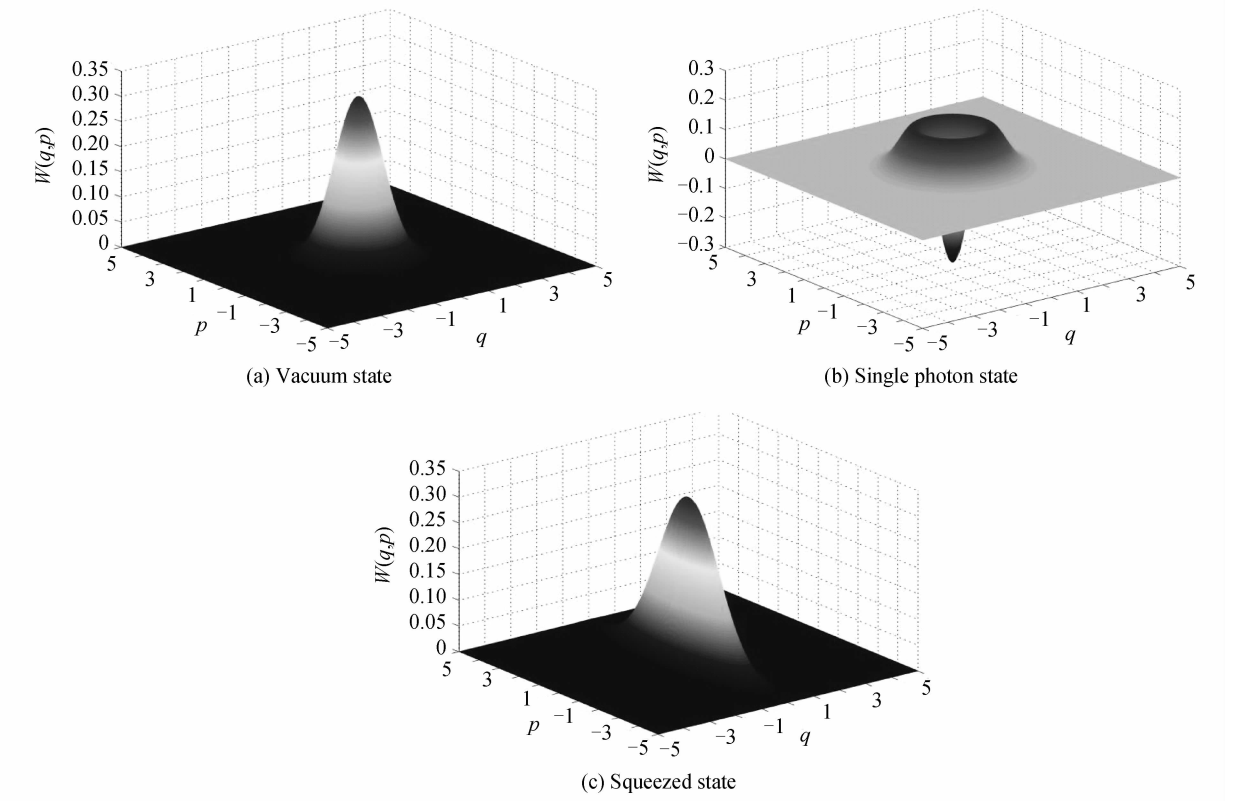


Fig. 2 Quantum noise distribution of three different quantum states of light

2 Quantum lidar based on the squeezed light

Generally speaking, as the special advantage of ultra-low noise (below the standard quantum limit), squeezed states of light is widely used in the area of optical precision measurements, where the measurements are limited by the quantum noise or light power. Research have shown that the power of the squeezed beam required is half of that of the coherent light to obtain a similar SNR, which is very important to detect a target object with certain damage threshold[11].

There has been an increasing interest in high SNR detection and imaging, for instance the amplitude squeezed state has an important application in absorption measurements or modulation measurement, as the quantum fluctuations in the amplitude quadrature are reduced below the shot noise. Similarly, the phase squeezed state can also be used to improve the SNR of the interferometry, in which the only noise contribution from the quadrature phase[12]. Considering different parts of the classical lidar with applications of the quantum technology, the quantum lidar based on the squeezed states of light can be divided into two groups[2, 16-18]: coherent quantum lidar and quantum lidar with squeezed vacuum injection. For coherent quantum lidar, the system has the same operating principles as the classical coherent lidar, the main difference is that an amplitude squeezed light is used as the local oscillator field in heterodyne detection to improve the detection sensitivity. An alternative approach to improve the SNR is based on the injection of the squeezed vacuum state, in which the squeezed vacuum is injected into the receiver to replace the vacuum state and a balanced homodyne detection system is employed to detect information returned from targets of interest.

2.1 Coherent quantum lidar

As we know, the classical lidar are classified as coherent or direct-detection lidar on the basis of detection methods. Compared with the direct detection, the coherent heterodyne detection is a detection technique with high sensitivity and it has wide applications in the domains of weak signal detection. Fig. 3 shows the schematic of the coherent detection system, the electric fields of the local light and signal light directed to the beam combiner following expressions are[19-20]

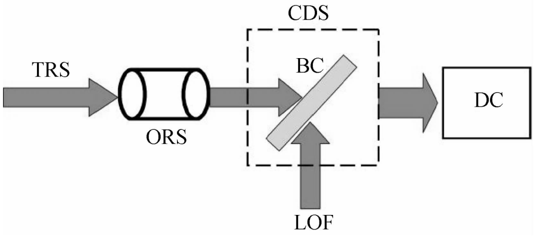


Fig. 3 Schematic of the coherent detection system

$$E_{\mathrm{LO}}(t)=A_{\mathrm{LO}}\cos\left(\omega_{\mathrm{LO}}t+\varphi_{\mathrm{LO}}\right)$$
$$E_{\mathrm{S}}(t)=A_{\mathrm{S}}\cos\left(\omega_{\mathrm{S}}t+\varphi_{\mathrm{S}}\right)$$

where $A_{\mathrm{LO}}/A_{\mathrm{S}}$, $\omega_{\mathrm{LO}}/\omega_{\mathrm{S}}$ and $\varphi_{\mathrm{LO}}/\varphi_{\mathrm{S}}$ are amplitudes, angular frequency and initial phase corresponding to the local field and signal field, respectively. Then the total current of the coherent heterodyne detection is given by

$$i\propto\left[E_{\mathrm{S}}(t)+E_{\mathrm{LO}}(t)\right]^2=\frac{\eta e}{h\nu}\{A_{\mathrm{S}}^2\cos^2(\omega_{\mathrm{S}}t+\varphi_{\mathrm{S}})+A_{\mathrm{LO}}^2\cos^2(\omega_{\mathrm{LO}}t+\varphi_{\mathrm{LO}})+A_{\mathrm{S}}A_{\mathrm{LO}}\cos\left[(\omega_{\mathrm{LO}}+\omega_{\mathrm{S}})t+(\varphi_{\mathrm{LO}}+\varphi_{\mathrm{S}})\right]+A_{\mathrm{S}}A_{\mathrm{LO}}\cos\left[(\omega_{\mathrm{LO}}-\omega_{\mathrm{S}})t+(\varphi_{\mathrm{LO}}-\varphi_{\mathrm{S}})\right]\} \quad (2)$$

It is known from Eq. (2) that, an average of 1/2 is obtained from the first two terms and the average of the third term is 0. The fourth term describes intermediate frequency signal and the intermediate frequency current is calculated as follows[19-20]

$$i_{\mathrm{IF}}=\frac{\eta e}{h\nu}A_{\mathrm{S}}A_{\mathrm{LO}}\cos\left[(\omega_{\mathrm{LO}}-\omega_{\mathrm{S}})t+(\varphi_{\mathrm{LO}}-\varphi_{\mathrm{S}})\right] \quad (3)$$

where η is quantum efficiency of detector and $h\nu$ is the photon energy. Obviously, the amplitude, phase and frequency of the incident signal field can be measured by the coherent heterodyne detection. From Eq. (3), it is beneficial to detect weak signal for the coherent heterodyne detection with a strong local oscillator field, as the amplitude of the i_{IF} is proportional to the product of the amplitudes of the local field and signal field. Then the power of the intermediate frequency signal is given by

$$P_{\mathrm{IF}}=2\left(\frac{\eta e}{h\nu}\right)^2P_{\mathrm{s}}P_{\mathrm{LO}}R_{\mathrm{L}}$$

where P_{s} and P_{LO} stand for the powers of the signal field and local field, respectively. R_{L} describe the load resistor. For the classical lidar, the noise sources mainly includes shot noise and thermal noise, which result from signal light, local light, background light, dark current of the detector and the load resistor. The noise sources are calculated as follows

$$N_{\mathrm{shot}}=2e\left[\frac{\eta e}{h\nu}(P_{\mathrm{S}}+P_{\mathrm{LO}}+P_{\mathrm{B}})+I_{\mathrm{D}}\right]BR_{\mathrm{L}}$$
$$N_{\mathrm{thermal}}=4kTB$$

where P_{B} is power of the background light and I_{D} is the dark current of the detector. B corresponds to the detection bandwidth. Considering a strong local field, the shot noise power of the local field becomes the dominant noise source and the SNRis given by

$$\mathrm{SNR}=\left(\frac{S}{N}\right)_{\mathrm{IF}}=\frac{\eta P_{\mathrm{s}}}{h\nu B} \quad (4)$$

which shows the maximal SNR of the classical coherent detection system, i. e. , Eq. (4) gives the quantum limit of the classical coherent detection[19-20].

In order to reduce the noise of the local field and improve the SNR of the classical coherent detection system, the amplitude squeezed light is used as the local light in the coherent quantum lidarsystem[16], as shown in Fig. 3. The shot noise is

$$N_{\mathrm{shot}}=2e\left[\frac{\eta e}{h\nu}(P_{\mathrm{S}}+P_{\mathrm{LO}}+P_{\mathrm{B}})+I_{\mathrm{D}}\right]BF_{\mathrm{LO}}R_{\mathrm{L}}$$

where F_{LO} is Fano factor, which describes the squeezing degree of the squeezed light. Here $F_{\mathrm{LO}}<1$ and the F_{LO} decrease with the increase of the squeezing degree. Then the SNR of the coherent quantum lidar system is given by[16]

$$\mathrm{SNR}'=\left(\frac{S}{N}\right)_{\mathrm{IF}}=\frac{\eta P_{\mathrm{s}}}{h\nu BF_{\mathrm{LO}}} \tag{5}$$

which shows that using squeezed local light instead of classical local light allows to improve the performance of the coherent quantum lidar and the SNR is increased by a factor of F_{LO}, as $F_{\mathrm{LO}}<1$. Fig. 4 shows the dependence of the relative signal-to-noise (SNR′/SNR) on the Fano factor. e. g. , as $F_{\mathrm{LO}}=0.16$, that is, a squeezing degree of 8dB would afford a 6. 25-fold improvement in SNR theoretically. As a result, the coherent quantum lidar based on the squeezed light benefits the increase of the detection sensitivity, which is very essential to detect extremely weak signal.

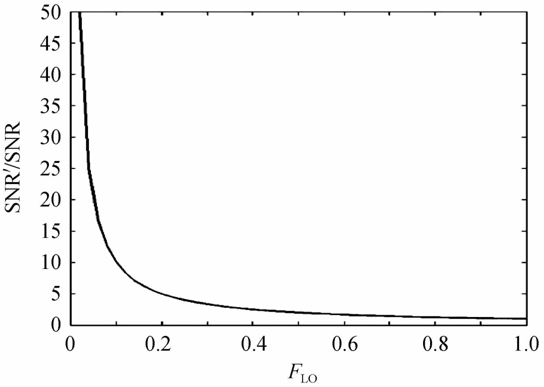
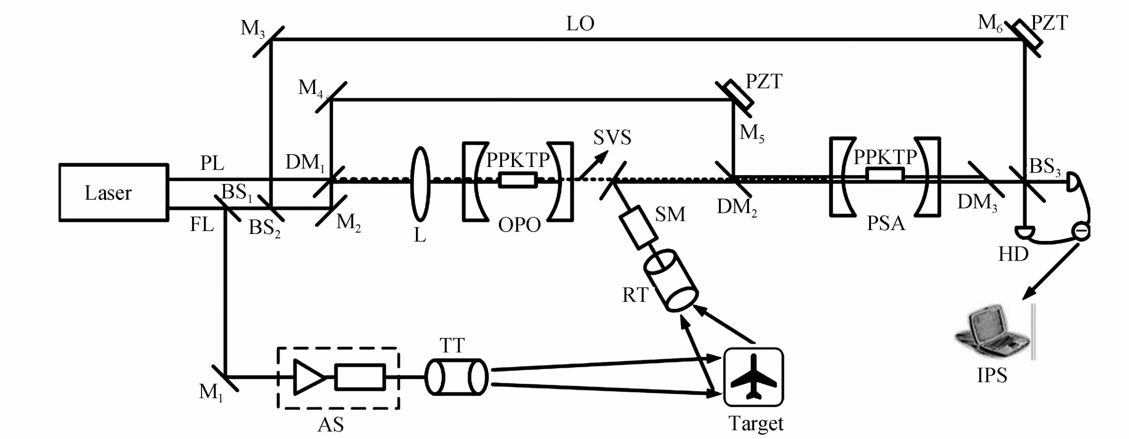


Fig. 4 The dependence of the relative signal-to-noise (SNR′/SNR) on the Fano factor

2. 2 Quantum lidar with squeezed vacuum injection

It is known that most of the non-classical states have drawbacks of sensitivity and fragility. Although the advantages of the quantum states are destroyed easily by low losses, receivers of the classical lidar still utilize the quantum effects. Studies of a Defense Advanced Research Project Agency (DARPA) directed Quantum Sensors Program (QSP) confirmed that an effective approach to improve the performance of the classical lidar is based on the injection of so-called squeezed vacuum state, in which target is illuminated by classical light and target-return signal is detected by a non-standard receiver[1]. The quantum properties of the squeezed vacuum essentially reduces the quantum noise of the receiver by replacing the vacuum state and makes the target-return signal easier to see.

Fig. 5 shows the schematic of the Squeezed-Vacuum Injected (SVI) quantum lidar. To avoid complexity and instability, the laser used has an output of double wavelength. The output fundamental light is divided into three parts. The first part is used to illuminate target and obtain spatial information about the target of interest. The second part mainly used for the generation of the squeezed vacuum and the third part is used as the local oscillator beam for homodyne detection. It is known that, the high-spatial- frequency signal returned from the target may be lost by a soft-aperture in the receiver of the classical lidar. In other words, the high- spatial- frequency information would be buried in the vacuum noise and can be recovered only by reducing the vacuum noise in a way. In fact, the injection of the squeezed vacuum aims to reduce the vacuum noise and regain the high-spatial-frequency information lost by the soft aperture attenuation and results in an improvement in the imaging resolution. The pump beam falls into two parts. One part is directed to the Optical Parametric Oscillator (OPO) together with the fundamental light to generate squeezed vacuum. The other part is guided to the Phase Sensitive Amplifier (PSA) to amplify the squeezed quadrature noiselessly. The PSA system is used to guarantee the quantum efficiency of the homodyne detection system. It is important to note that losses should be avoided in the process of squeezed vacuum injected as any kind of loss has a strong negative influence on squeezing. Injected from the signal port of the receiver, the squeezed vacuum replaces the vacuum state, thereby reducing the quantum noise of the target-return signal, which limit features extraction of the target[21].



M1~M6: reflecting mirror, BS1~BS3: beam splitter, PL: Pump light, FL: fundamental light, DM1~DM3: dichroic beam splitter, PZT: piezoelectric transducer, L: lens, PPKTP: periodically poled KTiOPO4, OPO: optical parametric oscillator, PSA: phase sensitive amplifier, AS: amplification system, TT: transmitting telescope, RT: receiving telescope, SM: scanning mirror, SVS: squeezed vacuum state, HD: homodyne detection, IPS: imaging processing system

Fig. 5 Schematic of the quantum lidar with SVI and PSA

In Refs. [1, 21], the authors have demonstrated that the classical field emerging from the soft aperture can be expressed as

$$E_{\mathrm{R}}'(\rho',t)=A(\rho')E_{\mathrm{R}}(\rho',t) \tag{6}$$

where ρ' represents the coordinates in the soft aperature plane, $A(\rho')$ is transmission of the soft aperture and is given by $A(\rho')=\mathrm{e}^{-2|\rho'|^2/R^2}$. $E_{\mathrm{R}}(\rho',t)$ indicates the target-return field directed to the soft aperture. In this case, the detected signal from the receiver can be described as follows, without taking into account affects of the absolute phase[1,21]

$$y(\rho)=\mathrm{Re}\left(\sqrt{\frac{\eta I_{\mathrm{T}}\tau_{\mathrm{p}}}{h\omega_{\lambda}}}\int T(\rho'')\,m(\rho-\rho')\,\mathrm{d}\rho''\right)+n_{\mathrm{d}}(\rho) \tag{7}$$

where η, I_{T}, τ_{p} and $T(\rho'')$ stand for the homodyne efficiency, the light intensity in the target plane, the pulse width and reflection coefficient, respectively. h is Plank's constant and ω_{λ} is frequency of the signal photon. $m(\rho-\rho')$ being the point spread function correlated to the soft aperature. It is worth mentioning that the target information is obtained from the first term in Eq. (7) and the second term contains the shot noise from the local light, which drown the high-space-frequency information and limits improvements of the spatial resolution.

When the squeezed vacuum state is employed at the soft aperture, the light field can be expressed by quantum field operators and Eq. (6) becomes

$$\hat{E}_{\mathrm{R}}'(\rho',t)=A(\rho')\hat{E}_{\mathrm{R}}(\rho',t)+\sqrt{1-A^2(\rho')}\hat{E}_{\mathrm{S}}(\rho',t) \tag{8}$$

where $\hat{E}_R(\rho',t)$ and $\hat{E}_{\mathrm{S}}(\rho',t)$ stand for the field operator of the classical coherent state and squeezed vacuum state, respectively. Note that the difference to the classical lidar system is that the squeezed vacuum injected to the soft aperture instead of the vacuum field and the squeezed quadrature is detected by the homodyne detectors. The effect of the PSA system is to amplify the squeezed quadrature noiselessly by an OPA, which is composed by an input coupling mirror, output coupling mirror and periodically poled $KTiOPO_4$ crystal. The OPA cavity can be used to amplify the interested signal by controlling the relative phase between the pump beam and the signal beam as the result of the OPA cavity is sensitive to the relative phase. The light field after the PSA is

$$\hat{E}(\rho,t)=\sqrt{G}\hat{E}_{\mathrm{R}}(\rho,t)+\sqrt{G-1}\hat{E}_{\mathrm{R}}'^{\dagger}(\rho,t)$$

where G is the OPA gain and $G>1$. When the SVI system and the PSA system are considered, the detected signal is[1,21]

$$y(\rho)=\mathrm{Re}\left(\sqrt{\frac{\eta G_{\mathrm{eff}}I_{\mathrm{T}}\tau_{\mathrm{p}}}{h\omega_{\lambda}}}\int T(\rho'')m(\rho-\rho')\,\mathrm{d}\rho''\right)+n_{\mathrm{d}}'(\rho) \tag{9}$$

where $n_{\mathrm{d}}'(\rho)$ indicates noise of the quantum lidar and contains quadrature quantum noise from the target, the vacuum noise related to the soft aperature and the homodyne efficiency, which is different from the noise term in Eq. (7). G_{eff} describes the gain of the PSA and is calculated by

$$G_{\text{eff}}=(\sqrt{G}+\sqrt{G-1})^2$$

As we know, the local oscillator of the classical lidar is limited by the shot noise and the spectral density is given by

$$S(f)=\frac{1}{4} \tag{10}$$

Taking the quantum effects of the squeezed vacuum and PSA into account, the spectral density takes the form[1,21]

$$S(f)=\frac{\eta G_{\text{eff}}}{4}\{A^2(\lambda Lf)+[1-A^2(\lambda Lf)]\mathrm{e}^{-2r}\}+\frac{1-\eta}{4} \tag{11}$$

whereη is the quantum efficiency of the homodyne detection, e^{-2r} indicates squeezed factor of the squeezed vacuum stateand r represents the degree of attenuation.

As presented in Refs. [1, 17], terms in Eq. (11) have the following physical meanings: the first term describes the quantum noise of the return signal from the target, the second term stands for the quantum noise of the injected squeezed vacuum field and the third term corresponds to the quantum noise resulted from the inefficient homodyne detection.

Eq. (11) shows that, for the squeezed-vacuum injected quantum lidar, i. e., $r>0$ and $G_{\text{eff}}\gg 1$, the SNR and spatial resolution could be improved by quantum effects of the SVI and PSA. Specifically, fromEqs. (7), (9), (10) and (11), it can be seen that the role of the SVI system is to generate the squeezed vacuum state, which has the quantum noise below the shot noise limit. As a result, the SVI system plays a significant role in reducing the vacuum noise on the high-spatial-frequency signal, that is, the high-spatial-frequency information can be recovered by the SVI system and the squeezed quadrature is detected by homodyne detectors. To avoid an inefficiency homodyne detection, the PSA system is used and aims to recover the lost SNR resulted from the inefficiency homodyne detection[1,21]. It is worthy noting that the SVI and PSA systems should act on the receiver of the classical lidar simultaneously to guarantee the SNR of the target-return signal.

3 Conclusion

In conclusion, we demonstrate quantum properties of the noise distribution of the vacuum state, single photon state and squeezed state in phase space by Wigner function theoretically. In addition, two types of the quantum lidar systems based on the squeezed light are presented, including the coherent quantum lidar and the quantum lidar with squeezed vacuum injection. By comparing the quantum lidar based on the squeezed light and the classical lidar, the results show that: 1) the SNR of the coherent quantum lidar increases with the decrease of the Fano factor F_{Lo}, that is, a higher squeezing degree benefits improvement of the SNR. For example, a squeezing degree of 8dB would afford a 6.25-fold improvement in SNR theoretically; 2) the SVI system and the PSA system are two essential aspects that affect the increasing of the spatial resolution. As a result, the quantum lidar with squeezed vacuum injection has higher spatial resolution, which depends on the squeezing level of the squeezed vacuum and the gain of the PSA. Briefly, thanks to the quantum property of having quantum noise below the shot noise, the SNR of the quantum lidar is improved and it is beneficial to detect weak signal and obtain high resolution image, and some experiments of the quantum lidar based on the squeezed light would be carried out latter on. Considering problems of the quantum lidar in engineering applications, it is likely that squeezed light would be used in the receiver to improve the performance of the classical lidar in future.

References

[1] DUTTON Z, SHAPIRO J H, GUHA S. Ladarresolution improvement using receivers enhanced with squeezed-vacuum injection and phase-sensitive amplification[J]. *Journal of the Optical Society of America B*, 2010, **27**(6): A63-A72.

[2] LANZAGORTA M. Quantumradar[M]. ZHOU Wan-xing, WU Ming-ya, HU Ming-chun, *et al*. transl. Beijing: Publishing House of Electronics Industry, 2013.

[3] WASILOUSKY P A, SMITH K H, GLASSER R, *et al*. Quantum enhancement of a coherent LADAR receiver using phase-sensitive amplification[C]. SPIE, 2011, **8163**: 816305.

[4] ZHOU Cheng-hong, QIN Wei-ping. An overview of radar system based on quantum technology[J]. *Radar Science and Technology*, 2015, **13**(5): 457-462.

[5] GE Yue-tao, JIANG Qi. Study of technology development of quantum radar[J]. *Tactical Missile Technology*, 2014, **4**(5): 5-9.

[6] JIANG Tao, SUN Jun. The principle and development of quantum radar detection target[J]. *Journal of CAEIT*, 2014, **9**(1): 10-16.

[7] XIAO Min, WU Ling-an, KIMBLE H J. Precision measurement beyond the shot-noise limit[J]. *Physical Review Letters*, 1987, **59**(3): 278-281.

[8] KHALAIDOVSKI A, VAHLBRUCH H, LASTZKA N, *et al*. Long-term stable squeezed vacuum state of light for gravitational wave detectors[J]. *Classical and Quantum Gravity*, 2011, **29**: 075001.

[9] CHELKOWSKI S. Squeezed light and laser interferometric gravitational wave detectors[D]. Ph. D. thesis, Hannover University, 2007.

[10] KHALAIDOVSKI A. Beyond the quantum limit a squeezed-light laser in GEO 600[D]. Ph. D. Thesis, Gottfried Wilhelm Leibniz University Hannover, 2011.

[11] JANOUSEK J. Investigation of non-classical light and its application in ultrasensitive measurements[D]. Ph. D. thesis, Technical University of Denmark, 2007.

[12] BACHOR H-A, RALPH T C. A guide to experiments in quantum optics[M]. Weinheim: Wiley-VCH Verlag GmbH & Co., KGa, 2003.

[13] CHELKOWSKI S. Squeezed light and laser interferometric gravitational wave detectors[D]. Ph. D. thesis, Hannover University, 2007.

[14] MCKENZIE K. Squeezing in the audio gravitational wave detection band[D]. Ph. D. thesis, Australian National University, 2008.

[15] MABUCHI H, POLZIK E S, KIMBLE H J. Blue-light induced infrared absorption in $KNbO_3$[J]. *Journal of the Optical Society of America B*, 1994, **11**(11): 2023-2029.

[16] LI Yong-qing, GUZUN D, XIAO Min. Sub-shot-noise-limited optical heterodyne detection using an amplitude-squeezed local oscillator[J]. *Physical Review Letters*, 1999, **82**(26): 5225-5228.

[17] LANZAGORTA M. Quantan radar crossscetions[C]. SPIE, 2010, **7727**: 77270K.

[18] LOPAEVA E D, DEGIOVANNI I P, OLIVARES S. Experimental realization of quantum illumination[J]. *Physical Review Letters*, 2013, **110**(15): 153-603.

[19] DAI Yong-jiang, LIDAR technology[M]. Beijing: Publishing House of Electronics Industry,2010.

[20] AN Yu-ying, ZENG Xiao-dong. Photoelectric detection principle[M]. Xi'an: http://www.xduph.com, 2004.

[21] BURDGE G, DEIBNER G, SHAPRIO J, *et al*. Quantum sensors program[R]. Final technical report, 2009, AFRL-RI-RS-TR-2009-208.