doi:10.3788/gzxb20174605.0512003

压缩真空光输入和光子计数法增强 Sagnac 效应

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摘 要:压缩真空光输入和平衡零拍探测可有效增强 Sagnac 效应,提高陀螺精度;考虑平衡零拍探测的相位精度与相位自身相关,仅在某特定相位能达到最佳灵敏度,设计了一种基于光子计数法提取 Sagnac 输出相位的方案,并利用贝叶斯理论估计相位.理论分析结果表明,该方法能突破散粒噪声极限,相位精度不再受限于相位自身,且当压缩真空光和相干激光功率相同时,精度在理论上能达到海森堡极限.

关键词:陀螺;光子计数;贝叶斯理论;散粒噪声极限;海森堡极限

中图分类号:O439; O431.2 **文献标识码**:A **文章编号**:1004-4213(2017)05-0512003-7

Sagnac-effect Enhancement with Squeezed Vacuum Light Input and Photon Counting Technique

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Abstract: Sagnac effect can be enhanced with squeezed vacuum light input and Quantum Balanced Homodyne Detection (QBHD), which can result in the improvement of the gyroscope precision. By considering the phase sensitivity obtained by QBHD is related to the phase itself, the best sensitivity is achieved only at some certain value of phase. A scheme based photon counting was designed to extract the Sagnac output phase with Bayesian theory. The theoretical analysis results show that, the proposed scheme can break through the Shot Noise Limit(SNL), and the phase accuracy can reach Heisenberg Limit(HL) theoretically when the power of squeezed vacuum light and coherent light are equal. Meanwhile, the best phase sensitivity can be achieved at any value of phase, which is advantageous for gyro to reach the best sensitivity at any output phase.

Key words: Gyroscope; Photon couting; Bayesian theory; Shot noise limit; Heisenberg limit

OCIS Codes: 120.3940; 060.2920; 030.5260

0 Introduction

Sagnac effect refers to a phase difference between two electromagnetic waves propagating in the opposite direction along a rotating circular path which costs the two electromagnetic waves different time for a circle propagation. Sagnac effect is applied in many important areas, such as detection[1-2], sensing[3] and high precision rotation measurement[4-5], among which optical gyro based on Sagnac effect is the mainstream device of the military and aerospace navigation. How to improve the output precision of Sagnac effect is an important research topic[6], and is also one of the most important ways to realize autonomous

Foundation item: The National Natural Science Foundation of China (Nos. 61573372, 61603413)

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Received: Jan. 8, 2017; **Accepted**: Feb. 27, 2017

navigation without the external signals, which is an essential technical support for deep space exploration, deep sea exploration and the military confrontation[7] with the characteristics of good concealment, strong anti-jamming and working all time and all weather.

An effective method for the improvement of Sagnac effect many scholars focus on is the utilization of particle waves such as superfluids[8] and cold atoms[9-10] based on the wave-particle duality theory[11]. In optical regime, quantum mechanics reckons that when laser used as light source is input in Sagnac interferometer, the theoretical limit of the output phase sensitivity is $1/\sqrt{\langle N\rangle}$ ($\langle N\rangle$ is the average number of photons input), which restricts the precision of optical gyro and is called Shot Noise Limit (SNL) or Standard Quantum Limit (SQL)[12]. Quantum metrology shows that SNL can be broken through and the precision even reaches $1/\langle N\rangle$ theoretically, which is called Heisenberg Limit (HL). In order to break through the restriction of SNL, Bertocchi et al. study the quantum Sagnac effect using single photon as light source and show that the output precision is below SNL[13], which conforms to the theory of quantum mechanics, because single photon doesn't have the correlation effect. Kolkiran et al subsequently take the two-photon entanglement source as light source, and prove that the output precision can be improved by adopting two-photon and four-photon coincidence measurement technique theoretically[14]. Nevertheless, the efficiency of two-photon entanglement generation in Kolkiran theory is relatively low. As squeezed vacuum light can be generated through the parametric down-conversion process efficiently, we utilize the squeezed vacuum light entering into the unused input port of Beam Splitter (BS) in gyro and Quantum Balanced Homodyne Detection (QBHD) for Sagnac effect enhancement[15]. However, the QBHD can only achieve the highest precision at $\theta=0$. In this manuscript, we suggest a new method for Sagnac phase extract by photon counting, which can achieve the highest precision at any value of phase θ.

1 Principle of Sagnac effect

Sagnac effect[16] is the optical path difference between two beams of light caused by the rotation of the ring. The optical path difference is linear with the rotation rate. By measuring the interference fringes of two beams of light, one can obtain the rotation rate. To be specific, as illustrated in the Fig. 1, a beam of light splits across the BS at point A, and then the two parts propagate respectively along the clockwise and anti-clockwise direction. If the ring doesn't rotate, the two beams of light will join and form the interference fringes at point A during the same time $t=2\pi r/c$ (r is the radius of interferometer, and c is the speed of light). If the ring rotates along the clockwise direction with angular rate Ω_r as shown in Fig. 1, the beam of light propagating for a circle along the clockwise will need more time $\Delta t=4\pi r^2\Omega_r/c^2$ than the other beam of light. Accordingly, the optical path difference of the two beams of light in the rotating ring for a circle is[17]

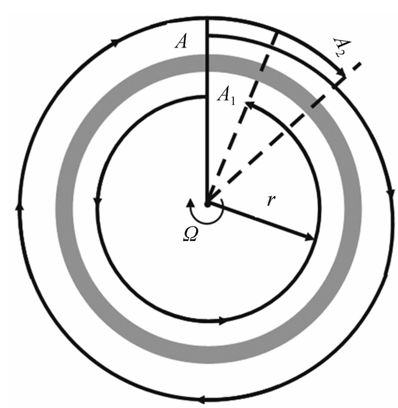


Fig. 1 A schematic diagram of Sagnac effect

$$\Delta L=\frac{4\pi r^2\Omega_r}{c} \tag{1}$$

Furthermore, we can write as

$$\Delta L=\frac{4\boldsymbol{\Omega}_r\boldsymbol{n}S}{c} \tag{2}$$

where $\boldsymbol{n}$ is the unit normal vector of interferometer surface and $\boldsymbol{\Omega}_r$ is the rotation vector. Eq. (2) shows that the optical path difference has nothing to do with the shape of the interferometer, but depends upon the flux of rotation vector $\boldsymbol{\Omega}_r$ which can be enhanced with more circular light path effectively. If the length of optical fiber is L, the phase difference can be written as

$$\theta=\frac{4\pi Lr\Omega_r}{c\lambda} \tag{3}$$

where λ is wavelength. One can see that the phase difference θ is linear with the rotation rate Ω_r. By measuring the phase difference θ, we can directly obtain the rotating rate Ω_r, whose accuracy depends on the measurement precision of θ.

Shot noise is caused by the vacuum zero-point fluctuation introduced through the unused input port of the BS. In order to break through the limitation of shot noise for the improvement of measurement precision, the unused port is suggested to be fed with squeezed vacuum light. As a result of the linear relationship between the rotation rate and relative phase difference, the key problem is how to effectively extract the phase information of the output light and utilize the squeezed properties of light field to improve the sensitivity of the Sagnac effect.

2 Squeezed vacuum technique

2.1 Scheme of squeezed vacuum generation and injection

The squeezed vacuum light is generated with degenerate parametric amplifier and fed into the unused port of BS instead of the vacuum for reaching the sub-shot-noise sensitivity with the precision measurement, as shown in Fig. 2 in detail. The laser source through BS A splits off one part, denoted by annihilation operator $\hat{c}_{in}$, to enter into the input port of BS C , and the other part serves as pump light after frequency doubled to generate the squeezed vacuum light $\hat{b}_{in}$. The pump light is left out through the dichroic mirror B, and the squeezed vacuum light $\hat{b}_{in}$ enters into the other input port of BS C in the Sagnac interferometer.

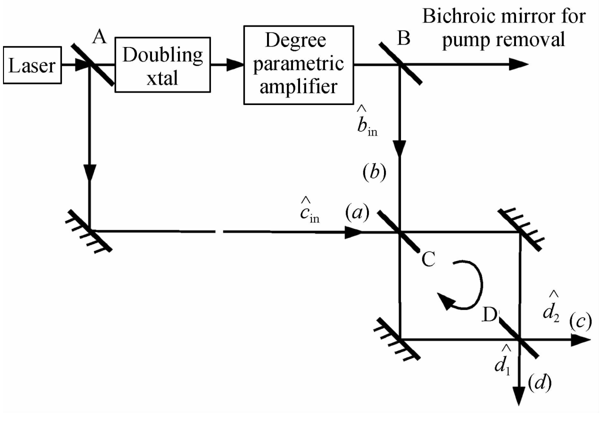


Fig. 2 The generation of squeezed vacuum light and fed into Sagnac interferometer

The physical distinction between the Sagnac interferometer and Mach-Zehnder Interferometer (MZI) is that the MZI has two independent BS while the Sagnac interferometer is only one BS but used twice by signal isolation between input and output with optical circulator. For the convenience of analysis, here Sagnac interferometer will be treated as MZI. Thus, the two beams of light will join at the BS D.

The process of the degenerate parametric amplification in Fig. 2 is a parametric down conversion process, in which the output frequency of light is half of the pump light. The purpose of frequency doubling for pump light is to keep squeezed vacuum $\hat{b}_{in}$ and laser $\hat{c}_{in}$ consistent in frequency. When no signal is fed into the amplifier, the output is squeezed vacuum light, which can be characterized as Bogolyubov transformation[18]

$$\hat{b}_s=\mu\hat{a}_s+\nu\hat{a}_s^{\dagger} \tag{4}$$

where $\mu=\cosh G$, $\nu=-\sinh G e^{i\psi}$ and $|\mu|^2-|\nu|^2=1$. We note that G is squeezed degree and ψ is the squeezed orientation, and the input operator $\hat{a}_s$ must satisfy the vacuum conditions

$$\langle\hat{a}_s^{\dagger}\hat{a}_s\rangle=\langle|\hat{a}_s^{\dagger 2}|\rangle=\langle|\hat{a}_s^2|\rangle=0$$
$$\langle\hat{a}_s\hat{a}_s^{\dagger}\rangle=1 \tag{5}$$

QBHD technique is introduced in the Ref. [15], here we show the scheme in the Fig. 3 and give the expectation value of homodyne detection data $\hat{M}$

$$\langle\hat{M}\rangle=-2|\alpha_L||\alpha_c|\cos\varphi_1\cos\left(\frac{\theta}{2}\right)\tag{6}$$

where φ_1 is the phase between the input coherent light $|\alpha_c\rangle$ and the local oscillator light $|\alpha_L\rangle$. Clearly, the expectation is a function of phase θ which can be effectively found out by adjusting the phase of local oscillator light, so that $\varphi_1=0$.

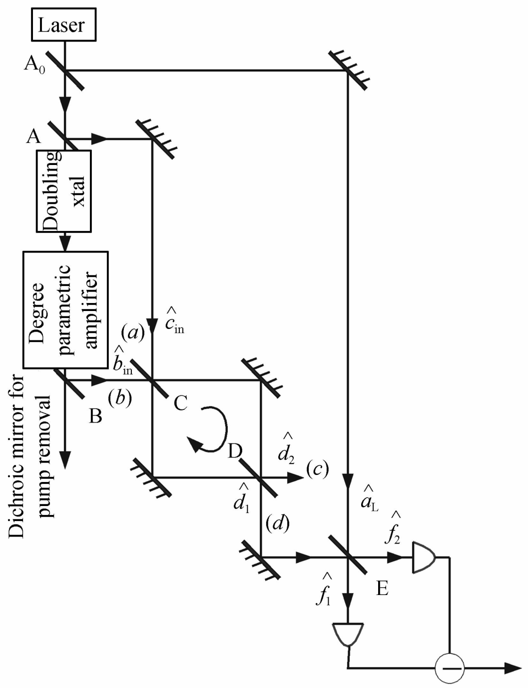


Fig. 3 Schematic diagram of QBHD

2.2 Photon counting with Bayesian phase inference

With the development of the photon counter, especially the production of single photon counter worked in the Geiger mode, the measurement of the number of particles at the output ports gradually becomes important detection method for the unknown value of the relative phase estimate[19-20]. Unlike the quantum balanced homodyne detection, the measurement method doesn't need the local oscillator, just need photon counting for the output particles of the BS D, as shown in Fig. 4.

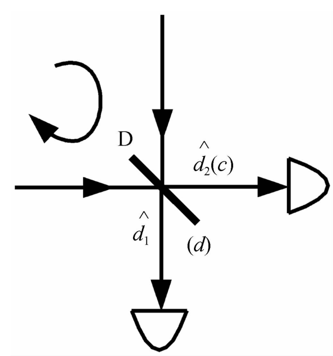


Fig. 4 Measurement of the number of particles

The quantum balanced homodyne detection technique doesn't take advantage of all information effectively, and is not optimal (see 'performance analysis' in section 3). In fact, the quantum fluctuations of the number of the particles at the output ports also contain the relative phase information[21]. The precision of the parameter estimated by the Bayesian inference strategy can reach the Heisenberg limit effectively by utilizing all the information.

The ideal laser in Coherent state $|\alpha\rangle$, and squeezed vacuum state $|\xi\rangle$ in particle number representation can be expressed as $|\alpha\rangle=\sum_n C_n|n\rangle$ and $|\xi\rangle=\sum_m S_m|2m\rangle$ respectively, where $C_n\equiv e^{-|\alpha|^2/2}\frac{\alpha^n}{\sqrt{n!}}$ and $S_m\equiv\frac{1}{\sqrt{\cosh r}}(-1)^m\left(\frac{1}{2}e^{i\vartheta_s}\tanh r\right)^m\frac{\sqrt{(2m)!}}{m!}$. Assuming that the output state of interferometer is $|\Psi(\theta)\rangle=e^{-i\theta\hat{J}_y}|\varphi_{in}\rangle$ where $\hat{J}_y=(\hat{b}_{in}^{\dagger}\hat{c}_{in}-\hat{c}_{in}^{\dagger}\hat{b}_{in})/2i$ and $|\varphi_{in}\rangle=|\alpha\rangle|\xi\rangle$[21], at the output port the conditional probability for measuring the number of particles N_c and N_d is

$$P(N_c, N_d \mid \theta) = \left| \langle N_c \mid \langle N_d \mid e^{-i\theta \hat{J}_y} \mid \varphi_{in} \rangle \right|^2 = \left| \sum_{n=0}^{N} C_{N-n} S_n d^{N/2}_{\mu, N/2-n}(\theta) \right|^2 \tag{7}$$

where $\mu = (N_c - N_d)/2$, $d^j_{\mu,v}(\theta) \equiv \langle j-\mu| \langle j+\mu| e^{-i\theta \hat{J}_y} |j+v\rangle |j-v\rangle$, and $N = N_c + N_d$ is the total number of particles in the system. $d^j_{\mu v}(\theta)$ are rotation matrix elements and can in detail be expressed as $d^j_{\mu,v}(\theta) = \sqrt{\frac{(j-v)!\ (j+v)!}{(j-\mu)!\ (j+\mu)!}} \left(\sin\frac{\theta}{2}\right)^{v-\mu} \left(\cos\frac{\theta}{2}\right)^{v+\mu} P^{v-\mu, v+\mu}_{j-v}(\cos\theta)$ where $P^{v-\mu,v+\mu}_{j-v}(\cos\theta) \equiv 2^{-(j-v)} \sum_{k=0}^{j-v} \binom{j-\mu}{k} \binom{j+\mu}{j-v-k} (1-\cos\theta)^{-(j-v-k)} (1+\cos\theta)^k$ is the Jacobi polynomial.

The derivation process is as follows

$$\begin{aligned} P(N_c, N_d \mid \theta) &= \left| \langle N_c \mid \langle N_d \mid e^{-i\theta \hat{J}_y} \mid \varphi_{in} \rangle \right|^2 = \left| \langle \frac{N}{2} - \mu \mid \langle \frac{N}{2} + \mu \mid e^{-i\theta \hat{J}_y} \sum\nolimits_n C_n \mid n \rangle \cdot \right. \\ &\left. \sum\nolimits_m S_m \mid 2m \rangle \right|^2 = \left| \langle \frac{N}{2} - \mu \mid \langle \frac{N}{2} + \mu \mid e^{-i\theta \hat{J}_y} \sum\nolimits_{m,n} C_n S_m \mid n \rangle \mid 2m \rangle \right|^2 = \left| \sum_{m=0}^{N/2} C_{N-2m} \cdot \right. \\ &\left. S_m \langle \frac{N}{2} - \mu \mid \langle \frac{N}{2} + \mu \mid e^{-i\theta \hat{J}_y} \mid N - 2m \rangle \mid 2m \rangle \right|^2 = \left| \sum_{m=0}^{N/2} C_{N-2m} S_m \langle \frac{N}{2} - \mu \mid \langle \frac{N}{2} + \right. \\ &\left. \mu \mid e_y^{-i\theta \hat{J}} \mid \frac{N}{2} + \frac{N}{2} - 2m \rangle \mid \frac{N}{2} - \left(\frac{N}{2} - 2m\right) \rangle \right|^2 = \left| \sum_{m=0}^{N/2} C_{N-2m} S_n d^{N/2}_{\mu, N/2-2m}(\theta) \right|^2 \end{aligned} \tag{8}$$

The posteriori probability function can be obtained based on Bayesian theory

$$P(\theta | N_c, N_d) = \frac{P(N_c, N_d | \theta) P(\theta)}{P(N_c, N_d)} \tag{9}$$

where $P(\theta)$ is the prior probability distribution of the angle θ. And usually we set $P(\theta) = \frac{1}{2\pi}(-\pi \leqslant \theta \leqslant \pi)$ which means that the prior probability of the angle θ in $[-\pi, \pi]$ obeys uniform distribution, namely the probability of each value of θ is the same. $P(N_c, N_d)$ can be set as a fixed constant for normalization. Thus under the condition of the output phase θ, we will be able to obtain a posteriori probability density function $P(\theta | N_c, N_d)$ by measuring the probability distribution of the number of particles at the two output ports. We can get the phase with the Bayesian inference $\theta_{es} = \int_0^\pi \theta P(\theta \mid N_c, N_d) d\theta$.

3 Performance analyses

3.1 Quantum balanced homodyne detection

We are more concerned about the phase sensitivity, which is an important index of the validity of the strategies for the enhancement of the Sagnac effect. In the Ref. [15], we show the phase uncertainty with QBHD technique by setting a phase bias is

$$\delta\theta = \frac{\sqrt{(|\mu| - |\nu|)^2 + \tan^2\left(\frac{\theta}{2}\right)}}{|\alpha_c|} \tag{10}$$

Obviously, the uncertainty of phase θ is related with phase θ. When $\theta = 0$ we can obtain the minimum uncertainty

$$\delta\theta_{min} = \frac{1}{2\sqrt{\langle n_{sq} \rangle}} \delta\varphi_{SNL} \tag{11}$$

where $\langle n_{sq} \rangle = \langle \hat{b}_s^\dagger \hat{b}_s \rangle = |\nu|^2$ is the average photon number of the squeezed vacuum light, and $\delta\varphi_{SNL} = 1/\sqrt{\langle n_c \rangle}$ where $\langle n_c \rangle = |\alpha_c|^2$ is the average photon number of coherent state light. As the average photon number $\langle_{sq}\rangle$ increases, the phase uncertainty will decrease.

If the Sagnac interference is input with only coherent state light, one can obtain the uncertainty of phase $\delta\varphi = 1/|\alpha_c| \left|\cos\left(\frac{\theta}{2}\right)\right|$ by adopting the homodyne detection technique. The minimum uncertainty of phase is the so-called standard quantum limit $\delta\varphi_{SNL} = \frac{1}{|\alpha_c|} = \frac{1}{\sqrt{\langle N \rangle}}$ at $\theta = 0$.

3.2 Photon counting with Bayesian phase inference

The estimation error (phase fluctuation) of the Bayesian inference based on statistical theories is no longer characterized with the error propagation formula, but with a more accurate expression called quantum Cramer-Rao Low Bound $\Delta\theta = 1/\sqrt{F(\theta)}$, where $F(\theta)$ is the quantum Fisher information. The quantum Fisher information of squeezed vacuum state and coherent state is[21]

$$F(\theta) = \sum_{N_c=0}^{+\infty}\sum_{N_d=0}^{+\infty} \frac{1}{P(N_c, N_d \mid \theta)} \left(\frac{\partial P(N_c, N_d \mid \theta)}{\partial \theta}\right)^2 = |\alpha_c|^2 e^{2G} + \sinh^2 G \tag{12}$$

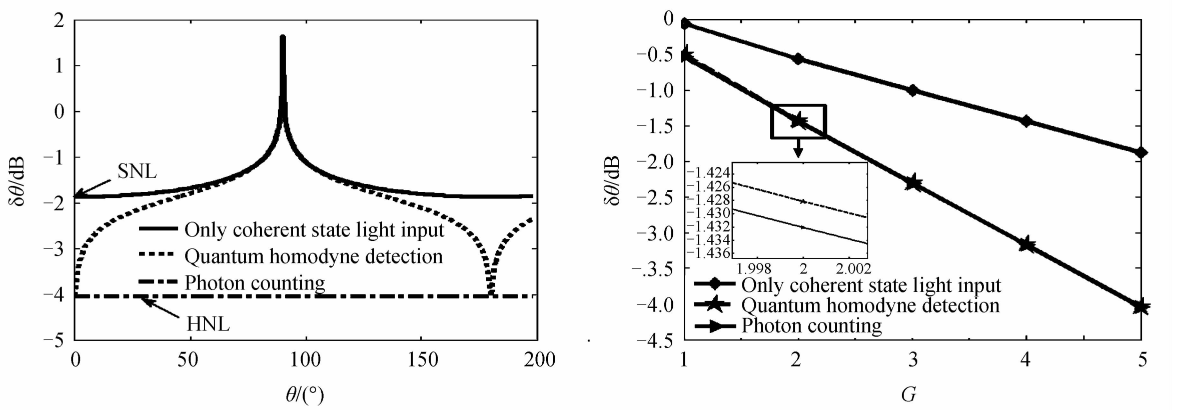
Then $\Delta\theta = 1/\sqrt{|\alpha_c|^2 e^{2G} + \sinh^2 G}$. Obviously, $1/\sqrt{|\alpha_c|^2 e^{2G} + \sinh^2 G} \leqslant 1/|\alpha_c| e^G$, if and only if $G=0$, the equation holds. When $|\alpha_c|^2 \geqslant \sinh^2 G$, $\Delta\theta \approx 1/|\alpha_c| e^G$ is in line with the optimal sensitivity of homodyne detection. If $|\alpha_c|^2 = \sinh^2 G = \bar{n}/2 \geqslant 1$, $\Delta\theta \approx 1/\bar{n}$ can reach the Heisenberg limit. We must note that the phase fluctuation $\Delta\theta$ is not dependent upon the phase θ with Bayesian inference, which is the optimal estimation for any value of phase. Furthermore, when the input power of the coherent light and squeezed vacuum light are equal, the Sagnac interferometer has the best output sensitivity, and can even reach the Heisenberg Limit (HL).

3.3 Comparison of each approach

To evaluate the theoretical performance of the designed scheme, we will make the numerical analysis and comparison among the Sagnac output sensitivities with only coherent state light input, coherent state and squeezed state light input with the QBHD and photon counting, as shown in Fig. 5.

We set the squeezed degree of the squeezed vacuum $G=5$ and the average photon number of coherent state $|\alpha_c|^2$ and squeezed state $\sinh^2 G$ to be equal. As shown in Fig. 5(a), the sensitivity of Sagnac interference output using quantum homodyne detection technique is related to the phase θ, and the best sensitivity is at $\theta=0$ (with phase bias). When the input signal is only the coherent state light, the sensitivity can just reach SNL at $\theta=0$. When the squeezed vacuum light fed into the other input port of Sagnac, the sensitivity is improved obviously. The results of Bayesian inference based on the statistics information of the photon number are independent on the phase θ, and for any value of output phase can get the best sensitivity. When the photon number of coherent states and squeezed state are equal, the Bayesian method can reach the HL.

The optimal performance of various methods with different squeezed degree G is compared with each other in Fig. 5(b) In order to make the comparison more clear, we set the maximum value of G is 5. Certainly, we can set any value of G for comparison theoretically. From Fig. 5(b) we can see that the precision of the QBHD and photon counting with Bayes analysis are very close to each other. As the average photon number of coherent state and the squeezed state are equal, the average photon number of coherent state light will increase as G increases. Fig. 5 reflects the phase estimation precision is proportional to the average photon number of coherent state. In other words, the sensitivity increases as the input photon number increases. However, the increase of the photon number means the increase of the input power which increases the burden of equipment. As the input power increases, the radiation pressure



(a) G=5 with different phase (b) Different G

Fig. 5 Performance comparison among each approach noise will dominate. We cannot improve the sensitivity by only increasing the input power. The Sagnac sensitivity will be effectively improved with the squeezed vacuum light input, with little increase of optical input power. Meanwhile, the photon counting with Bayesian phase inference can make full use of the statistical information, so it can achieve best sensitivity at any value of phase.

4 Conclusion

On the basis of the squeezed vacuum technique for improving the precision of optical gyro, considering the phase sensitivity of QBHD is related to the phase itself, photon counting is introduced for the effective measurement of the relative phase of the Sagnac interferometer. It can be realized in experiment. There exist some challenges for implementation: 1) the higher requirement for experimental environment and the strict requirement for photon number accuracy, due to the photon number counting error will have a big impact on the result of the phase estimation accuracy; 2) the input power of light cannot be too high to damage the photon counter. Consequently, The photon counting method is suitable for small amount of photons input and can achieve the best sensitivity at any value of phase. When the power density of coherent state light and squeezed vacuum light are equal, it has the optimal performance which can reach HL.

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