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线性和二次电光效应共同主导的非相干耦合空间孤子族

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摘　要:基于加偏压的单光子光折变晶体,理论推导了线性和二次电光效应共同主导下的亮孤子族和暗孤子族的解,数值研究了亮孤子族和暗孤子族的强度包络和稳定特性,讨论了线性和二次电光效应在孤子族形成中的不同作用.结果表明:线性和二次电光效应的相互作用能够增强亮孤子族的光折变非线性,而减弱暗孤子族的光折变非线性.此外,在传输过程中,亮孤子族的各个分量能够稳定传输;暗孤子族各个分量在较长传输距离时表现出不稳定性.

关键词:非线性光学;光折变效应;电光效应;非相干耦合;孤子族
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Incoherently Coupled Spatial Soliton Families Governed by Both the Linear and Quadratic Electro-optic Effects

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Abstract: Based on the biased one-photon photorefractive crystals, the solutions of incoherently coupled bright and dark soliton families governed simultaneously by linear and quadratic electro-optic effects were derived theoretically. The intensity profile and stability of soliton families were investigated. The different functions of the linear and quadratic electro-optic effects were discussed and the results show that, the interplay between the linear and quadratic electro-optic effects can enhance the photorefractive nonlinearity for bright soliton families but weaken the photorefractive nonlinearity for dark soliton families. Moreover, the bright soliton components can remain invariant during propagation process, and the dark soliton components show instability for long propagation distance.

Key words: Nonlinear optics; Photorefractive effect; Electro-optic effect; Incoherently coupled; Soliton family
OCIS Codes: 190.0190; 190.5330; 190.6135

0 Introduction

Photorefractive (PR) spatial solitons have been widely investigated during the last two decades since their unique features of formation at low laser power and potential important applications. There are two nonlinear effects to support the generation of PR spatial solitons. One is the Pockels effect i. e. linear Electro-Optic (EO) effect, in which the refractive index change is proportional to the electric field. Usually, non-centrosymmetry crystals, such as SBN and $LiNbO_3$ crystals show the large linear EO effect

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and then support various spatial solitons[1-3], soliton pairs[4] and soliton families[5]. The other is the Kerr effect i. e. quadratic EO effect, in which the refractive index change is proportional to the square of the electric field. The centrosymmetric crystal KLTN is the typical example. Various PR spatial solitons[6-7], soliton pairs[8] and soliton families[9] supported by only quadratic EO effect have also been reported. However, a number of new EO crystals that have simultaneously large linear and quadratic EO effects near the phase-transition temperature have been found, such as ferroelectric PZN-PT and PMN-PT single crystals, and so on[10-12]. Very recently, Hao et al. have demonstrated the properties of PR spatial solitons and soliton pairs governed by both the linear and quadratic EO effects in biased one-photon PR crystals[13-15]. Simultaneously, we have reported that PR spatial soliton pairs supported by both linear and quadratic EO effects can exist in biased two-photon PR crystals[16].

In this paper, we theoretically investigate the characteristics of incoherently coupled bright and dark spatial soliton families in biased one-photon PR crystals with both the linear and quadratic EO effects. Our results predict that bright, dark spatial soliton families can exist under appropriate conditions. These soliton families are formed by multiple incident beams that have the same polarization, wavelength, and are mutually incoherent. Moreover, we also discuss the difference between the linear and quadratic EO effects. The results show that the quadratic EO effect can enhance the nonlinearity for bright solitons type but decrease the nonlinearity for dark solitons type under given conditions. Finally, we investigate the stability of those soliton families.

1 Soliton coupling equation

To study the incoherently coupled soliton family supported by both the linear and quadratic EO effects, let us consider N optical beams that propagate collinearly in biased PR crystals along the z axis and are permitted to diffract only along the x direction. These N optical beams have the same polarization, wavelength, and are mutually incoherent. The polarization of the incident optical beam is parallel to the optical axis of the crystal, and the external bias electric field is applied in the same direction. As shown in Ref. [5], the optical fields are expressed in terms of slowly varying envelopes φ_1, φ_2, $\cdots$ and φ_N, i. e., $E_1=\hat{x}\varphi_1(x,z)\exp(ikz)$, $E_2=\hat{x}\varphi_2(x,z)\exp(ikz)$, $\cdots$ and $E_N=\hat{x}\varphi_N(x,z)\exp(ikz)$, where $k=k_0n_e$, $k_0=2\pi/\lambda_0$, λ_0 is the free-space wavelength, n_e is the unperturbed index of refraction. Under these conditions, the envelope of soliton family components φ_1, φ_2, $\cdots$ and φ_N satisfy the following equations[5, 13]

$$\mathrm{i}\frac{\partial\varphi_j}{\partial z}+\frac{1}{2k}\frac{\partial^2\varphi_j}{\partial x^2}-\frac{k_0n_e^3r_{33}E_{sc}}{2}\varphi_j-\frac{k_0n_e^3gE_{sc}^2}{2}\varphi_j=0\qquad(j=1,2,\cdots,N)\tag{1}$$

where, r_{33} and g are the linear and quadratic EO coefficients, respectively, E_{sc} is the induced space-charge field. In the case that the diffusion effect is neglected, the space-charge field can be approximately given by[13]

$$E_{sc}=E_0\frac{I_\infty+I_b+I_d}{I+I_b+I_d}\tag{2}$$

where I_d and I_b are the optical intensity of dark irradiance and background beam, respectively. $I=I(x,z)=(n_e/2\eta_0)\sum_{j=1}^{N}|\varphi_j|^2$ is the total optical intensity of the N component, and $\eta_0=\sqrt{\mu_0/\varepsilon_0}$. I_∞ is the total optical intensity in the area far away from the center of the crystal, and $I_\infty=I(x\rightarrow\pm\infty,z)$. Substituting Eq. (2) into Eq. (1), and after appropriate normalization, we can obtain the normalized envelope U_1, U_2, $\cdots$, and U_N of the Nincident optical beam satisfy the following evolution equations[5]

$$\mathrm{i}\frac{\partial U_j}{\partial\xi}+\frac{1}{2}\frac{\partial^2U_j}{\partial s^2}-\beta_1\frac{(1+\rho)U_j}{1+\sum_{j=1}^{N}|U_j|^2}-\beta_2\frac{(1+\rho)^2U_j}{\left(1+\sum_{j=1}^{N}|U_j|^2\right)^2}=0\qquad(j=1,2,\cdots,N)\tag{3}$$

where the dimensionless parameters $\varphi_j=[2\eta_0(I_b+I_d)/n_e]^{1/2}U_j$, $s=x/x_0$, and $\xi=z/(kx_0^2)$. x_0 is an arbitrary spatial width for scaling, $\rho=I_\infty/(I_d+I_b)$, $\beta_1=(k_0x_0)^2n_e^4r_{33}E_0/2$, $\beta_2=(k_0x_0)^2n_e^4gE_0^2/2$, E_0 is the value of the external bias field.

In what follows, we will solve Eq. (3) and present the numerical solutions of bright, dark and gray soliton families in the steady-state regime.

2 Bright soliton family

Firstly, we begin our analysis from the bright soliton family. In this case, the soliton intensity in the area far away from the center of the crystal is zero i. e. $I_\infty=0$, so $\rho=I_\infty/(I_d+I_b)=0$. The Eq. (3) reduced to

$$\mathrm{i}\frac{\partial U_j}{\partial \xi}+\frac{1}{2}\frac{\partial^2 U_j}{\partial s^2}-\beta_1\frac{U_j}{1+\sum_{l=1}^{N}|U_l|^2}-\beta_2\frac{U_j}{\left(1+\sum_{l=1}^{N}|U_l|^2\right)^2}=0\qquad (j=1,2,\cdots,N)\tag{4}$$

We can obtain the numerical solution of bright soliton families from Eq. (4) by expressing the normalized envelope U_j in the form of $U_j(s,\xi)=r^{1/2}c_j y(s)\exp(\mathrm{i}\nu\xi)$, where ν represents a nonlinear shift of the propagation constant, r is defined as $r=I(0)/(I_b+I_d)$, $I(0)$ stands for the total optical intensity of the center area for the soliton family. $y(s)$ is a normalized real function bounded between $0\leqslant y(s)\leqslant 1$. The parameter c_j^2 corresponds to the ratio of the intensity of the jth soliton component to the total intensity of the soliton family, and satisfies the conditions of $\sum_{j=1}^{N}c_j^2=1$. For the bright soliton family, $y(s)$ is required to satisfy the boundary conditions of $y(0)=1$, $y'(0)=0$, $y(s\rightarrow\pm\infty)=0$, and all the derivatives of $y(s)$ are zeros when $s\rightarrow\pm\infty$. Substitution of this form of U_j into Eq. (4), we have

$$\frac{\mathrm{d}^2 y}{\mathrm{d}s^2}-2\nu y-2\beta_1\frac{y}{1+ry^2}-2\beta_2\frac{y}{(1+ry^2)^2}=0\tag{5}$$

By integrating and employing the boundary conditions of $y(s)$, we can obtain the following expression

$$s=\pm\int_y^1\left\{\mathrm{d}\tilde{y}\Big/\left\{\frac{2\beta_1}{r}\left[\ln(1+r\tilde{y}^2)-\tilde{y}^2\ln(1+r)\right]+\frac{2\beta_2}{1+r}\frac{r\tilde{y}^2(1-\tilde{y}^2)}{1+r\tilde{y}^2}\right\}^{1/2}\right\}\tag{6}$$

From Eq. (6), $y(s)$ can be easily obtained by numerical integration and then the soliton family component can be obtained through $U_j(s,\xi)=r^{1/2}c_j y(s)\exp(\mathrm{i}\nu\xi)$.

To illustrate our results, we adopt PMN-0.33PT single crystal as the example, the relative parameters as following[10-13]: $n_e=2.562$, $r_{33}=182\times10^{-12}$ m/V, $g=1.38\times10^{-16}$ m^2/V^2, $x_0=20$ μm, $\lambda_0=632.8$ nm, $E_0=2\times10^5$ V/m, from these parameters, we can obtain $\beta_1=30.9$, $\beta_2=4.7$. The other parameters are $c_1^2=0.3$, $c_2^2=0.25$, $c_3^2=0.2$, $c_4^2=0.15$, $c_5^2=0.1$, and $r=10$. Fig. 1 depicts the intensity profile of the bright soliton family with five components and all components have the same Full Width at Half Maximum (FWHM) i. e. 12 μm. The soliton family is supported by both the linear and quadratic EO effects.

However, if only the linear or quadratic EO effect exists, how the FWHM will change? We take the soliton family component U_1 as the example, in Fig. 2, find that the FWHM will increase slightly when the soliton is supported only by the linear EO effect ($\beta_1=30.9$, $\beta_2=0$) and the FWHM will increase more when the soliton is supported only by the quadratic EO effect ($\beta_1=0$, $\beta_2=4.7$). That is to say, the interplay between the linear and quadratic EO effects can enhance the PR nonlinearity corresponding to the narrowest FWHM and the linear EO effect plays a central role. The physical mechanism of this phenomenon is that the linear and quadratic EO effects all are positive which can support the bright soliton in self-focusing medium.

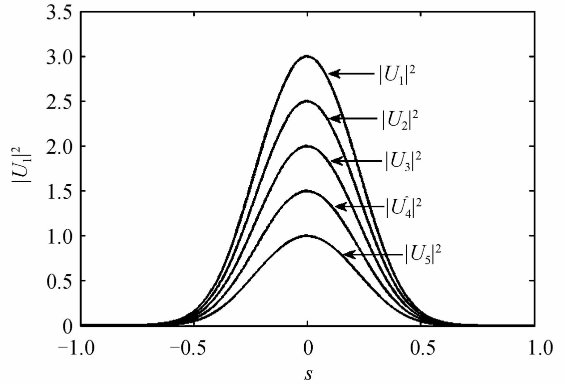

Fig. 1 The intensity profiles of the bright soliton family with five components

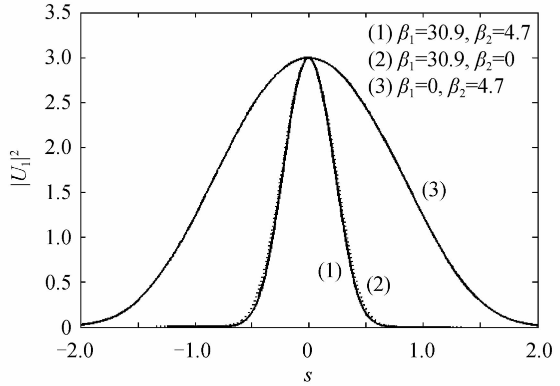

Fig. 2 The intensity profiles of the bright soliton component U_1 at different conditions

The stability of the soliton family also is an important part. The propagations of five bright soliton components are shown in Fig. 3, which is obtained by solving numerically the Eq. (4). We can find that the soliton family is stable with distance.

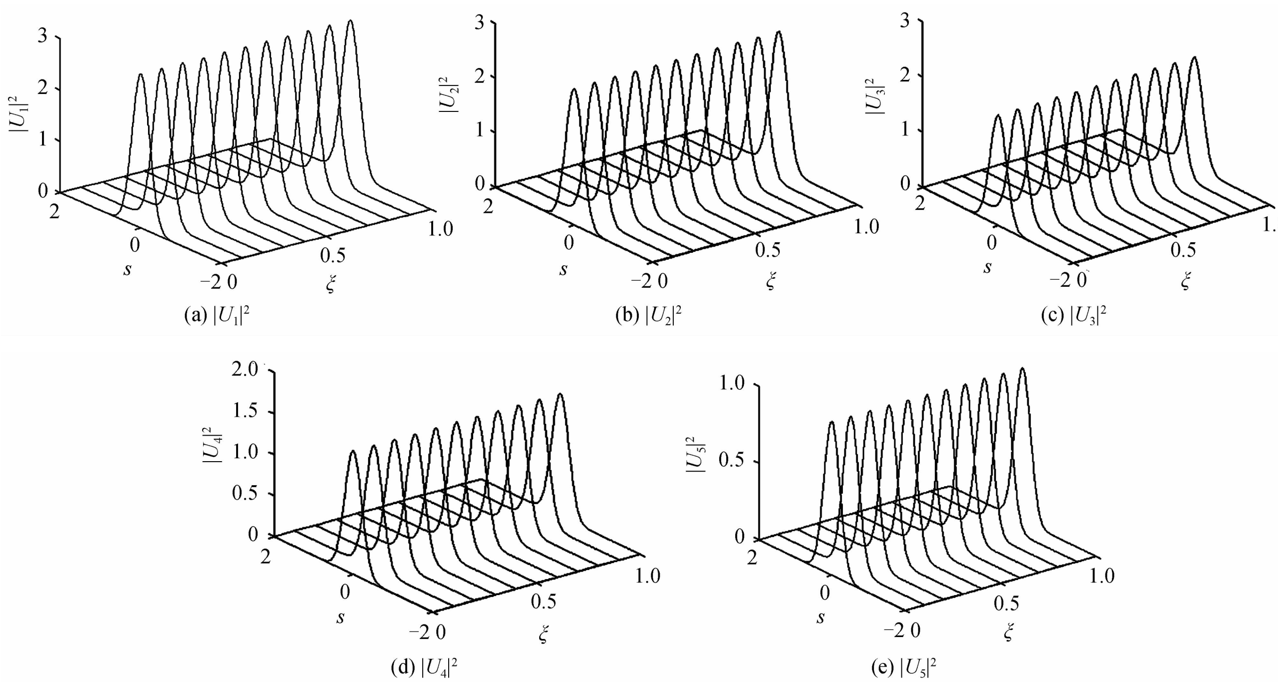

Fig. 3 The stable propagation of the bright soliton family

3 Dark soliton family

Taking the similar method as above, we express the solution of the dark soliton family as the form $U_j(s,\xi)=\rho^{1/2}c_j y(s)\exp(\mathrm{i}\mu\xi)$, μ is a nonlinear shift of the propagation constant, $y(s)$ is the normalized odd function of s and satisfies the boundary conditions: $y(0)=0$, $y(s\to\pm\infty)=\pm1$, $y^{(n)}(s\to\pm\infty)=0(n\geqslant1)$. Substitution U_j into Eq. (3) lead to the following expression

$$\frac{\mathrm{d}^2y}{\mathrm{d}s^2}-2\mu y-2\beta_1(1+\rho)\frac{y}{1+\rho y^2}-2\beta_2(1+\rho)^2\ \frac{y}{(1+\rho y^2)^2}=0 \tag{7}$$

By integrating and employing the boundary conditions of $y(s)$, we can obtain

$$s=\pm\int_0^y\left\{\mathrm{d}\tilde{y}\Big/\left\{-2\beta_1\left[(\tilde{y}^2-1)-\frac{1+\rho}{\rho}\ln\left(\frac{1+\rho\tilde{y}^2}{1+\rho}\right)\right]-2\beta_2\rho\frac{\tilde{y}^4-2\tilde{y}^2+1}{1+\rho\tilde{y}^2}\right\}^{1/2}\right\} \tag{8}$$

The intensity profiles of the dark soliton family U_j can be obtained by simple numerical integration as shown in Fig. 4. Here, we take the external bias field $E_0=-2\times10^5$ V/m, $\rho=10$, the other parameters are same as the bright soliton family. These components have the same FWHM 8 μm.

Similarly, we take the dark soliton component U_1 to analyze the role of linear and quadratic EO effects . In Fig. 5, the curve (1) represents the soliton component supported by both the linear and

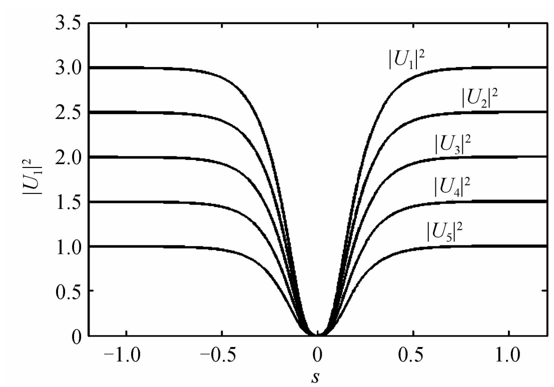

Fig. 4 The intensity profiles of the dark soliton family with five components

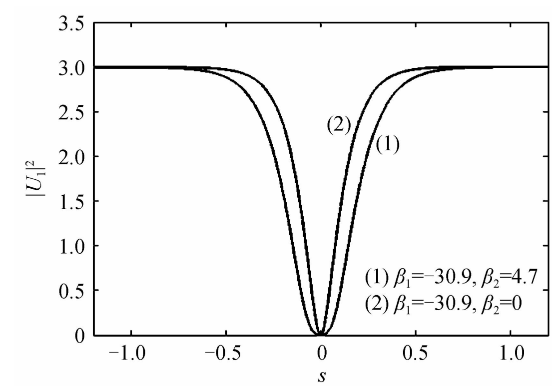

Fig. 5 The intensity profiles of the dark soliton component $|U_1|^2$ at different conditions

quadratic EO effects. The curve (2) represents the soliton component supported by only linear EO effect. We find that the interaction between the linear and quadratic EO effects can weaken the PR nonlinearity and so the soliton width of curve (1) is bigger than that of curve (2). The reason is that the quadratic EO coefficient of PMN-0.33PT single crystal is positive, which is irrelevant to the direction of the external bias field. Of course, the dark soliton family cannot exist in PMN-0.33PT single crystal if it only has positive quadratic EO effect.

We also investigate the stability of the dark soliton family as shown in Fig. 6. It shows that the dark soliton family is stable in comparatively short distance (~3 mm) and will emerge instability for long propagation distance (>3 mm), which can be mainly attributed to the truncation in the process of numerical calculation[13].

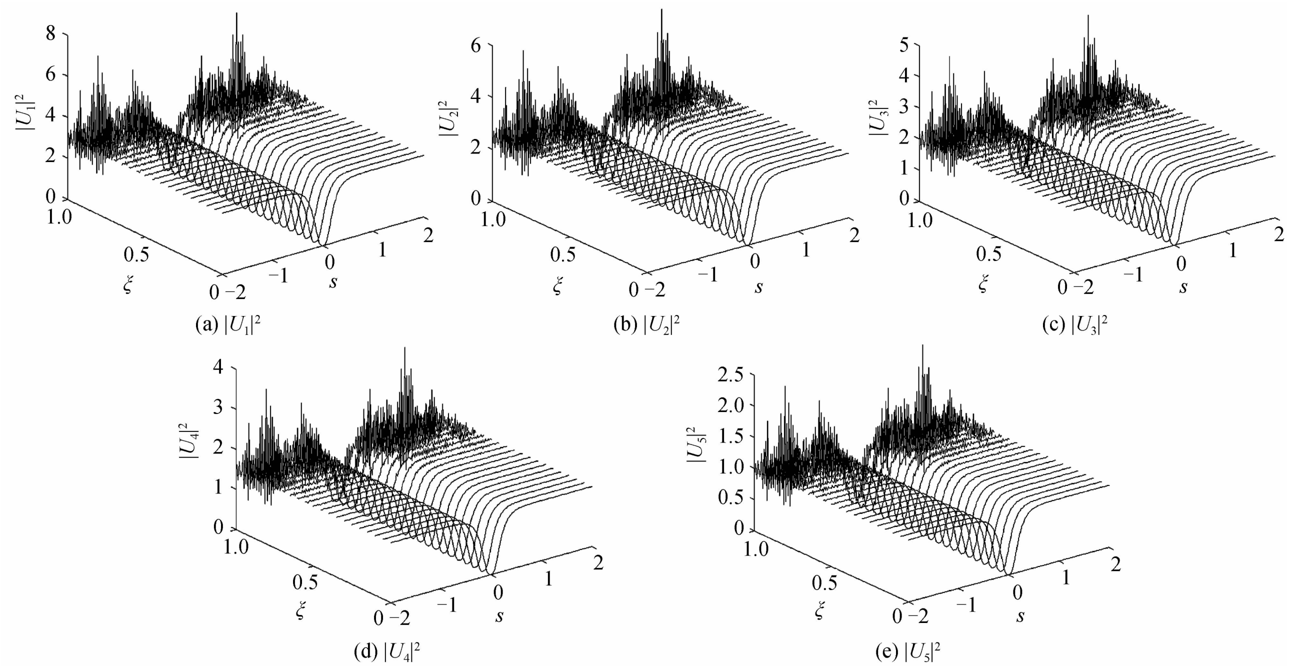

Fig. 6 The unstable propagation of the dark soliton family

4 Conclusion

In conclusions, we have theoretically shown that incoherently coupled bright and dark soliton families governed simultaneously by linear and quadratic electro-optic effects can exist in biased one-photon photorefractive crystals. The interaction between the linear and quadratic EO effects can enhance or weaken the PR nonlinearity under certain conditions. The bright soliton family is stable with distance but dark soliton family show instability for long propagation distance. Moreover, our bright and dark soliton families can degrade into corresponding soliton pairs under certain conditions. The results have potential application in all-optical switching.

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