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色噪声对光学双稳态的影响

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摘　要:采用统一色噪声近似构建了由乘法色噪声驱动的纯吸收型光学双稳态状态方程,并分析了色噪声对光学双稳态的影响,将结果与白噪声驱动的光学双稳态进行比较.结果表明:当乘法噪声与加法噪声处在正关联时,增加乘法色噪声的自关联时间 τ,光学双稳性的区域显著变宽,即磁滞回线面积变大;当乘法噪声与加法噪声处在负关联时,只有乘法噪声较小时,改变乘法色噪声的自关联时间 τ,光学双稳态才发生改变;当乘法噪声的自相关时间等于零时,本文模型退化为乘法白噪声驱动的光学双稳性状态方程.

关键词:非线性光学;光学双稳态; 统一色噪声近似; 色噪声; 状态方程
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Effect of Colored Noise on Optical Bistable System

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Abstract: A state equation of absorptive optical bistable system driven by colored multiplicative noise was built by using the unified colored noise approximation. The results were discussed and compared with the system with white noise. The results show that, when the multiplicative noise and the additive noise are in positive correlation, the area of optical bistability broadens outstandingly, that also is the hysteresis cycle width is broaden with the increasing of self-correlation time τ; whereas when the multiplicative noise and additive noise are in negative correlation, and only when the multiplicative noise strength is weak, the change of self-correlation time τ can induce the change of the area of optical bistability; when the self-correlation time τ of multiplicative noise is equal zero, the model of the system with colored noise falls back to the model of the system with white noise.

Key words: Nonlinear optics; Optical bistability; Unified colored noise approximation; Colored noise; State equation
OCIS Codes: 140.0540; 270.2500; 030.1670; 140.3430

0 Introduction

Optical Bistability (OB) has attracted a great amount of interest and has given rise to numerous experimental and theoretical studies for years[1-4], due to its widely potential applications in optical switches, optical memories, and optical amplification, etc. Simultaneously, optical bistable element is core

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component of next generation computer: superspeed optical digital computer, and to realize fast and efficient optical digital computer, it is essential to investigate influencing factors on the stability of optical bistable element.

In this paper we only discuss the impact of stochastic fluctuations on optical bistable system. In 1978, at the fourth session of the coherence and quantum optics in Rochester, the stochastic fluctuations were first introduced to the bistable system by way of adding a noise phenomenologically[5-6]. In terms of physics it can be interpreted as fluctuations of imput light field, spontaneous emission of non-linear medium (quantum noise) or thermal fluctuations. The purely absorptive OB Langevin equation only with additive noise was adopted as the mathematical model in that research[7]. Hanggi et al, in "good cavity" approximation, deduced an Ito stochastic differential equation driven by multiplicative Gaussian white noise as absorptive OB model[8]. In 1994, Bartussek et al studied the stochastic resonance in the optical bistable system which was driven by multiplicative noise[9], that is, took into account fluctuations in the inversion of the population in the atomic levels. Recently the influence of stochastic phase fluctuations on OB was researched[10], as well as optical bistable state and multiple-state caused by coherence[11]. Zhang *et al* researched the effects of correlation intensity between multiplicative and additive noises on OB[12]. However, in the Ref. [12], the multiplicative noise in state equation that described OB is Gaussian white noise, which is only ideal condition. The objective of this paper is to deduce a state equation of OB which contains colored noise rather than white noise to better describe the real situation. The results of this paper show that, the self-correlation time of multiplicative noise can obviously change the hysteresis cycle (S shape curve) width of the bistable curve, if correlation intensity is positive value. Whereas, if correlation intensity is minus value, the effect of self-correlation time of multiplicative noise on the area of OB is different for different multiplicative noise intensity. Specifically, for the weak multiplicative noise strength, the self-correlation time of multiplicative noise has marked effect on the change of the area of OB. For the strong multiplicative noise strength, the self-correlation time has little effect on the change of the area of OB.

The rest of this paper is organized as follows. In Section 1, the state equation for Absorptive Optical Bistability (AOB) driven by colored multiplicative noise is deduced. Due to colored nature of multiplicative noise and the system driven by colored multiplicative noise undergoing a non-Markovian process, we cannot obtain the precise Fokker-Planck Equation (FPE). So in Section 1, Unified Colored Noise Approximation (UCNA)[13-14] and the method of Stratonovich equivalent stochastic differential equation[15] are used to derive the corresponding FPE and the stationary probability distribution. The state equation of AOB system driven by multiplicative colored noise is further obtained. Section 2 is the discussion. We mainly analyze the change of the state equation of AOB when the multiplicative white noise is replaced by the multiplicative colored noise. And based on the numerical results, we discuss the effects of the "color" of multiplicative noise on the curves of the transmitted light amplitude versus the input light amplitude (or S shape curve). It is found that the self-correlation time τ of multiplicative noise can markedly change the width of S shape curves. Section 3 concludes this paper.

1 State equation for AOB with colored multiplicative noise

When laser, optical cavity and atomics of medium are in resonance, and the bistability is interpreted as a result of the intrinsic nonlinear and feedback of the absorption process, the OB is modeled by a purely AOB. The corresponding evolution equation is given by[16]

$$\dot{x}=y-x-\frac{2\tilde{c}x}{1+x^2} \tag{1}$$

where $\tilde{c}$ is cooperation parameter (parameter of OB), $\tilde{c}>0$, y is the amplitude of the input light, x is the transmitted amplitude, x, y and $\tilde{c}$ have been rendered to be dimensionless variables.

We take into account fluctuations of the input light $y\to y+\eta(t)$, which are embodied in the form of additive noise in the evolution Eq. (1)[7]. Then fluctuations of the cooperation parameter $\tilde{c}$ due to spontaneous emission process, collisions of atoms, or fluctuations of the atomic density in the cavity are considered, $\tilde{c}\to\tilde{c}+\xi(t)$ which behaves in the form of multiplicative noise[8]. The corresponding evolution is thus dominated by the following Stratonovich Langevin equation

$$\dot{x}=y-x-\frac{2\tilde{c}x}{1+x^2}+\frac{2x}{1+x^2}\xi(t)+\eta(t) \tag{2}$$

In Ref. [12], multiplicative and additive noises embodied in the Langevin equation all are Gaussian white noise. Here in this paper, to see the effect of colored noise, we will substitute Gaussian colored multiplicative noise for Gaussian white multiplicative noise, and its statistic properties are

$$\begin{cases}\langle \xi(t)\rangle=0\\ \langle \xi(t)\xi(t')\rangle=\dfrac{Q}{\tau}\exp\left[-\dfrac{1}{\tau}|t-t'|\right]\end{cases} \tag{3}$$

where Q is the strength of multiplicative noise, τ is the self-correlation time of multiplicative noise. Additive noise $\eta(t)$ is Gaussian white noise with zero mean and correlation function

$$\langle \eta(t)\eta(t')\rangle=D\delta(t-t') \tag{4}$$

where D is the strength of additive noise.

Because of the colored multiplicative noise, the Langevin Eq. (2) undergoes a non-Markovian process, and we cannot obtain the FPE directly. There are many approximate theoretical treatments to be used to obtain an approximate FPE. In this paper, we use UCNA and the method of Stratonovich equivalent stochastic differential equation developed in Ref. [15] to derive the approximate FPE.

First, we change the colored multiplicative noise into the following process

$$\begin{cases}\dot{\xi}(t)=-\dfrac{1}{\tau}\xi(t)+\dfrac{1}{\tau}\eta_1(t)\\ \langle \eta_1(t)\rangle=0\\ \langle \eta_1(t)\eta_1(t')\rangle=Q\delta(t-t')\end{cases} \tag{5}$$

Obviously, this is a two-dimensional Markovian process of x and ξ. Due to Multiplicative noise and additive noise have a common origin: the fluctuation of input light, they have cross-correlation between noises. The cross-correlation function between multiplicative noise and additive noise is represented by

$$\langle \xi(t)\eta(t')\rangle=\langle \xi(t')\eta(t)\rangle=\lambda\sqrt{QD}\,\delta(t-t') \tag{6}$$

where the cross-correlation coefficient $-1\leqslant\lambda\leqslant1$.

To clearly express the derivation, Eq. (2) is transformed into the following form

$$\dot{x}=f(x)+g_1(x)\xi(t)+g_2(x)\eta(t) \tag{7}$$

with

$$\begin{cases}f(x)=y-x-\dfrac{2\tilde{c}x}{1+x^2}\\ g_1(x)=\dfrac{2x}{1+x^2}\\ g_2(x)=1\end{cases} \tag{8}$$

Using UCNA, the two-dimensional Markovian process Eq. (5) and Eq. (7) is changed into an approximate one-dimensional Markovian process[13-14]

$$\dot{x}=\frac{1}{C(x,\tau)}[f(x)+g_1(x)\eta_1(t)+g_2(x)\eta(t)] \tag{9}$$

where

$$C(x,\tau)=1-\tau\left[f'(x)-\frac{g_1'(x)}{g_1(x)}f(x)\right]=1+\tau-\tau(x-y)(1-x^2)/x(1+x^2) \tag{10}$$

The prime in Eq. (10) and below denotes the derivative with respect to x.

To obtain FPE, using a simple rule of Ref. [15], Eq. (9) with Eq. (10) is equivalent to a Stratonovich stochastic differential equation in the following form

$$\dot{x}=\frac{f(x)}{C(x,\tau)}+\frac{g(x)}{C(x,\tau)}\tilde{\eta}(t) \tag{11}$$

with

$$g(x)=[Qg_1^2(x)+2\lambda\sqrt{QD}\,g_1(x)g_2(x)+Dg_2^2(x)]^{1/2} \tag{12}$$

in which $\tilde{\eta}(t)$ is Gaussian white noise with zero mean and

$$\langle \tilde{\eta}(t)\tilde{\eta}(t')\rangle=2\delta(t-t') \tag{13}$$

Thus the FPE corresponding to Eq. (11) with Eq. (12) and Eq. (13) is obtained as[17]

$$\frac{\partial P(x,t)}{\partial t}=-\frac{\partial A(x)P(x,t)}{\partial x}+\frac{\partial^2 B(x)P(x,t)}{\partial x^2} \tag{14}$$

where

$$A(x)=\frac{f(x)}{C(x,\tau)}+\frac{g(x)}{C(x,\tau)}\left(\frac{g(x)}{C(x,\tau)}\right)' \tag{15}$$

and

$$B(x)=\frac{g^2(x)}{C^2(x,\tau)} \tag{16}$$

The stationary probability distribution of FPE is accordingly given by

$$P_{\mathrm{st}}(x)=\frac{N}{B(x)}\exp\left[\int\frac{A(x)}{B(x)}\mathrm{d}x\right] \tag{17}$$

where N is integral constant.

Eq. (17) is rewritten in the following form

$$P_{\mathrm{st}}(x)=N\exp\left[-U_{\mathrm{FP}}(x)\right] \tag{18}$$

where $U_{\mathrm{FP}}(x)$ is the generalized potential, and reads

$$U_{\mathrm{FP}}(x)=\ln B(x)-\int\frac{A(x)}{B(x)}\mathrm{d}x \tag{19}$$

In order to see the extreme distribution of $U_{\mathrm{FP}}(x)$, we first take the derivative of generalized potential with respect to x, then substitute it into the extreme equation $U'_{\mathrm{FP}}(x)=0$, and get

$$A(x)=B'(x) \tag{20}$$

From this, for the AOB system with colored multiplicative noise and correlated noises, we can derive the state equation for the input light amplitude y and the transmitted light amplitude x as follows

$$y=x+\frac{2\tilde{c}x}{1+x^2}+\frac{1}{C(x,\tau)}\left[4Q\frac{x(1-x^2)}{(1+x^2)^3}+2\lambda\sqrt{QD}\frac{(1-x^2)}{(1+x^2)^2}\right]-\frac{C'(\tau,x)}{C^2(\tau,x)}\left[D+Q\frac{4x^2}{(1+x^2)^2}+\frac{4\lambda\sqrt{QD}x}{1+x^2}\right] \tag{21}$$

2 Discussions

From Eq. (10), we know that, when $\tau=0$, namely multiplicative noise is white noise (the case of Ref. [12]), $C(x,\tau)=1$, and the fist derivative of $C(x,\tau)$, $C'(x,\tau)=0$, accordingly the state Eq. (21) falls back to

$$y=x+\frac{2\tilde{c}x}{1+x^2}+4Q\frac{x(1-x^2)}{(1+x^2)^3}+2\lambda\sqrt{QD}\frac{(1-x^2)}{(1+x^2)^2} \tag{22}$$

which is just the state Eq. (10) of Ref. [12]. So the model of Ref. [12] is only a special case of this paper. The conclusions of Ref. [12] are included in this paper, for brevity, we will not repeat the same results of the two papers. Here, we only discuss the new phenomena when OB is in the case of $\tau\neq 0$, that is the case of colored noise.

The effect of τ on the S shape curves are discussed in two cases: complete positive cross-correlation between multiplicative noise and additive noise $\lambda=1$, and complete minus cross - correlation between multiplicative noise and additive noise $\lambda=-1$.

2.1 The case of $\lambda=1$

Fig. 1 depicts the transmitted light amplitude x versus the input light amplitude y curve with the self-correlation time of multiplicative noise τ as the parameter. The parameters are chosen as: $\lambda=1$, $Q=1$, $D=0.5$, $\tilde{c}=6$. All parameters are dimensionless. It can be clearly seen that the self-correlation time of multiplicative noise τ has a great influence on the change of the area of OB. Increasing the cross-correlation time of multiplicative noise τ leads to significance increasing of the width of S shape curves and

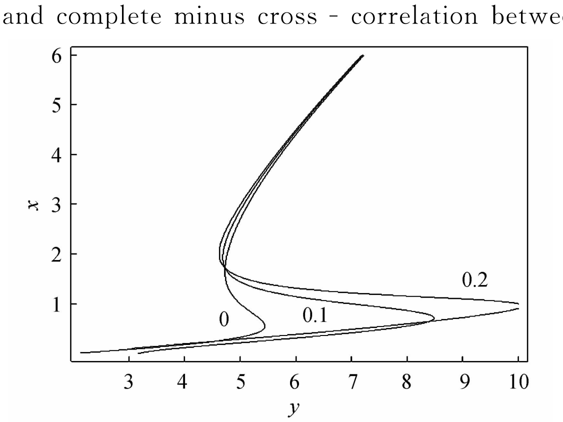

Fig. 1 Transmitted light amplitude x as a function of the input light amplitude y according to Eq. (21)

slightly decreasing of the bistable threshold y_{th}.

2.2 The case of $\lambda=-1$

In Fig. 2, the parameters are chosen as: $D=1$, $\tilde{c}=6$, all parameters are dimensionless. We plot the S shape curves with τ as the parameter for $\lambda=-1$ in two different strength of multiplicative noise $Q=1$ and $Q=0.1$, separately. It is clearly seen from Fig. 2(a) that in the situation of strong multiplicative noise strength, the S shape curves are almost overlap with the increase of τ, the bistable threshold y_{th} is also almost at the same value for different τ. This means when $\lambda=-1$, and the multiplicative noise strength Q is taken in bigger value, the self-correlation time τ has little effect on OB. However, Fig. 2 (b) shows when multiplicative noise strength Q is taken in small value, Increasing τ, the area of OB is obviously broadened , the bistable threshold y_{th} decreases. From Fig. 2 (a) and Fig. 2 (b), It can be concluded that, only when multiplicative noise is small, τ has marked effect on the change of the width of OB. However, when multiplicative noise is big, τ affects OB tinily.

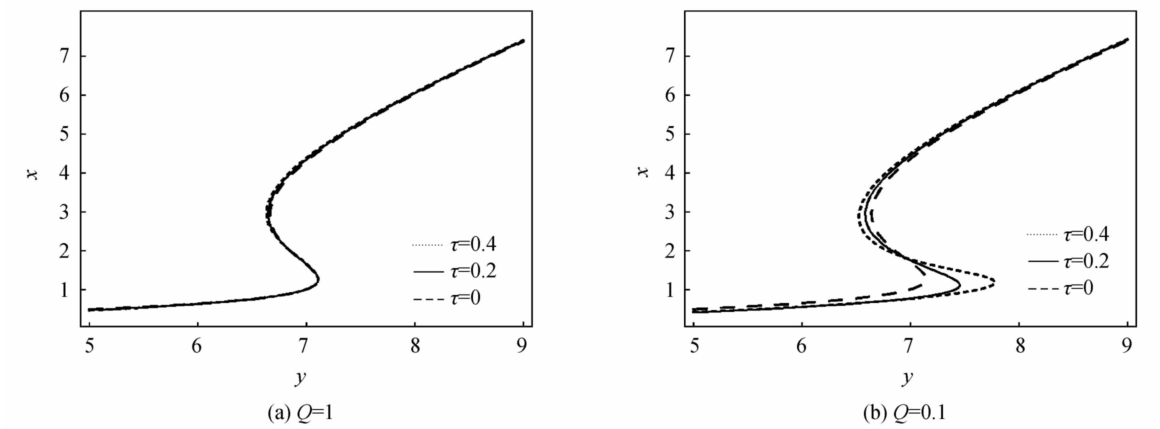

Fig. 2 Transmitted light amplitude x as a function of the input light amplitude y according to Eq. (21)

In Fig. 3(a) and 3(b), the parameters are chosen as: $\lambda=-1$, $D=1$, $\tilde{c}=6$. We show the effect of multiplicative noise strength Q on the S shape curves for Gaussian white noise $\tau=0$ and Gaussian colored noise $\tau\neq 0$, respectively. Seen from Fig. 3(a), the white noise case, with the increase of Q, the evolutional process of the area of OB is divided into two stages, first increasing Q, the width of OB is slightly changed, increasing Q unceasingly, the width of OB begins to increase. Whereas, seen from Fig. 3(b), the colored noise case, taken the same other parameters as Fig. 3(a), the area of OB increases with the increase of Q. Fig. 3 also shows, whether white noise or colored noise, the bistable threshold y_{th} decreases with the increase of Q.

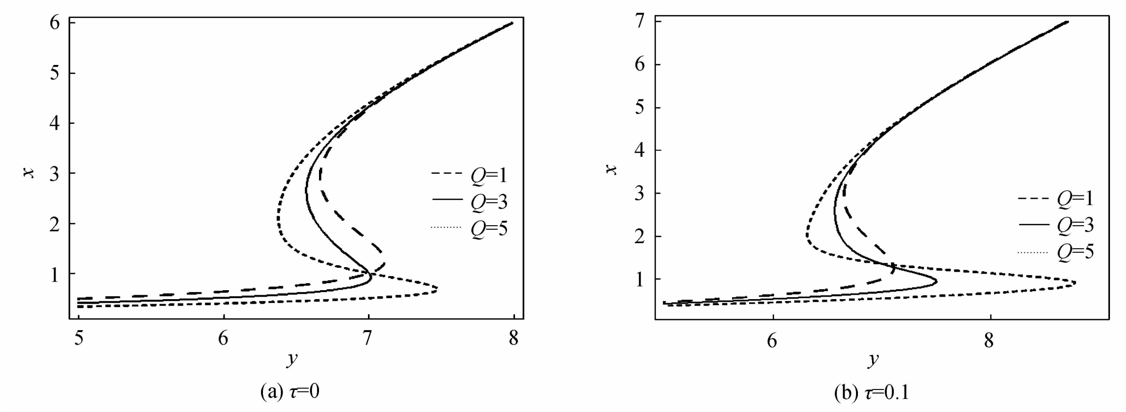

Fig. 3 Transmitted light amplitude x as a function of the input light amplitude y according to Eq. (21)

3 Conclusion

In conclusion, we have illustrated the OB behaviors through the absorptive optical bistable system with cross-correlated colored multiplicative noise and white additive noise. On the conclusions of Ref. [12] and this paper, we find that the self-correlation time τ of multiplicative noise and cross-correlation coefficient between multiplicative noise and additive noise λ as well as the cooperation parameter $\tilde{c}$ can affect the OB behavior dramatically, which can be used to control the bistable threshold intensity and the hysteresis cycle width of the bistable curve.

The state equation obtained in this paper for AOB system driven by colored multiplicative noise is more universal, when $\tau=0$, the state equation of this paper will fall back into the state equation driven by cross-correlated white noises in the Ref. [12], that is, the state equation obtained in Ref. [12] is just a special example of the state equation of this paper.

When multiplicative noise and additive noise is complete positive cross-correlation $\lambda=1$, the area of OB is broaden with the increase of the self-correlation time τ. However, when multiplicative noise and additive noise is complete negative cross-correlation $\lambda=-1$, only the strength of multiplicative noise Q is taken in small value (weak noise), the change of τ can markedly affect the area of OB. For the big value of Q (strong noise), τ has little effect on OB. The multiplicative noise strength Q has different effect on OB for $\tau=0$ and $\tau\neq0$. It is known that, white noise is only an ideal condition, yet colored noise approaches reality. So the result of colored noise is more reliable. Increasing Q, the width of OB increases and the bistable threshold y_{th} decreases.

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