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无周期光学超晶格中误差函数研究

张丽娟[1],赵丽明[2]

(1 晋中学院 信息技术与工程学院,山西 晋中 030600)

(2 首都师范大学 物理系 理论物理中心,北京 100048)

摘 要:利用非线性共轭梯度算法设计了无周期光学超晶格结构,研究了文献[12]提出的误差函数在该结构中的适用性.研究结果表明:误差函数能够很好地适用于无周期光学超晶格结构,且该结构比文献[12]中所用的非周期光学超晶格更具有一般性;当 $u_2'(x_n)$ 和 n 保持很好的线性关系时,无周期光学超晶格结构中计算所得的误差函数曲线与文献[12]中的误差函数曲线几乎是重合的,说明了在无周期光学超晶格中达到了很好的准相位匹配.通过在样品中引入随机误差进一步研究了相位失配情况下误差函数在无周期光学超晶格结构中的适用性,结果表明:相位失配时无周期光学超晶格结构中计算所得的误差函数曲线与标准曲线是有偏离的,且相位失配程度越大,偏离也越大;对于一些误差函数曲线与标准曲线偏离不大的情况,误差函数仍可近似地用来估计无泵浦损耗近似的适用范围.

关键词:二次谐波产生;误差函数;非线性共轭梯度算法;无周期光学超晶格;准相位匹配

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Research on the Relative Tolerance Function in Non-periodic Optical Superlattice

ZHANG Li-juan[1], ZHAO Li-ming[2]

(1 *School of Information Technology and Engineering, Jinzhong University, Jinzhong, Shanxi* 030600, *China*)

(2 *Department of Physics, Capital Normal University, Beijing* 100048, *China*)

Abstract: The Non-periodic Optical Superlattice (NOS) structure was designed by using the the Nonlinear Conjugate-Gradient(NCG) algorithm, and the applicability in the NOS structure of the relative tolerance function mentioned in Ref. [12] was studied. The results show that, this relative tolerance function can apply to the NOS structure well, and the NOS structure is more general to compare with the Aperiodic Optical Super-lattice(AOS) which is used in Ref. [12]. When $u_2'(x_n)$ and n keep a good linear relationship, the relative tolerance function curve calculated in the NOS structure is nearly coincident with that mentioned in Ref. [12], which shows that a Quasi-Phase-Matching(QPM) is well satisfied in the NOS structure. The applicability in the NOS structure of the relative tolerance function which is under the case of phase mismatching was also furtherly investigated by introducing the random error in the sample. The results show that, the relative tolerance function curve calculated in the NOS structure is not coincident with the standard curve under the case of phase mismatching, and as the mismatching degree increases, the relative tolerance function curves exhibit a larger deviation from the standard one. The applicable scope of Un-depleted Pump Approximation (UPA) can be approximately calculated by using the relative tolerance function when this deviation is not large.

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First author: ZHANG Li-juan (1984—), female, lecturer, M. S. degree, mainly focuses on nonlinear optics. Email:adl3085@163.com

Contact author: ZHAO Li-ming (1978—), female, professor, Ph. D. degree, mainly focuses on nonlinear material frequency conversion and photonic crystal. Email:me_zlm@sohu.com

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0 Introduction

In the past decades, Second Harmonic Generation (SHG) has been extensively studied due to its advantage for obtaining a compact and high-power laser emitting in the visible or ultraviolet spectral regions. Since the so-called Quasi-Phase-Matching (QPM) technology was proposed, the SHG conversion efficiency can be enhanced significantly. The QPM technique is used to compensate for the mismatch among the wave vectors of the interacting light beams by modulating the nonlinear susceptibility in a period of coherence length[1]. To date the QPM has been extensively used to various super-lattices, including the Quasi-Period Super - lattice(QOS)[2-4], Non-periodic Optical Super - lattice(NOS)[5], Aperiodic Optical Super-lattice(AOS)[6-9], and disorder domain configuration[10-11]. It is well known that QPM is obtained under the Un-depleted Pump Approximation(UPA). However when the conversion efficiency of SHG is high enough, the depletion of the pumping light power cannot be ignored and the coupled wave equation needs an exact solution[12-14]. Accordingly, more attention is focused on achieving compact and high power laser via SHG under taking pump depletion into account. Some analytical methods have been proposed to study the SHG considering pump depletion[15-17], in addition, a lot of effort has been made to study SHG considering pump depletion by numerical methods[18-20]. Recently , more attention has been paid to SHG of short laser pulses in the regime of pump depletion[21-22]. But, most of the previous studies focused on the feasibility of the methods in dealing with the SHG considering pump depletion, in the Refs. [12-14], Zhao et al. researched the effects brought about by the pump depletion itself and defined a relative tolerance to estimate the applicable scope of UPA. It is found that the relative tolerance is only determined by the conversion efficiency and it is unrelated to the pump intensity, incidental wavelength, nonlinear media and sample configuration. However, the relative tolerance curves are different for different nonlinear optical process, such as SHG[12] and Difference Frequency Generation(DFG)[14]. It is worth pointing out that the sample is devised using the Simulated Annealing(SA) method in Refs. [12-14]. However, whether the conclusions proposed in above mentioned references are valid in the other configurations is still in question.

In this article, we investigate the applicability of the relative tolerance for SHG in a disorder configuration with the use of Nonlinear Conjugate-Gradient (NCG)[23] method. Our work aims to research whether the results obtained in Ref. [12] is still applicable for more general configuration, thereby obtaining more detailed information and providing directly guidance for practical samples.

1 The nonlinear conjugate-gradient method

The NCG method was proposed by R. Fletcher and C. Reeves in 1964, since then much effort has been devoted to develop this algorithm and many achievements have been obtained. It is introduced to design the NOS structure in this article.

The object function is assumed as

$$E=\sum_{\alpha=1}^{N_\omega}\omega_\alpha|1-\eta_\alpha|^2+\beta\sum_{\alpha=1,\beta=1}^{N_\omega}\sum_{\alpha\neq\beta}^{N_\omega}|\eta_\alpha-\eta_\beta|^2 \tag{1}$$

where N_ω represents the number of the pre-assigned wavelength, ω_α and β is the modulating parameter, η is the SHG conversion efficiency, t_n describes the thickness of every domain. The polarization directions in successive domains are opposite. The optimum design is to find a set of variables in order to minimize the object function E. The variables can be obtained by an iteration algorithm in the form of

$$T^{(k+1)}=T^{(k)}+\tau^{(k)}d^{(k)} \tag{2}$$

where $T^{(k)}$ represents the thickness of domain in the k-th iteration, $\tau^{(k)}$ is the step size, $d^{(k)}$ is the search direction, and $d^{(k)}$ is given by

$$d^{(k)}=-\nabla E(T^{(k)})+\gamma^{(k-1)}d^{(k-1)},k=1,2,3,\cdots \tag{3}$$

where $\nabla E(T^{(k)})$ denotes the gradient of the object function E with respect to T, and $\gamma^{(k-1)}$ is given by

$$\gamma^{(k-1)}=\frac{|\nabla E(T^{(k)})|^2}{\nabla E(T^{(k-1)})|^2},k=1,2,3,\cdots \tag{4}$$

To ensure the convergence of iterations, the step size $\tau^{(k)}$ should be chosen in order to guarantee

$$E(T^{(k+1)})<E(T^{(k)}) \tag{5}$$

The gradient of the object function E with respect to t_n can be derived as

$$\frac{\partial E(t)}{\partial t_n}=\sum_{\alpha=1}^{N_\omega}2\left(1-\frac{1}{|\eta_\alpha|}\right)\mathrm{Re}\left(\eta_\alpha^*\frac{\partial|\eta_\alpha|}{\partial t_n}\right)+\beta\sum_{\alpha=1,\beta=1,\alpha\neq\beta}^{N_\omega}2(|\eta_\alpha|-|\eta_\beta|)\left(\frac{\partial\eta_\alpha}{\partial t_n}-\frac{\partial\eta_\beta}{\partial t_n}\right) \tag{6}$$

$$\frac{\partial|\eta|}{\partial t}=\frac{1}{|\eta|}\mathrm{Re}\left(\frac{\partial\eta}{\partial t}\eta^*\right) \tag{7}$$

The iteration process is terminated until the object function′s gradient reaches a small value or the number of iterations exceeds a given value.

For the NCG method, the polarization directions in successive domains are opposite, but the thickness of each domain is arbitrary, and further optimized using NCG method. However, the main idea of SA method is to give a coherent thickness for each domain, and then optimize the domain configuration by modulating the polarization direction. Undoubtedly, the construct devised by NCG is more general than that devised by SA method because there is no any restriction on the thickness of single domain for NCG method.

2 Results and analyses

The relative tolerance is defined as

$$\sigma=\frac{u_2'(x_n)-u_2(x_n)}{u_2(x_n)} \tag{8}$$

in Ref. [12], where $u_2'(x_n)$ represents the value in the UPA case and $u_2(x_n)$ represents the value in the exact solution case, and the conversion efficiency is given by $\eta=u_2^2(L)$, where L is the length of the sample. In Ref. [12] the curve of relative tolerance vs. $u_2(x_n)$ was obtained for single wavelength case and three wavelengths case when the so-called QPM was satisfied. And it was found that the relative tolerance was solely determined by the SHG conversion efficiency. According to the curves the expression of the relative tolerance was given in Ref. [12] as below

$$\sigma=0.39u_2^2(x_n)-0.0115u_2(x_n)\qquad u_2(x_n)\leqslant 0.4 \tag{9}$$

$$\sigma=0.0097\mathrm{e}^{u_2(x_n)/0.217}\qquad 0.4<u_2(x_n)<0.9 \tag{10}$$

The model is convenient to estimate the SHG conversion efficiency when pump depletion can not be ignored and furthermore the applicability of UPA can also be well estimated.

We consider a one-dimensional disorder structure made by (LN) crystal layers, consisting of laminar ferroelectric domains. Each domain is parallel to the yz plane, the propagation of the incident light is along the x axis, the polarization direction is along the z axis. And the directions of polarization in successive domains are opposite as are the signs of the corresponding nonlinear optical coefficient. The thickness of each domain should be arbitrary and further optimized using NCG method. In this devised sample, a laser beam with the frequency of ω is perpendicularly incident to the surface, and the Second Harmonic Wave (SHW) with the frequency of 2ω is generated by the nonlinear optical process.

In order to research whether the relative tolerance is valid in the disorder structure which is devised by the NCG method, we first consider the single wavelength case. The following parameters are adopted in the sample devised by the NCG method: the pre-assigned wavelength of Fundamental Wave (FW) is $\lambda=1.064$ μm; the corresponding refractive indices in $T=25$℃ are 2.155 for the FW and 2.233 for the SHW; the nonlinear optical coefficient of LN crystal is 27.0 pm/V according to Ref. [24]. The intensity of the incident light is set to $I=1.0\times10^{10}$ W/m^2, and the number of domains is 600. For the single wavelength case, the periodic configuration with its period l_c (the coherence length) is optimal. For the perfect period configuration, the SHG conversion efficiency is 8.769×10^{-2}[17], and the SHG conversion efficiency obtained from the sample which is devised by the NCG method can be achieved to 8.763×10^{-2}[25] which is nearly equal to the value of the perfect period configuration. So, although the structure devised by the NCG method is not the perfect period configuration, we also can calculate the relative tolerance in such a sample. Fig. 1 gives the variation of relative tolerance with $u_2(x_n)$, the dashed curve denotes the relative tolerance curve expressed by the Eqs. (1) and (2) which is called the standard curve in this article, the

straight curve is obtained from the devised disorder configuration. It is clearly seen that the straight curve is nearly coincident with the standard curve. We also calculate the case that the disorder configuration is designed by NCG method in order to achieve multiple SHGs with high and nearly identical conversion efficiency. As an example, we assume that the pre-assigned FW wavelengths are $\lambda_1=0.972\ \mu m$, $\lambda_2=1.064\ \mu m$, $\lambda_3=1.283\ \mu m$. Other parameters are exactly the same with that in Fig. 1. It can be found that the curve of σ vs. $u_2(x_n)$ is also nearly coincident with the standard curve. This denotes that Eqs. (1) and (2) are applicable for the disorder structure. It is obvious that the configuration is more general than the structure devised by using SA method in which the thickness of polarization domain should be the integer time of the coherence length.

In order to further study the characteristics of SHG in the constructed samples, Fig. 2 shows the curves of $u_2'(x_n)$ vs. n for the above mentioned two cases. It is found that $u_2'(x_n)$ increases linearly with n which implies that the good phase matching between FW and SHW can be achieved. So the curves for the above mentioned two cases are nearly coincident with the standard curve which is obtained when the so-called QPM was satisfied.

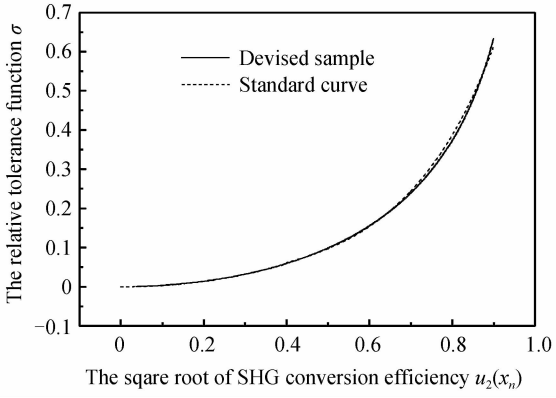

Fig. 1 The variation of relative tolerance with $u_2(x_n)$ for single wavelength case in SHG

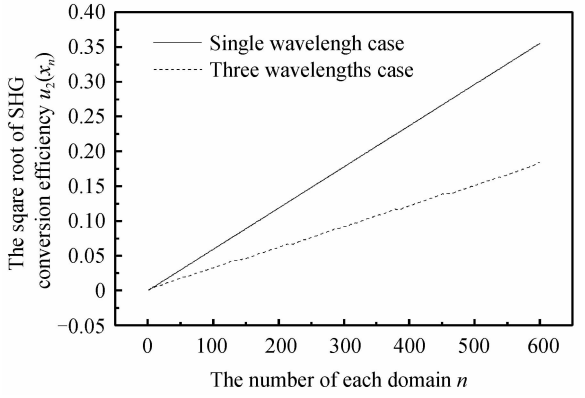

Fig. 2 The variation of $u_2'(x_n)$ with the number of domain n for single wavelength case and three wavelengths case

In order to investigate the relative tolerance of Eqs. (1) and (2) for different structures of sample, we introduce the random error function to simulate the fabrication error for an actual sample. We impose a random distribution function aa in the range of [0,1] on the thickness of each domain, the thickness of n-th domain in an actual sample can be written as $x_n'=x_n+(aa-0.5)/0.5\times x_n\times\delta$. Here, the fabrication error is set as δ, x_n is the thickness of n-th domain designed in Fig. 1, and x_n' is the one after introducing the fabrication error. Fig. 3 gives the variation of relative tolerance with $u_2(x_n)$, the straight line, dashed line, dotted line and dash-dotted line denote the case of $\delta=0.01, 0.03, 0.05, 0.07$ respectively, the dash-dot-dotted line represents the standard curve. It is clearly observed that the curves are inclined to deviate from the standard curve with the increase of δ, except for $\delta=0.01$, which is nearly coincident with the standard curve. With the increase of δ, the deviation is more larger.

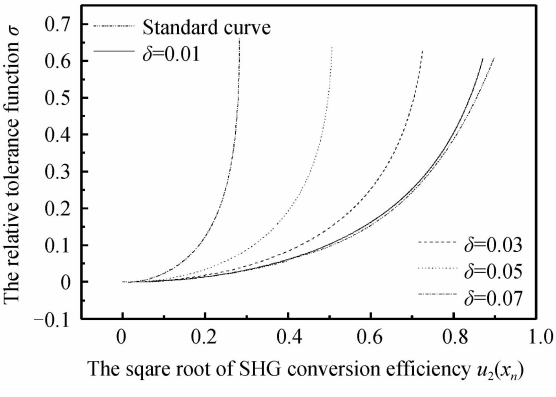

Fig. 3 The variation of relative tolerance with $n_2(x_n)$ for different fabrication errors

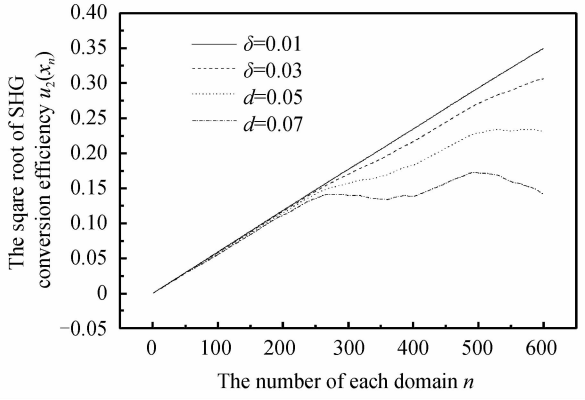

Fig. 4 The variation of $u_2'(x_n)$ with the number of domain n for different fabrication errors

Fig. 4 shows the curves of $u_2'(x_n)$ vs. n when $\delta=0.01, 0.03, 0.05, 0.07$. It is clearly observed that $u_2'(x_n)$ increases nonlinearly with n except for$\delta=0.01$, which is nearly increasing linearly with n. These results obtained here indicate that the random fluctuation leads to the mismatch of phase between the FW and SHW. The bigger the fabrication error is, the stronger mismatch between the FW and SHW will suffer from. These results imply that Eqs. (1) and (2) are applicable for the case of QPM sample for SHG. When the QPM is destroyed by the fabrication error, the relative tolerance is deviated from the standard one.

However, there are still interesting questions requiring further study. For example, how big is the difference between $u_2(x_n)$ estimated by the standard relative tolerance curve and the exact solution. It is assumed that $u_2'(x_n)=0.75$, which can be easily obtained in UPA in the case of $\delta=0.03$. By standard relative tolerance curve, we can calculate that $u_2(x_n)=0.635$. At the same time, we can obtain the exact solution as $u_2(x_n)=0.6$. Correspondingly, the error of $u_2(x_n)$ is about 5.8%. Therefore, although it appears the large deviation of relative tolerance curve from the standard one for $\delta=0.03$, Eqs. (1) and (2) are still used to assess the value of $u_2(x_n)$. So, although the σcurves when fabrication error is introduced are different from the standard curve, Eqs. (1) and (2) are still approximately applicable when the fabrication error is not too big.

3 Conclusion

In summary, we investigate the relative tolerance in the more general disorder structure which is devised by the NCG method and the results show that the relative tolerance defined in Ref. [12] is well applied to the disorder structure. When the QPM is satisfied, the σ curve is nearly coincident with the standard one. We also investigate the phase mismatching cases, it is found that the σ curves deviate from the standard one. With the degree of mismatch is large, the deviation is larger. However it is found that the standard curve is still used to assess the exact solution. These results will provide direct guidance for practical application, such as the design of domain configuration and the estimate for the conversion efficiency of SHG.

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