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基于拉盖尔-高斯光束的单光子捕获概率研究

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摘　要:单光子源通常采用基于高斯光束的高度衰减激光脉冲,假设激光束具有初始高斯时域脉冲波形和 TEM_{01} 模拉盖尔-高斯空域分布.基于折射率起伏的 Rytov 近似和修正 von Karman 谱模型,研究了大气湍流对星地量子通信单光子捕获概率的影响;建立了上行信道和下行信道的单光子捕获概率理论模型;针对低轨卫星-地面站间激光链路,对单光子捕获概率进行了分析.结果表明:上行信道的单光子捕获概率强烈依赖于地面折射率结构常数 $C_n^2(0)$,且随着 $C_n^2(0)$ 的增加而减小;然而,下行信道的单光子捕获概率并不依赖于 $C_n^2(0)$,即大气湍流对其没有影响.

关键词:单光子捕获概率;拉盖尔-高斯光束;折射率结构常数;星地量子通信;大气湍流

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Study on Single-photon Acquisition Probability Based on Laguerre-Gaussian Beams

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Abstract: Highly attenuated laser pulses based on Gaussian beams are used as single-photon sources. It is assumed that this laser beam has an initial Gaussian temporal pulse shape and a TEM_{01} mode Laguerre-Gaussian spatial profile. Based on the Rytov approximation and the modified von Karman spectrum model of refractive-index fluctuations, the influence of atmospheric turbulence on a single-photon acquisition probability in satellite-ground quantum communications was studied. Theoretical models of the single-photon acquisition probability in an uplink channel and a downlink channel were established. The single-photon acquisition probability was analyzed for laser links between a ground station and a satellite in a low earth orbit. The results show that, the single-photon acquisition probability in the uplink channel depends strongly on the refractive-index structure parameter at the ground $C_n^2(0)$ and decreases as $C_n^2(0)$ increasing. However, in the downlink channel the single-photon acquisition probability does not depend on $C_n^2(0)$, that is, atmospheric turbulence has little influence on it.

Key words: Single-photon acquisition probability; Laguerre-Gaussian beams; Refractive-index structure parameter; Satellite-ground quantum communications; Atmospheric turbulence

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0 Introduction

As one of the most promising technologies in the fast developing field of quantum information, free-

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space quantum key distribution[1-3] has been successfully demonstrated by distributing single-photons[4-6] and entangled photon pairs[7-9]. Terrestrial free-space quantum communication based on current technology cannot surpass on the order of 100 km. In order to implement global-scale quantum communication networking, orbiting satellite can be used as a secure relay station[10-12]. Feasibilities of satellite-based quantum communications have also been studied[13-15].

Single-photon acquisition technology is one of the most important technologies in satellite-ground quantum communications. According to the uncertainty principle of quantum mechanics and the corpuscular property of photon, it is impossible to accurately acquire a single-photon on transverse section of laser beam propagation, and only the probability that the single-photon is acquired can be obtained. In order to acquire the single-photon in satellite-ground quantum communications, an optical link need first be established by pointing-acquisition-tracking technology in satellite laser communications. Besides, the influence of atmospheric turbulence on single-photon acquisition must be taken into consideration[16-17].

Erdmann[18] proposed a symmetry idea that if a conventional laser signal is sent to a satellite with appropriate telescopic optics at each end, then the net system light gathering efficiency through atmosphere equals to the probability for a single-photon to reach the detector in the same optical system. Zhang *et al*[19-21] developed this idea and established theoretical models of single-photon acquisition probability based on the fundamental-mode Gaussian beam, Hermite-Gaussian beams and Laguerre-Gaussian beams. Bi *et al*[22] established a physical model of time filtering of single-photon acquisition based on Hermite-Gaussian beams. In the theoretical models established by Zhang *et al* and Bi *et al*, the influence of atmospheric turbulence on single-photon acquisition has not been considered. Recently, Zhang *et al*[23] studied the effect of turbulence on detection probability of single-photon propagation in atmospheric communication channel based on the fundamental-mode Gaussian beam.

In this paper, based on the Rytov approximation and the modified von Karman spectrum model of refractive-index fluctuations, the influence of atmospheric turbulence on a single-photon acquisition probability in satellite-ground quantum communications is studied theoretically. Expressions of the single-photon acquisition probability based on a TEM_{01} mode Laguerre-Gaussian beam are derived in an uplink channel and a downlink channel. The single-photon acquisition probability is analyzed numerically for laser links between a ground station and a satellite in a low earth orbit. Finally, the conclusions are given.

1 Theoretical models of single-photon acquisition probability

In satellite-ground quantum communications, highly attenuated laser pulses based on Gaussian beams are used as single-photon sources. It is assumed that a laser beam has an initial Gaussian temporal pulse shape and a TEM_{01} mode Laguerre-Gaussian spatial distribution. According to quantum electrodynamics, the photon wave function is equivalent to the optical mode. Based on the Rytov approximation and the approximation of spatiotemporal coherence function of laser field as the product of spatial and temporal coherence function[24], the photon wave function $\psi(r,\varphi,z,t)$ based on Laguerre-Gaussian beams can be expressed as

$$\psi(r,\varphi,z,t)=\psi_{mn}(r,\varphi,z)T(t)=C_{mn}\frac{1}{w(z)}\left[\frac{\sqrt{2}r}{w(z)}\right]^m\times L_n^m\left[\frac{2r^2}{w^2(z)}\right]\exp\left[-\frac{r^2}{w^2(z)}\right]\exp[\tilde{\psi}(r,\varphi,z)]\times$$
$$\exp\left\{-\mathrm{i}\left[k\left(z+\frac{r^2}{2R(z)}\right)-(1+m+2n)\arctan\frac{z}{z_R}\right]\right\}\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix}\sqrt{\frac{1}{T_0}\sqrt{\frac{2}{\pi}}}\exp\left[-\left(\frac{t}{T_0}\right)^2-\mathrm{i}\omega t\right] \quad (1)$$

where

$$\psi_{mn}(r,\varphi,z)=C_{mn}\frac{1}{w(z)}\left[\frac{\sqrt{2}r}{w(z)}\right]^m\times L_n^m\left[\frac{2r^2}{w^2(z)}\right]\exp\left[-\frac{r^2}{w^2(z)}\right]\exp[\tilde{\psi}(r,\varphi,z)]\cdot$$
$$\exp\left\{-\mathrm{i}\left[k\left(z+\frac{r^2}{2R(z)}\right)-(1+m+2n)\arctan\frac{z}{z_R}\right]\right\}\times\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix} \quad (2)$$

$$T(t)=\sqrt{\frac{1}{T_0}\sqrt{\frac{2}{\pi}}}\exp\left[-\left(\frac{t}{T_0}\right)^2-\mathrm{i}\omega t\right] \quad (3)$$

and $k=2\pi/\lambda$ is the wave number, λ is the wavelength, T_0 is the pulse time duration, ω is the angular frequency, $w(z)=w_0\sqrt{1+z^2/z_R^2}$ is the beam radius at the distance z, w_0 is the waist radius of Gaussian *beams*, $z_R=\pi w_0^2/\lambda$ is the Rayleigh length, $R(z)=z(1+z_R^2/z^2)$ is the phase front radius of curvature, $\tilde{\psi}(r,\varphi,z)=\tilde{\psi}_1(r,\varphi,z)+\tilde{\psi}_2(r,\varphi,z)+\cdots$ is the complex phase perturbation of Rytov approximation, $L_n^m[2r^2/w^2(z)]$ is the associated Laguerre function of degree n and C_{nm} is the normalized constant.

According to the statistical annotation of the wave function, the probability density function of the photon $\rho(r,\varphi,z,t)$ is given by

$$\rho(r,\varphi,z,t)=\langle|\psi(r,\varphi,z,t)|^2\rangle=\langle|\psi_{nm}(r,\varphi,z)|^2\rangle\langle|T(t)|^2\rangle=\frac{C_{nm}^2}{T_0}\sqrt{\frac{2}{\pi}}\frac{1}{w^2(z)}\left[\frac{\sqrt{2}r}{w(z)}\right]^{2m}\left\{L_n^m\left[\frac{2r^2}{w^2(z)}\right]\right\}^2\times$$
$$\exp\left[-\frac{2r^2}{w^2(z)}\right]\langle\exp[\tilde{\psi}(r,\varphi,z)+\tilde{\psi}^*(r,\varphi,z)]\rangle\times\begin{Bmatrix}\cos m\varphi\\\sin m\varphi\end{Bmatrix}^2\langle\exp\left[-2\left(\frac{t}{T_0}\right)^2\right]\rangle \tag{4}$$

Based on the modified von Kaman spectrum of refractive-index fluctuations,

$$\frac{1}{w^2(z)}\left[\frac{\sqrt{2}r}{w(z)}\right]^{2m}\left\{L_n^m\left[\frac{2r^2}{w^2(z)}\right]\right\}^2\exp\left[-\frac{2r^2}{w^2(z)}\right]\times\langle\exp[\tilde{\psi}(r,\varphi,z)+\tilde{\psi}^*(r,\varphi,z)]\rangle=$$
$$\frac{1}{w_{LT}^2(z)}\left[\frac{\sqrt{2}r}{w_{LT}(z)}\right]^{2m}\left\{L_n^m\left[\frac{2r^2}{w_{LT}^2(z)}\right]\right\}^2\times\exp\left[-\frac{2r^2}{w_{LT}^2(z)}\right] \tag{5}$$

where $w_{LT}(z)$ is the long-term beam radius in atmospheric turbulence.

Applying the usual von Karman spectrum of refractive-index fluctuations, the temporal averaging of pulse in weak turbulence can be estimated by

$$\langle\exp\left[-2\left(\frac{t}{T_0}\right)^2\right]\rangle=\frac{T_0}{T_1}\exp\left[-2\left(\frac{t-\frac{z}{c}}{T_1}\right)^2\right] \tag{6}$$

where $T_1=\sqrt{T_0^2+26.31c^{-2}L_0^{5/3}\int_0^z C_n^2(z')\mathrm{d}z'}$ is the temporal broadening, L_0 is the outer scale of turbulence, $C_n^2(z)$ is the refractive-index structure parameter and c is the speed of light in atmosphere.

By Eqs. (4) , (5) and (6), the probability density function of the photon is given by

$$\rho(r,\varphi,z,t)=\frac{C_{nm}^2}{T_1}\sqrt{\frac{2}{\pi}}\frac{1}{w_{LT}^2(z)}\left[\frac{\sqrt{2}r}{w_{LT}(z)}\right]^{2m}\times\left\{L_n^m\left[\frac{2r^2}{w_{LT}^2(z)}\right]\right\}^2\exp\left[-\frac{2r^2}{w_{LT}^2(z)}\right]\times$$
$$\begin{Bmatrix}\cos m\varphi\\\sin m\varphi\end{Bmatrix}^2\exp\left[-2\left(\frac{t-z/c}{T_1}\right)^2\right] \tag{7}$$

The single-photon acquisition probability $P(z)$ is given by

$$P(z)=\int_0^{d/2}\int_0^{2\pi}\int_{-T_0/2+z/c}^{T_0/2+z/c}\rho(r,\varphi,z,t)r\mathrm{d}r\mathrm{d}\varphi\mathrm{d}t=\frac{C_{nm}^2}{T_1}\sqrt{\frac{2}{\pi}}\frac{1}{w_{LT}^2(z)}\int_0^{d/2}\int_0^{2\pi}\int_{-T_0/2+z/c}^{T_0/2+z/c}\left[\frac{\sqrt{2}r}{w_{LT}(z)}\right]^{2m}\times$$
$$\left\{L_n^m\left[\frac{2r^2}{w_{LT}^2(z)}\right]\right\}^2\exp\left[-\frac{2r^2}{w_{LT}^2(z)}\right]\begin{Bmatrix}\cos m\varphi\\\sin m\varphi\end{Bmatrix}^2\times\exp\left[-2\left(\frac{t-\frac{z}{c}}{T_1}\right)^2\right]r\mathrm{d}r\mathrm{d}\varphi\mathrm{d}t \tag{8}$$

where d is the diameter of detector.

For the TEM_{01} mode Laguerre-Gaussian beam, $C_{01}=\sqrt{2/\pi}$, $L_1^0(x)=1-x$. Therefore, the probability density function of the photon is

$$\rho(r,\varphi,z,t)=\frac{2}{\pi T_1}\sqrt{\frac{2}{\pi}}\frac{1}{w_{LT}^2(z)}\left[1-\frac{2r^2}{w_{LT}^2(z)}\right]^2\times\exp\left[-\frac{2r^2}{w_{LT}^2(z)}\right]\exp\left[-2\left(\frac{t-\frac{z}{c}}{T_1}\right)^2\right] \tag{9}$$

The single-photon acquisition probability is

$$P(z)=\frac{1}{T_1}\sqrt{\frac{2}{\pi}}\left\{1-\exp\left[-\frac{d^2}{2w_{LT}^2(z)}\right]-\frac{d^4}{4w_{LT}^2(z)}\exp\left[-\frac{d^2}{2w_{LT}^2(z)}\right]\right\}\times\int_{-T_0/2+z/c}^{T_0/2+z/c}\exp\left[-2\left(\frac{t-\frac{z}{c}}{T_1}\right)^2\right]\mathrm{d}t \tag{10}$$

2 Numerical simulation for satellite-ground uplink and downlink channels

For quantum communications between a ground station and a satellite in a low earth orbit, the influence of atmospheric turbulence on single-photon acquisition needs to be considered. In Eq. (10), $w_{LT}(z)$ is the long-term beam radius in atmospheric turbulence. For satellite-ground downlink channel, the long-term beam radius is

$$w_{LT}(z)\cong w(z) \tag{11}$$

For satellite-ground uplink channel, the long-term beam radius is

$$w_{\mathrm{LT}}(z)\cong\begin{cases} w(z)\sqrt{1+\left(\dfrac{1.35w_0}{\tilde{\rho}_0}\right)^{5/3}} & 0\leqslant\dfrac{1.35w_0}{\tilde{\rho}_0}<1 \\ w(z)\left[1+\left(\dfrac{1.35w_0}{\tilde{\rho}_0}\right)^{5/3}\right]^{3/5} & 0\leqslant\dfrac{1.35w_0}{\tilde{\rho}_0}<\infty \end{cases} \tag{12}$$

where $\tilde{\rho}_0^2=\rho_0^2[1-0.715\kappa_0^{1/3}]^{-1}$, $\rho_0^2=\left[1.45k^2\int_0^z C_n^2(z')\left(1-\dfrac{z'}{z}\right)^{5/3}\mathrm{d}z'\right]^{-6/5}$ is the long-term lateral coherence length[25], $\kappa_0=2\pi/L_0$.

According to the Hufnagel-Valley (H-V) model, the refractive-index structure parameter is given by

$$C_n^2(h)=0.005\,94\left(\frac{v}{27}\right)^2(10^{-5}h)^{10}\exp\left(-\frac{h}{1000}\right)+2.7\times10^{-16}\exp\left(-\frac{h}{1500}\right)+C_n^2(0)\exp\left(-\frac{h}{100}\right) \tag{13}$$

where h is the altitude, v is the wind speed and $C_n^2(0)$ is the refractive-index structure parameter at the ground.

Fig. 1 shows the single-photon acquisition probability in satellite-ground uplink channel for different link distance and refractive-index structure parameter at the ground. It can be seen from Fig. 1 that the single-photon acquisition probability in satellite- ground uplink channel depends strongly on the refractive-index structure parameter at the ground $C_n^2(0)$ and decreases as $C_n^2(0)$ increasing.

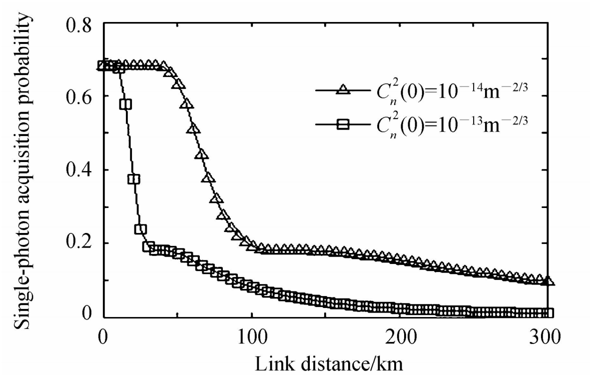

Fig. 1 Single-photon acquisition probability in satellite- ground uplink channel for different link distance and refractive-index structure parameter at the ground with $\lambda=850$ nm, $w_0=0.05$ m, $T_0=10^{-6}$ s, $d=1$ m, $L_0=5$ m, $v=21$ m/s

Fig. 2 shows the single-photon acquisition probability in satellite-ground downlink channel for different link distance. It can be seen from Fig. 2 that the single-photon acquisition probability only depends on the link distance effect due to the beam spreading and decreases as the link distance increases. In satellite-ground downlink channel, the single-photon acquisition probability does not depend on $C_n^2(0)$ and atmospheric turbulence has little influence on it. Therefore, the downlink channel should be used for practical quantum communications between the satellite in the low earth orbit and the ground station, that is, a transmitter is on the satellite in the low earth orbit and a receiver is in the ground station, because the atmospheric turbulence effect can be ignored.

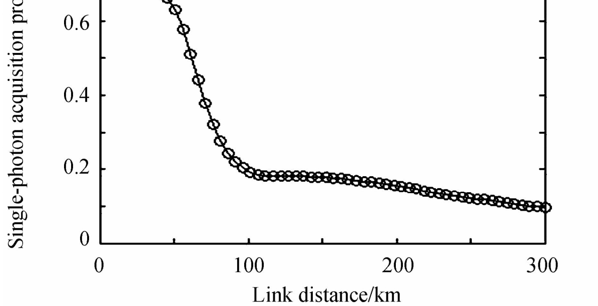

Fig. 2 Single-photon acquisition probability in satellite- ground downlink channel for different link distance with $\lambda=850$ nm, $w_0=0.05$ m, $T_0=10^{-6}$ s, $d=1$ m, $L_0=5$ m, $v=21$ m/s.

3 Conclusion

In summary, based on the Rytov approximation and the modified von Karman spectrum model of refractive-index fluctuations, theoretical models of a single-photon acquisition probability in satellite-ground uplink channel and downlink channel are established. Expressions of the single-photon acquisition probability based on a TEM_{01} mode Laguerre-Gaussian beam are given. The single-photon acquisition probability is analyzed numerically for laser links between a ground station and a satellite in a low earth orbit. Theoretical analysis and numerical simulation show that it is feasible for satellite-ground quantum communications.

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