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# 复杂脉冲单发频域干涉测量

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摘 要:以傅里叶变换极限脉冲作参考脉冲,利用单次测量分析法对复杂的皮秒脉冲进行测量,用窗口 傅里叶变换代替傅里叶变换对干涉条纹进行时间-频率分析,直接提取出复杂脉冲的啁啾特性或光谱成 分,将光谱图中的功率密度 S(w, t) 沿 w 轴求和重建脉冲的时域包络.分别用该方法和传统频域干涉测 量法测量一个线性啁啾脉冲和一个复杂脉冲.结果表明,该方法可实现复杂整形脉冲的实时测量,且时 间分辨率为 70 fs.

关键词:超快光学;超短脉冲测量;频域干涉测量法;窗口傅里叶变换;线性啁啾脉冲 中图分类号:TN247 文献标识码:A 文章编号:1004-4213(2016)06-0612004-5

# Analyzing Spectral Interferometer Fringe Using a Short Time Fourier Transform for Single Shot Complex Pulses Measurement

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Abstract: A single-shot measurement and analysis technique with a Fourier transform limited pulse as a reference pulse was presented, which can be used for measuring the complex picoseconds pulses. Substituting the Fourier transform with a short time Fourier transform, a time-frequency analysis of the spectral fringe was achieved to directly extract the chirp characteristics or composition of complex pulse. Furthermore, the temporal envelope of pulse was reconstructed from the spectrogram by summing the power density  $S(\omega,t)$  along the spectrum  $\omega$  axis. To verify the utility of this method, this method and the traditional spectral interferometry was used to measure a linear chirped pulse and a complex pulse respectively. The results show that this method can be used to measure the characterization of complex shaped pulses in real-time, and with a temporal resolution of 70 fs.

Key words: Ultrafast optics; Measurements of ultrashort pulses; Spectral interferometry; Short time Fourier transform; Linear chirped pulse

OCIS Codes: 120.2650; 120.3180; 320.0320; 320.7100; 070.0070; 320.1590

# 0 Introduction

The chirped or arbitrarily shaped pulses are widely used in these fields such as laser driven shock dynamics<sup>[1]</sup>, ultrafast photography<sup>[2]</sup>, multidimensional spectroscopy<sup>[3]</sup>, biological imaging<sup>[4]</sup>, compression of optical pulses<sup>[5]</sup> and optical communications<sup>[6]</sup>. In these applications, the temporal and spectral qualities of the pulses must be accurately controlled or tailored to meet the especial demands. So, in these adjusting processes, the real-time characterization of these pulses is very important for the feedback control.

At present, the two widely used characterization techniques for ultra-short pulses are Frequency-

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Resolved Optical Grating (FROG)<sup>[7]</sup> and Spectral Phase Interferometry of Direct Electric Field Reconstruction (SPIDER)<sup>[8]</sup>. They cannot measure the complex or high Time-Bandwidth Product (TBP) pulses. Cross-correlation FROG<sup>[9]</sup> (XFROG) is suitable for the characterization of relatively long pulses, but it requires nonlinear optical materials with large nonlinearities, fast response time and relatively high intensity laser pulses. Grating-Eliminated No-Nonsense Observation of Ultrafast Incident Laser Light e-fields (GRENOUILLE) is a simplified single-shot Second-Harmonic Generation Frequency-Resolved Optical Grating (SHG-FORG) and a really single shot spectrum resolved autocorrelation<sup>[10]</sup>. However, it can only measure the simple pulses (TBP < 10). For the relatively long pulses, a combination of a spectrometer with a streak camera could measure the timewavelength relationship<sup>[11]</sup>. But the result is easily affected by the sweep nonlinearity of the streak camera. A frequency domain interference method<sup>[12]</sup> can measure the chirp characteristics of a linearly picosecond chirped pulse without nonlinear optical materials and pulse intensity restrictions. But it requires scanning the delay time between the reference pulse and the test pulse, which leads to a low work efficiency.

In practice, the chirped or shaped pulses commonly start with a known Fourier transform limited pulse. Using the known pulse as a reference pulse, Fourier Transform Spectral Interferometry (FTSI)<sup>[13]</sup> can be expediently used to provide a measurement of both the amplitude  $A(\omega)$  and the phase  $\phi(\omega)$ , from which the electric field E(t) can use the Fourier transform to gain  $A(\omega)$  and  $\phi(\omega)$  and reconstruct the temporal waveform by Inverse Fast Fourier Transform (IFFT). The more complex the pulse is, the more irregular  $\phi(\omega)$  gets, and the bigger the noise of the spectrum phase from Fourier transform extraction becomes. After IFFT, the time domain reconstruction error is inevitable.

In this paper, we still use the spectral fringes as a signal carrier to maintain the ability of spatial resolution. Firstly, the Fourier transform was substituted by a short time Fourier transform to make a time-frequency analysis of the spectral interferogram. Compared with the standard FTSI, the spectrograms of Short Time Fourier Transform Spectral Interferometry (STFTSI) can intuitively give some key features such as the presence of chirp or multi-pulses, without resorting to the IFFT algorithm. Especially, it can straightforwardly extract the chirp characteristics or composition of complex pulses, and greatly reduce the noise. Secondly, by using the Fourier transform limited pulse as a reference pulse, we apply the Fourier transform to spectral fringe to directly get the temporal envelop of test pulse, without retrieving the  $A(\omega)$  and  $\phi(\omega)$  at first. In this way, this method reduces the spectral phase error effects in the E(t) retrieving process.

## 1 Theory

Ι

### 1.1 Spectral interferometry

Spectral interferometry<sup>[13]</sup> is a linear-optical technique for measuring the spectral intensity and phase of a pulse when a characterized reference pulse is available. The unknown pulse E(t) and the reference pulse  $E_{\rm ref}(t)$  are recombined into a spectrometer with a delay  $\tau$ . The signal exhibits fringes in spectral domain, due to the interference between both pulses. The spectrum intensity is related to the square of the sum of the Fourier transform of the two fields <sup>[13]</sup>

$$(\omega) = |E(\omega) + E_{\text{ref}}(\omega) e^{i\omega\tau}|^2 = A^2(\omega) + A^2_{\text{ref}}(\omega) +$$

 $2A(\omega)A_{\rm ref}(\omega)\cos[(\phi(\omega)-\phi_{\rm ref}(\omega)-\omega\tau)]$  (1) where the cosine term which is responsible for the fringes contains two contributions. The main one,  $\omega\tau$ , sets the overall fringe carrier frequency. The second one,  $\phi(\omega) - \phi_{\rm ref}(\omega)$ , locally modulates the fringe frequency along with the varying of spectrum. Expanding the spectral phase  $\phi(\omega)$  as a Taylor series [14]

$$\phi(\omega) = \phi_0 + \phi'_0 (\omega - \omega_0) + \frac{1}{2} \phi_0^{"} (\omega - \omega_0)^2 + \frac{1}{6} \phi_0^{"} (\omega - \omega_0)^3 + L$$
(2)

where the first order coefficient,  $\phi'_0$ , is the arrival time of the pulse. The second order coefficient,  $\phi'_0$ , is referred to as a linear chirp. If this term is present, the pulse's arrival time will vary linearly with the spectrum. The most commonly used algorithm for spectral intensity  $E(\omega)$  and phase  $\phi(\omega)$  reconstruction is the Fourier transform<sup>[16]</sup>. Once  $\phi(\omega)$  is obtained, the group delay is calculated by differencing  $\phi(\omega)$  with respect to the spectrum. Further, in order to attain the temporal intensity E(t) and phase  $\phi(t)$  of the pulse, invert Fourier transform is applied for  $E(w) \cdot$  $\exp[i\phi(\omega)]$ . In practice, there are always noises on the retrieved phase, resulting in the errors of  $d\phi/d\omega$ , E(t)and  $\phi(t)$ .

#### 1.2 Short time Fourier transform

We proposed a direct method for measuring the chirp or time characterization of pulses, for assumption that the reference pulse was a Fourier transform limited pulse. In Eq. 1, the spectrum  $\omega$ , not t, was a variable. Meantime, the phase  $\phi_{ref}(\omega)$  of the reference pulse was invariable and  $\omega \tau$  was linear. So, besides a time offset  $\tau$ , the fringe carrier frequency was identified as  $\phi(\omega)/d\omega$ . If we use a time-frequency tool such as a short time

Fourier transform to analyze the fringe carrier frequency, we could get the distribution of power density,  $S(\omega, t)$ , with the spectrum resolution (corresponding to time domain in traditional analysis) and temporal resolution (corresponding to frequency domain in traditional analysis) simultaneously. Compared with normal Fourier transform, a short time Fourier transform is to choose a time window with certain width and intercept signals section by section, and then apply normal Fourier transform to each section to acquire the frequency of the signal in it. The window Fourier transform to Eq. 1 can be expressed as

$$f(\omega,t) = \int I(w)g(w-w')e^{-2\pi ut} dw$$
(3)

where  $g(\omega, t)$  represents the window function, as which Hamming window is usually selected. The fringe frequency here is related to the chirped characteristics. For each spectrum  $\omega_i$ , a certain  $t_i$  corresponding to the maximum modulus  $S(\omega_i, t_i)$  was found. The certain  $t_i$ was corresponding to the fringe carrier frequency, i.e. the  $\phi(\omega)/d\omega$ , which indicated the arrival time of each spectrum component respect to the reference pulse or the pulse chirp characteristics indicated in Eq. 2. The modulus S ( $\omega_i, t_i$ ) was proportional to amplitude of unknown pulse at time  $t_i$ . When an appropriate spectrum window was adopted, we could get a temporal distribution preview of the pulse by summing the power density  $S(\omega_i, t_i)$  along the spectrum  $\omega$  axis. According to Heisenberg's uncertainty principle  $\Delta\omega\Delta t \ge$ 1/2, there was a compromise between spectrum window and time resolution<sup>[15]</sup>. In order to obtain a higher time resolution, a wider spectrum window must be chosen. Furthermore, owing to the advantage of analyzing multiple frequency component of signal, STFTSI could simultaneously analyze the chirp characteristics of multiple pulses in single shot. There was a potential capability for complicated pulse measurement. We could use a filtering window to separate individual pulse and to independently retrieve each pulse temporal waveform at the sacrifice of certain temporal resolution.

We also proposed a method for directly retrieving the temporal profiles of the unknown pulse from the spectrogram. The amplitude  $A_{ref}(\omega)$  and phase  $\phi_{ref}(\omega)$ of the reference pulse could be measured using FROG, so Eq. 1 divided out by  $A_{ref}(\omega)$  can be rewritten as

$$I_{\text{new}}(\omega) = A_{\text{ref}}(\omega) + A^2(\omega) / A_{\text{ref}}(\omega) + 2A(\omega)$$

$$\cos\left[\left(\phi(\omega) - \phi_{\text{ref}}(\omega) - \omega\tau\right)\right] \tag{4}$$

And, the unknown pulse actually was

$$\widetilde{E}(\omega) = A(\omega) \cos[\phi(\omega)]$$
(5)

Since the amplitude  $A(\omega)$ ,  $A_{ref}(\omega)$ ,  $A(\omega)/A(\omega)$ varied slowly along with the carrier frequency  $\tau$ , the Fourier spectrum of the third term in Eq. 4 is separated from unwanted background spectrum by the carrier frequency  $\tau$ . Here, a linear phase term  $\omega \tau$  played an important role to avoid the test pulse overlap with background in time domain<sup>[16]</sup>. The carrier frequency or the delay time  $\tau$  was commonly chosen from the range of 5 ps $\sim$ 15 ps. We considered that the Fourier transform of Eq. 4 was to some extent equivalent to the Fourier transform of Eq. 5, except for the time shift due to a linear phase term  $\omega \tau$  in the spectral domain and the temporal shift of the pulse envelope with respect to the underlying carrier oscillation due to the constant phase  $\phi_{ref}(\omega)$ . The absolute temporal phase is not important if the pulse envelope A(t) does not significantly vary within one oscillation period. Accordingly, the envelope of  $E_{\text{new}}(t)$  resulted from the Fourier transform of  $I_{\text{new}}(\omega)$  is identified as E(t). Consequently, we could retrieve the temporal envelope of the unknown pulse by directly applying Fourier transform to the interferogram.

# 2 Experiment and results

## 2.1 Chirped pulse

To demonstrate this technique, we performed experiments using a femtosecond fiber laser. The laser were centered at 790 nm, with a FWHM(Full-Width at Half-Maximum) bandwidth of 15 nm and a pulse width of <120 fs, at a repetition rate of 76 MHz. Fig. 1(a) shows the experimental setup for measuring the chirp characteristics of a linearly chirped pulse. A portion of fs pulse was used as a reference pulse; another portion of fs pulse was temporally stretched to about 10 ps by a pair of gratings. The spectral interferogram of these two pulses were recorded by an imaging spectrometer. In this experiment, our spectrometer had a spectral range of about 30 nm and a spectral resolution of 0.05 nm.

Fig. 1 (b) shows these spectral interferograms of a fs pulse and the corresponding chirped pulse at four different time delays ( $\tau_1 = 8.1 \text{ ps}, \tau_2 = 10.8 \text{ ps}, \tau_3 =$ 13.5 ps,  $\tau_4 = 16.2$  ps), which was achieved by moving M1. We used STFT to analyze these spectral interferogram. Fig. 1(c) shows an example of STFT spectrogram. The ridge of STFT modulus in Fig. 1 (c) is exactly the group delay with respect to  $\omega$ . The  $\omega$ -t relationships at different time delays were deduced from the ridges and plotted in Fig. 1 (d). Besides some offset resulted from different delays  $\tau$ , these data were coincident with each other. Overlapping these data in Fig. 1(e), we could find that the maximum error was less than 70fs, except for some points at end. Using the method mentioned above, we reconstructed the temporal waveform of the unknown pulse. Comparing our retrieved temporal waveform to that of the conventional FTSI method in Fig. 1(f), the agreement between the two independently reconstruction confirms the validity of our method. The temporal measure range at present is about 35 ps, which is determined by the spectrometer resolution of 0.05 nm and the pixel spectrum of 0.19 nm.



Fig. 1 The measure results of a chirped pulse

#### 2.2 Complex pulse

Our next experiment highlighted the measuring and analyzing the ability for multiple pulses at single shot experiment. We measured a complicated pulse generated by a Michelson interferometer and a pair of grating to produce a fs pulse and two chirped pulses. The experimental setup is shown in Fig. 2 (a). Fig. 2 (b) is a typical interferogram and fringe. The fringe curve shows visual modulations which indicate the existence of several pulses with different relative time delays. STFT was performed with an optimal window width (relative short), and the spectrogram is shown in Fig. 2 (c). In the spectrogram, every single pulse spectrum component and pulse duration can be easily seen. We use some filtering windows to separately filter each pulse from spectrogram and carefully retrieve



Fig. 2 The measure result of a complex pulse

the temporal profile of each pulse, which is illustrated in Fig. 2 (d). Each pulse temporal profile is in agreement with experimental setup. The fs pulse duration is about 150 fs and the two chirped pulse duration is about 10 ps. The delay time of these two chirped pulses is about 1.75 ps, which is consistent with the arrival time of the fs pulse. We performed STFT with a longer window and summed the power density  $S(\omega_i, t_i)$  along the spectrum  $\omega$  axis to get a whole temporal profile of complicated pulse in detail. The retrieved waveform is shown in Fig. 2 (e). The figure demonstrates a phenomenon known as chirped pulse beating, which corresponds to a constant beat frequency between these two chirped pulses at each point in time. The comparison between our method and FTSI method indicated the validity of our method again. There is a distorted spike due to the effects of the spectral phase in FTSI method.

# 3 Conclusion

In summary, we have used a short time Fourier transform to analyze the spectral interferogram for the complete characterization of complex pulses up to 35 ps duration with 70 fs resolution. A precondition for this method is that we could use a Fourier transform limited pulse as a reference pulse. We substituted the Fourier transform by a short time Fourier transforms to make a time-frequency analysis of the spectral interferogram. From the STFT spectrogram, we directly extract the chirp characteristics of simple pulse or time-spectrum composition of complex pulse, without retrieving the spectrum amplitude and phase. Furthermore, we could achieve one-step reconstruction of the pulse temporal waveform by directly performing Fourier transform or short time Fourier transform on the interference fringe. We believe that it is a promising approach for measuring and analyzing the complex pulses with picoseconds duration on a single shot.

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