doi:10.3788/gzxb20164505.0501003

海洋湍流中的量子高斯-谢尔光束偏振模型

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摘　要：研究湍流对海水光通信信道中传输的量子高斯-谢尔光束偏振度的影响.运用量子化场的惠更斯-菲涅尔原理，构建湍流海水中量子化高斯谢尔光束的产生和湮灭算符. 基于海水折射率的空间功率谱，导出了量子高斯-谢尔光束的偏振度.数值结果表明：在参数给定条件下，温度与盐度的比值从－4.5 变化到－0.5 时，偏振度从 0.75 下降到 0.20；接收光子数从 20 提高到 50 时，偏振度从 0.91 提高到 0.96；光源横向尺寸从 0.02 m 增大到 0.12 m 时，偏振度从 0.82 提高到 0.97；温度起伏对偏振度的影响高于盐度起伏的影响；提高发射光子数和增大发射孔径是扼制湍流干扰的有效方法.

关键词：偏振起伏；高斯-谢尔模型；海洋湍流；产生算符与湮灭算符；数值模拟

中图分类号：O439　　　　**文献标识码**：A　　　　**文章编号**：1004-4213(2016)05-0501003-7

Polarization Model of Quantized Gaussian Schell-model Fields in an Oceanic Turbulence

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Abstract: The effects of turbulence on the performance of the polarization degree of quantized Gaussian-Schell model beams propagating through oceanic optical communication channel were studied. The annihilation and creation operator of the linearly polarization quantized Gaussian Schell-beam were structured based on Huygens-fresnel principle of quantized field in oceanic water. An expression for the polarization properties of Gaussian Schell-model quantization fields propagating through the oceanic channel is derived based on the spatial power spectrum of the refractive index of ocean water. The numerical experimental results show that under given parameters, as the ratio of temperature and salinity contributions to the refractive index spectrum varies from －4.5 to －0.5, the polarization degree decreases from 0.75 to 0.21; when the number of receiving photon increases from 20 to 50, the polarization degree also increases from 0.91 to 0.96; when the source's transverse size varies from 0.02 m to 0.12 m, the polarization degree changes from 0.82 to 0.97; the effects of temperature-induced polarization decrease is surpassing the effects of salinity fluctuations; increasing the launch photon number and the radius of the aperture are an effective way to mitigate turbulent disturbance.

Key words: Polarization fluctuation; Gaussian Schell-mode; Oceanic turbulence; Creation and annihilation operator; Numerical simulation

OCIS Codes: 010.3310; 010.4450; 010.4455; 010.7060 ; 010.7340

Foundation item: The Natural Science Foundation of Jiangsu Province of China (No. BK20140128).

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Received: Nov. 30, 2015; **Accepted**: Feb. 25, 2016

0 Introduction

Underwater or atmospheric optical wireless communication has been proposed as an innovative and good alternative communication[1-3]. The effects of oceanic turbulence on the beam propagation in sea water have been discussed. Such as, Wei Lu, et al.[4] studied the influence of temperature and salinity fluctuations of turbulence on propagation behaviour of partially coherent beams, and pointed out that partially coherent beams exhibit robust turbulence resistance for the rate of dissipation of the mean-squared temperature being smaller. Y. Ata and Y. Baykal[5] investigated the features of the Wave Structure Function (WSF) of spherical wave in turbulent water and found that as the rate of the dissipation of turbulent kinetic energy decreases, wave structure function increases. Some authors revealed the scintillation index of plane, spherical[6-7] and Gaussian[6] light waves propagating in the clear-water weakly turbulent ocean. It is found that strong oceanic turbulence can occur at distances as short as several meters. Y. Ata and Y. Baykal[8] investigated the absolute field correlation of the spherical wave in an underwater turbulence at the receiver plane by using the extended Huygens-Fresnel principle. M. Tang and D. Zhao[9] investigated the effects of astigmatism on the spectral density and the spectral degree of polarization of electromagnetic nonuniformly correlated beam with astigmatic aberration, propagating in isotropic and homogeneous oceanic turbulence and shown that the polarization at the intensity centeris very sensitive to the astigmatism index, as well as to the source coherence length, and the turbulence parameters. O. Korotkova and N. Farwell[10] analyzed the changes in the second-order ingle-point statistics of a typical random electromagnetic beam propagating in oceanic turbulence and indicated that the polarization of the propagating beam is very sensitive to slight changes in the size and the correlations of the source, and also on the parameters of turbulence. H. Zhang and W. Fu[11] studied the changes in polarization properties of partially polarized, partially coherent vectorial cosh-Gaussian beams propagating in oceanic turbulence with the unified theory of coherence and polarization. Y. Huang, et al.[12] studied the influence of oceanic turbulence on the beam quality parameters of partially coherent Hermite-Gaussian linear array beams. M. Yousefi, et al.[13] investigated the propagation behaviour of partially coherent divergent Gaussian beams through oceanic turbulence and found that beam′s statistical propagation behaviour is affected by both environmental and source parameters variations. Based on the extended Huygens-Fresnel principle and the unied theory of coherence and polarization, the behavior of the spectral composition of a typical stochastic beam[14] and stochastic anisotropic electromagnetic beams[15-17] in a turbulent ocean environ was revealed. E. Shchepakina, et al.[14] found that the source correlation induced spectral shift is compensated by turbulence at sufficiently large distances. M. Tang, et al.[15] shown that the on-axis spectrum is always blue-shifted along with the propagation distance, however, for the off-axis positions, red-blue spectral switch can be found , Y. Zhou, et al.[16-17] shown that different strengths of astigmatism have different effects on the spectral degree of polarization and the changes in the statistical properties of the anisotropic source on propagation are qualitatively different from those of the isotropic source. H. Gercekcioǧlu[18] evaluated bit error rate of focused Gaussian beams in weak oceanic turbulence. X. Yi, et al.[19] studied the aperture-averaged of scintillation under weak turbulence conditions and shown that the large-aperture receiver leads to a remarkable decrease of scintillation of and consequently asignificant improvement on the system performance. J. Xu et al. investigated the propagation of an electromagnetic non-uniformly correlated beam[20] and vortex beam[21] propagating through oceanic turbulence, and found that the intensity self-focusing will be affected under the influence of oceanic turbulence and the spectral degree of polarization of a stochastic electromagnetic vortex beam composed by isotropic sources on propagation in far zone will return to its value in the source plane. Y. Huang, et al.[22] studied the evolution behavior of Gaussian Schell-model vortex beams through oceanic turbulence and shown that the evolution behavior of coherent vortices and average intensity depends on the rate of dissipation of turbulent kinetic energy per unit mass of fluid, rate of dissipation of mean-square temperature, relative strength of temperature salinity fluctuations, and beam parameters including the spatial correlation length and topological charge of the beams, as well as the propagation distance. Polarization is one of the most important properties of light with a large number of applications, both in the quantum and classical domains. M. Yao, et al.[23] explored the effects of the anisotropic parameter on the spectral density, the spectral degree of coherence and on the spectral degree of polarization of the GSM beam. The closed-form expression for the mean square temporal width of Gaussian-beam-wave pulses passing horizontally through strong anisotropic atmospheric turbulence is developed based on the extended Huygens-Fresnel principle[24]. To the best of our knowledge, there is no report on the polarization fluctuations of quantization Gaussian Schell fields propagating in a turbulent ocean.

The paper develops a theoretical model for the polarization fluctuations of Gaussian Schell photon beams in an oceanic turbulence channel, which includes the effects of the source′s transverse size and the transverse coherent width of source on the degree of polarization of Gaussian Schell photon beams.

1 Degree of polarization for Gaussian Schell-model quantum field

The degree of polarization for linearly polarized quantum field of Gaussian Schell-model in turbulent ocean can be taken the form[25]

$$P=\frac{\sqrt{\langle\langle\xi|\hat{S}_0(\boldsymbol{\rho},\boldsymbol{q},z)|\xi\rangle\rangle_{o,s}}}{\sqrt{\langle\langle\xi|\hat{S}_0(\boldsymbol{\rho},\boldsymbol{q},z)|\xi\rangle\rangle_{o,s}+2}} \tag{1}$$

Where, $\hat{S}_0(\boldsymbol{\rho},\boldsymbol{q},z)=\hat{a}_x^\dagger(\boldsymbol{\rho},\boldsymbol{q},z)\hat{a}_x(\boldsymbol{\rho},\boldsymbol{q},z)$ is quantum Stokes operators of field propagating in ocean, $\hat{a}_x(\boldsymbol{\rho},\boldsymbol{q})$ and $\hat{a}_x^\dagger(\rho,q)$ are the photon annihilation and creation operators in the model $(\boldsymbol{q},\hat{\boldsymbol{x}})$ respectively, $\boldsymbol{q}$ is the momentum of photon and $\hat{\boldsymbol{x}}$ denotes the unit vector of the polarization direction of photon along x axis; $\boldsymbol{\rho}$ denotes transverse coordinate of the photon in the (x,y) plane at $z=z$ (shown in Fig. 1). The operators $\hat{a}_x$ ($\hat{a}_x^\dagger$) obey the communication relations $[\hat{a}_x,\hat{a}_y^\dagger]=\hat{\delta}_{xy}$, with x, $y=1$, 2. $\langle\xi|\hat{S}_0(\boldsymbol{\rho},\boldsymbol{q},z)|\xi\rangle=s_0(\boldsymbol{\rho},\boldsymbol{q},z)$ is the Stokes parameter, $|\xi\rangle$ is a coherent state of two polarization modes, $\langle\cdots\rangle_{o,s}=\langle\cdots\rangle_o\langle\cdots\rangle_s$, $\langle\cdots\rangle_o$ and $\langle\cdots\rangle_s$ denote average over the ensemble of the turbulent ocean and the source, respectively.

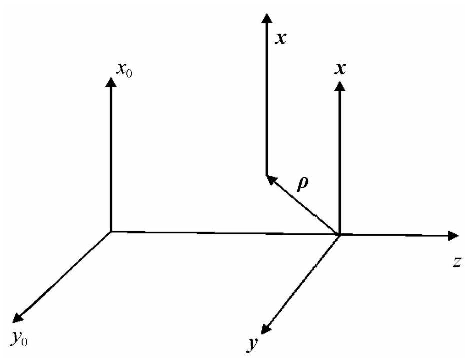

Fig. 1 The explanation of $\hat{x}$ and the (x,y) plane at $z=z$

The propagating quantum linearly polarized field in the turbulent and paraxial ocean channel can be expressed as[25]

$$\hat{E}_x^\dagger(\boldsymbol{\rho},z)=-\frac{\mathrm{i}k\mathrm{e}^{\mathrm{i}kz}}{2\pi z}\sqrt{\tau}\int\hat{E}_x^\dagger(\boldsymbol{\rho}',0)\cdot \exp\left[\frac{\mathrm{i}k}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2+\psi_o(\boldsymbol{\rho},\boldsymbol{\rho}',z)\right]\mathrm{d}^2\boldsymbol{\rho}' \tag{2}$$

where τ is transmittance of channel, $\boldsymbol{\rho}'$ denotes transverse coordinate of the photon at the source plane, the function $\psi_o(\boldsymbol{\rho},\boldsymbol{\rho}',z)=\chi_o(\boldsymbol{\rho},\boldsymbol{\rho}',z)+\mathrm{i}s_o(\boldsymbol{\rho},\boldsymbol{\rho}',z)$ describes the turbulent effects of ocean on the propagating spherical wave, $\chi_o(\boldsymbol{\rho},\boldsymbol{\rho}',z)$ and $s_o(\boldsymbol{\rho},\boldsymbol{\rho}',z)$ terms account for the stochastic log-amplitude and phase fluctuations, respectively, imposed by ocean turbulence. For the approximation of the passive ocean medium, the quantization field $\hat{E}_x(\boldsymbol{\rho}',0)$ at $z=0$ is given by [25]

$$\hat{E}_x^\dagger(\boldsymbol{\rho}',0)=(2\pi)^{-1}\int\hat{a}_x(\boldsymbol{q},\boldsymbol{\rho}')\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{\rho}'}\mathrm{d}^2\boldsymbol{q} \tag{3}$$

where $\hat{a}_x(\boldsymbol{q},\boldsymbol{\rho}')=\hat{a}_{0x}(\boldsymbol{q})u(\boldsymbol{\rho}')$ is the effective photon annihilation operator, $u(\boldsymbol{\rho}')$ is the transverse beam amplitude function for the beam modes in turbulent ocean, $\hat{n}(\boldsymbol{q})=\hat{a}_x^\dagger(\boldsymbol{q})\hat{a}_x(\boldsymbol{q})$ is effective number operator in ocean, $\hat{n}_0(\boldsymbol{q})=\hat{a}_{0x}^\dagger(\boldsymbol{q})\hat{a}_{0x}(\boldsymbol{q})$ is the initial number operator in source plane and $\hat{n}_0(\boldsymbol{q})|\xi\rangle=n_0(\boldsymbol{q})$.

Using Eq. (2) and Eq. (3), we can obtain the photon annihilation operator of quantum linearly polarized field in the turbulence of ocean channels

$$\hat{a}_x(\boldsymbol{q},\boldsymbol{\rho},z)=-\frac{\mathrm{i}k\mathrm{e}^{\mathrm{i}kz}}{2\pi z}\sqrt{\tau}\int\hat{a}_x(\boldsymbol{q},\boldsymbol{\rho}'')\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{\rho}''-\boldsymbol{\rho})}\cdot \exp\left[\frac{\mathrm{i}k}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}'')^2+\psi_o(\boldsymbol{\rho},\boldsymbol{\rho}'',z)\right]\mathrm{d}^2\boldsymbol{\rho}'' \tag{4}$$

The photon creation operator $\hat{a}_x^\dagger(\boldsymbol{q},\boldsymbol{\rho},z)$ of quantum field in ocean channel is expressed as

$$\hat{a}_x^\dagger(\boldsymbol{q},\boldsymbol{\rho},z)=\frac{\mathrm{i}k\mathrm{e}^{-\mathrm{i}kz}}{2\pi z}\sqrt{\tau}\int\hat{a}_x(\boldsymbol{q},\boldsymbol{\rho}')\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{\rho}'-\boldsymbol{\rho})}\cdot \exp\left[-\frac{\mathrm{i}k}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}')^2+\psi_o^*(\boldsymbol{\rho},\boldsymbol{\rho}',z)\right]\mathrm{d}^2\boldsymbol{\rho}' \tag{5}$$

Inparaxial optical channel $(\boldsymbol{k}\cdot(\boldsymbol{\rho}''-\boldsymbol{\rho}')\approx 0)$ and by Eq. (4) and Eq. (5), we can obtain the average photon number of turbulence ensemble for quantum light in ocean and in z plane

$$\langle s_0(\boldsymbol{q},\boldsymbol{\rho},z)\rangle_{o,s}=\langle\langle\xi|\hat{a}_x^\dagger(\boldsymbol{q},\boldsymbol{\rho},z)\hat{a}_x(\boldsymbol{q},\boldsymbol{\rho},z)|\xi\rangle\rangle_{o,s}= n\left(\frac{k}{2\pi z}\right)^2\iint\mathrm{d}^2\boldsymbol{\rho}'\mathrm{d}^2\boldsymbol{\rho}''\langle u^*(\boldsymbol{\rho}',\omega)u(\boldsymbol{\rho}'',\omega)\rangle_s\cdot \exp\left[\mathrm{i}k\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2-(\boldsymbol{\rho}-\boldsymbol{\rho}'')^2}{2z}\right]\cdot \langle\exp[\psi_o^*(\boldsymbol{\rho},\boldsymbol{\rho}',z)+\psi_o(\boldsymbol{\rho},\boldsymbol{\rho}'',z)]\rangle_o \tag{6}$$

where $n=\tau n_0$ is the number of transmissive received photon.

in Eq. (6), the angular bracket $\langle\exp[\psi_o^*(\boldsymbol{\rho},\boldsymbol{\rho}',z)+\psi_o(\boldsymbol{\rho},\boldsymbol{\rho}'',z)]\rangle_o$ can be approximated as[4]

$$\langle\exp[\psi_o^*(\boldsymbol{\rho},\boldsymbol{\rho}',z)+\psi_o(\boldsymbol{\rho},\boldsymbol{\rho}'',z)]\rangle_o= \exp(-4\pi^2k^2z)\exp\left[\int_0^1\int_0^\infty\mathrm{d}\kappa\mathrm{d}\xi\kappa\varphi_n(\kappa)\cdot \mathrm{J}_0(\kappa|\xi(\boldsymbol{\rho}'-\boldsymbol{\rho}'')|)\right]=\exp\left[-\frac{\pi^2k^2z}{3}(\rho'^2+\rho''^2- 2\boldsymbol{\rho}'\cdot\boldsymbol{\rho}'')\int_0^\infty\kappa^3\varphi_n(\kappa)\mathrm{d}\kappa\right]=\exp\left[-\frac{\pi^2k^2z}{3}(\rho'^2+\rho''^2-\right.$$

$$2\boldsymbol{\rho}'\cdot\boldsymbol{\rho}'')T(\eta,\varepsilon,\chi_{\mathrm{T}},\bar{\omega})\Big] \tag{7}$$

and

$$T(\eta,\varepsilon,\chi_{\mathrm{T}},\bar{\omega})=\int_0^{\infty}\kappa^3\varphi_n(\kappa)\,\mathrm{d}\kappa \tag{8}$$

where κ is the spatial frequency of turbulent fluctuations and $\varphi_n(\kappa)$ is the turbulence spectrum of ocean.

In our analysis we will employ the developed model[3-4] for clear-water oceanic turbulence which combines effects of temperature and salinity fluctuations in the water column. For the oceanic turbulence of eddy thermal diffusivity equaling to the diffusion of salt, the turbulence spectrum is represented as

$$\varphi_n(\kappa)=0.388C_m^2\kappa^{-11/3}\left[1+2.35(\kappa\eta)^{2/3}\right]\varphi(\kappa,\bar{\omega}) \tag{9}$$

where $C_m^2=10^{-8}\varepsilon^{-1/3}\chi_{\mathrm{T}}$ is the refractive- index structure parameter of ocean turbulence with units $\mathrm{m}^{-2/3}$ from $3.16\times10^{-18}\ \mathrm{m}^{-2/3}$ to $9.49\times10^{-7}\ \mathrm{m}^{-2/3}$, ε is the rate of dissipation of kinetic energy per unit mass of fluid ranging from $10^{-10}\ \mathrm{m}^2/\mathrm{s}^3$ to $10^{-1}\ \mathrm{m}^2/\mathrm{s}^3$, χ_T is the dissipation rate of temperature variance and has the range $10^{-10}\ \mathrm{K}^2/\mathrm{s}$ to $10^{-2}\ \mathrm{K}^2/\mathrm{s}$, $\eta=10^{-3}$ m is the Kolmogorov microscale (inner scale) of turbulence

$$\varphi(\kappa,\bar{\omega})=\left[\exp(-A_{\mathrm{T}}\delta)+\bar{\omega}^{-1}\exp(-A_{\mathrm{S}}\delta)-2\bar{\omega}^{-2}\bar{\omega}\exp(-A_{\mathrm{TS}}\delta)\right] \tag{10}$$

and $A_{\mathrm{T}}=1.863\times10^{-2}$, $A_{\mathrm{S}}=1.9\times10^{-4}$, $A_{\mathrm{TS}}=9.41\times10^{-3}$, $\delta=8.284(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2$, $\bar{\omega}$ is the relative strength of temperature and salinity fluctuations, which in the seawaters can vary in the interval$[-5;0]$, 0 value corresponding to the case when temperature-driven turbulence dominates, -5 value corresponding to the situation when salinity-driven turbulence prevails.

By Eq. (9), the integration in Eq. (8) can been written as

$$\begin{aligned}T={}&0.388C_{\mathrm{m}}^2\bar{\omega}^{-2}\int_0^{\infty}\kappa^{-2/3}\mathrm{d}\kappa\times\{\bar{\omega}^2\exp[-A_{\mathrm{T}}(8.284(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2)]+\exp[-A_{\mathrm{S}}(8.284(\kappa\eta)^{4/3}+\\&12.978(\kappa\eta)^2)]-2\bar{\omega}\exp[-A_{\mathrm{TS}}(8.284(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2)]\}+0.912\cdot C_{\mathrm{m}}^2\bar{\omega}^{-2}\eta^{2/3}\int_0^{\infty}\mathrm{d}\kappa\{\bar{\omega}^2\cdot\\&\exp[-A_{\mathrm{T}}(8.284(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2)]+\exp[-A_{\mathrm{S}}(8.284(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2)]-\\&2\bar{\omega}\exp[-A_{\mathrm{TS}}(8.284(\kappa\eta)^{4/3}+12.978(\kappa\eta)^2)]\}\end{aligned} \tag{11}$$

In the following, we will use the integral relatioonship[17] Eq. (12) and (13) to simplify Eq. (11)

$$\begin{aligned}\int_0^{\infty}\kappa^{2n-8/3}\exp(-\alpha\kappa^{4/3}-\beta\kappa^2)\,\mathrm{d}\kappa={}&\frac{1}{4}\beta^{-1/2-n}\Big[2R^{4/3}\Gamma\left(n-\frac{5}{6}\right)\cdot{}_2\mathrm{F}_2\left(\frac{n}{2}-\frac{5}{12},\frac{n}{2}+\frac{1}{12};\frac{1}{3},\frac{2}{3};-\frac{4\alpha^3}{27\beta^2}\right)-\\&2\alpha\beta^{3/2}\Gamma\left(n-\frac{1}{6}\right)\cdot{}_2F_2\left(\frac{n}{2}-\frac{1}{12},\frac{n}{2}+\frac{5}{12};\frac{2}{3},\frac{4}{3};-\frac{4\alpha^3}{27\beta^2}\right)+\alpha^2\Gamma\left(n+\frac{1}{2}\right)\cdot\\&{}_2\mathrm{F}_2\left(\frac{n}{2}+\frac{1}{4},\frac{n}{2}+\frac{3}{4};\frac{4}{3},\frac{5}{3};-\frac{4\alpha^3}{27\beta^2}\right)\Big]\end{aligned} \tag{12}$$

and

$$\begin{aligned}\int_0^{\infty}\kappa^{2n-2}\exp(-\alpha\kappa^{4/3}-\beta\kappa^2)\,\mathrm{d}\kappa={}&\frac{1}{4}\beta^{-5/6-n}\Big[2\beta^{4/3}\Gamma\left(n-\frac{1}{2}\right)\cdot{}_2\mathrm{F}_2\left(\frac{n}{2}-\frac{1}{4},\frac{n}{2}+\frac{1}{4};\frac{1}{3},\frac{2}{3};-\frac{4\alpha^3}{27\beta^2}\right)-\\&2\alpha\beta^{3/2}\Gamma\left(n+\frac{1}{6}\right)\cdot{}_2\mathrm{F}_2\left(\frac{n}{2}+\frac{1}{12},\frac{n}{2}+\frac{7}{12};\frac{2}{3},\frac{4}{3};-\frac{4\alpha^3}{27\beta^2}\right)+\alpha^2\Gamma\left(n+\frac{5}{6}\right)\cdot\\&{}_2F_2\left(\frac{n}{2}+\frac{5}{12},\frac{n}{2}+\frac{11}{12};\frac{4}{3},\frac{5}{3};-\frac{4\alpha^3}{27\beta^2}\right)\Big]\end{aligned} \tag{13}$$

where n is a integer, α and β are the constants, $\Gamma(\cdot)$ is the Gamma function and ${}_p\mathrm{F}_q(a_1,\cdots a_p;b_1\cdots b_q;x)$ is the generalized hypergeometric function, where p and q are positive integers. In Eqs. (12) and (13), $-4\alpha^3/27\beta^2\ll1$ is always satisfied on by the asymptotic relation[4]

$${}_2\mathrm{F}_2(a,b;c,d;-x)\approx1-\frac{abx}{cd},\ (|x|\ll1)$$

and a, b, c and d are the constants.

Performing the integration, one have the lateral coherence length of the spherical wave[4]

$$\rho_{\mathrm{om}}=(18.01k^2zC_{\mathrm{m}}^2)^{-3/5}\left[(0.419-0.838\bar{\omega}^{-1}+4.19\bar{\omega}^{-2})\right]^{-3/5} \tag{14}$$

for $\rho_0\gg\eta$

For a planar GSM source occupying a finite domain, the elements of the electric cross-spectral density matrix can be expressed in the form[26]

$$w^{(0)}(\boldsymbol{\rho}',\boldsymbol{\rho}'',\omega)=\langle u^*(\boldsymbol{\rho}',\omega)u(\boldsymbol{\rho}'',\omega)\rangle=\sqrt{S^{(0)}(\boldsymbol{\rho}'',\omega)}\sqrt{S^{(0)}(\boldsymbol{\rho}'',\omega)}\mu^{(0)}(\boldsymbol{\rho}''-\boldsymbol{\rho}',\omega) \tag{15}$$

where $S^{(0)}(\boldsymbol{\rho}',\omega)$ and $S^{(0)}(\boldsymbol{\rho}'',\omega)$ are the spectral densities of the Gaussian Schell-model field in the source plane [27]

$$S^{(0)}(\boldsymbol{\rho}',\omega)=A^2(\omega)\exp\left(-\frac{\rho'^2}{2w_0^2}\right) \tag{16}$$

$$S^{(0)}(\boldsymbol{\rho}'',\omega)=A^2(\omega)\exp\left(-\frac{\rho''^2}{2w_0^2}\right) \tag{17}$$

and $\mu^{(0)}(\boldsymbol{\rho}''-\boldsymbol{\rho}',\omega)$ denotes the spectral degree of coherence of the field across the source which is given by the expression

$$\mu^{(0)}(\boldsymbol{\rho}''-\boldsymbol{\rho}',\omega)=B\exp\left[-\frac{(\boldsymbol{\rho}''-\boldsymbol{\rho}')^2}{2\rho_{s0}^2}\right] \tag{18}$$

in which the coefficients A and B are independent of position but they generally depend on the frequency ω. w_0 is the width of the source beam and ρ_{s0} is the source correlation coefficient. The cross-spectral density matrices of a Gaussian Schell-model source are given by expression in the form

$$w^{(0)}(\boldsymbol{\rho}',\boldsymbol{\rho}'',\omega)=\exp\left[-\frac{\rho'^2+\rho''^2}{4w_0^2}-\frac{|\boldsymbol{\rho}'-\boldsymbol{\rho}''|^2}{2\rho_{s0}^2}\right] \tag{19}$$

where w_0 represents the source′s transverse size, ρ_{s0} represents the source′s transverse coherent width, and $A=B=1$.

On making use of the Eqs. (6), (7) and (19), we get the average photon number of turbulence ensemble for quantum fields in ocean

$$\langle s_0(q,\rho,z)\rangle_{o,s}=\left(\frac{\sqrt{n}k}{2\pi z}\right)^2\times\iiiint\exp\left[-\frac{\rho'^2+\rho''^2}{4w_0^2}-\frac{(\boldsymbol{\rho}'-\boldsymbol{\rho}'')^2}{2\rho_{s0}^2}-\frac{(\boldsymbol{\rho}'-\boldsymbol{\rho}'')^2}{\rho_{om}^2}+\mathrm{i}k\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2-(\boldsymbol{\rho}-\boldsymbol{\rho}'')^2}{2z}\right]\cdot \mathrm{d}^2\boldsymbol{\rho}'\mathrm{d}^2\boldsymbol{\rho}'' \tag{20}$$

Using the integral formula[28]

$$\int_{-\infty}^{\infty}\exp\left(-p^2x^2\pm qx\right)\mathrm{d}x=\frac{\sqrt{\pi}}{p}\exp\left(\frac{q^2}{4p^2}\right) \tag{21}$$

we have the analytical expression of Stokes parameter $\langle s_0\rangle_{o,s}$ for a linearly polarized quantum beam in the oceanic turbulence

$$\langle s_0(q,\rho,z)\rangle_{o,s}=n\left(\frac{k}{2\pi z}\right)^2\frac{\pi^2}{\alpha^2\beta^2}\times\exp\left[-\left(\left(\frac{1}{2\alpha z}\right)^2-\left(\frac{1}{4z\beta\alpha^2\rho_{s0}^2}+\frac{1}{2z\beta\alpha^2\rho_{om}^2}-\frac{1}{2z\beta}\right)^2\right)k^2\rho^2\right] \tag{22}$$

where $\alpha=\sqrt{\frac{1}{4w_0^2}+\frac{1}{2\rho_{s0}^2}+\frac{1}{\rho_{om}^2}-\frac{\mathrm{i}k}{2z}}$ and $\beta=\sqrt{\frac{1}{4w_0^2}+\frac{1}{2\rho_{s0}^2}+\frac{1}{\rho_{om}^2}+\mathrm{i}k\frac{1}{2z}-\left(\frac{1}{2\alpha\rho_{s0}^2}+\frac{1}{\alpha\rho_{om}^2}\right)^2}$.

By Eq. (1) and (22), we have the polarization degree of Gaussian Schell-model quantization fields in turbulent ocean channels

$$P=\frac{\sqrt{n\left(\frac{k}{2\pi z}\right)^2\frac{\pi^2}{\alpha^2\beta^2}\exp\left[-Yk^2\rho^2\right]}}{\sqrt{n\left(\frac{k}{2\pi z}\right)^2\frac{\pi^2}{\alpha^2\beta^2}\exp\left[-Yk^2\rho^2\right]+2}} \tag{23}$$

where $Y=\left(\frac{1}{2\alpha z}\right)^2-\left(\frac{1}{4z\beta\alpha^2\rho_{s0}^2}+\frac{1}{2z\beta\alpha^2\rho_{om}^2}-\frac{1}{2z\beta}\right)^2$.

2 Numerical experiments and discussions

In this section, we discuss the relationship between the polarization degree of Gaussian-Schell model beams P and the several parameters which are the temperature structure constant C_m^2, the ratio of temperature and salinity contributions to the refractive index spectrum $\bar{\omega}$, the number of transmissive received photon n, the dissipation rate of temperature variance χ_T, the rate of dissipation of kinetic energy per unit mass of fluid ε, the transverse `coordinate of the photon at the plane ρ, the wavelength λ and the source′s transverse size w_0.

To comprehend the regularity of P, we calculate the influence of C_m^2 and $\bar{\omega}$ (see Fig. 2) for given values of the initial radius of $z=1\ 000$ m, $w_0=0.05$ m, $\lambda=417.3$ nm, the source′s transverse coherent width $\rho_{s0}=0.02$ m, $\rho=0.02$ m and $n=40$. We deduce from Fig. 1 that for any given $\bar{\omega}$, there is an decrease of P along with increasing C_m^2. And we find that, for any given C_m^2, the effects of temperature-induced polarization decrease is surpassing the effects of salinity fluctuations.

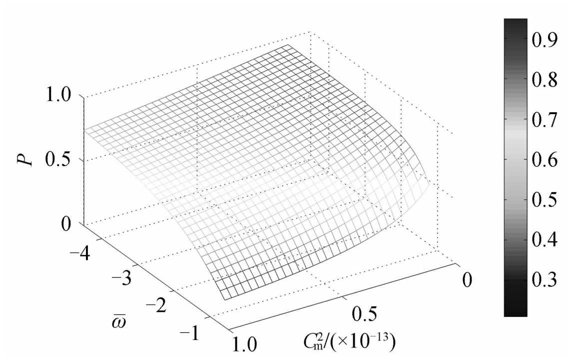

Fig. 2 The polarization degree of Gaussian-Schell model beams P as the functions of C_m^2 and $\bar{\omega}$

In Fig 3, we show the effect of n and χ_T on P. The system parameters of links are installed as the initial radius of $w_0=0.05$m, $z=1\ 000$ m, $\lambda=417.3$ nm, $\rho_{s0}=0.02$ m, $\bar{\omega}=-2$, $\rho=0.02$ m and $\varepsilon=10^{-7}\ \mathrm{m^2s^{-3}}$. From Fig. 2 we see that P decreases with the increase of χ_T, and increases with the increase of n.

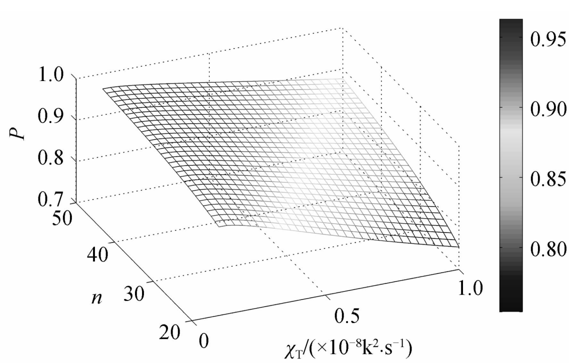

Fig. 3 The polarization degree of Gaussian-Schell model beams P as the functions of n and χ_T

P as functions of ε and ρ is shown in Fig 4. The system parameters of links are given by the initial radius of $w_0=0.05$ m, $z=1\ 000$ m, $\lambda=417.3$ nm, $\rho_{s0}=0.02$ m, $\bar{\omega}=-2$, $n=40$ and $\chi_T=10^{-8}\cdot\mathrm{k^2}\cdot\mathrm{s^{-1}}$.

As a consequence, P increases with the increase of ε and decreases with the increase of ρ.

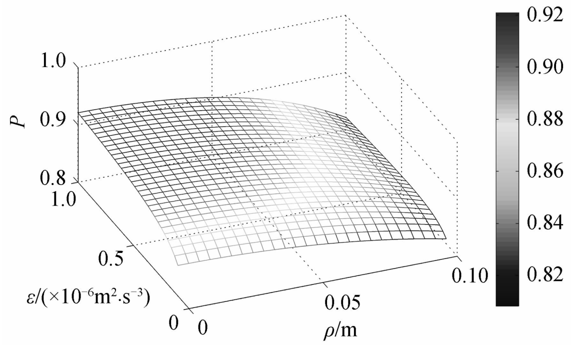

Fig. 4 The polarization degree of Gaussian-Schell model beams P as the functions of ε and ρ

Fig. 5 plots P as functions of z and w_0. The system parameters are given by $\varepsilon = 10^{-6} m^2 s^{-3}$, $n=40$, $\rho_{s0}=0.02$ m, $\bar{\omega}=-2$, $\lambda=417.3$ nm, $\rho=0.02$ m and $\chi_T = 10^{-8}$ $k^2 s^{-1}$. Fig. 4 shows that P decreases with increasing propagation distance z. But as w_0 increases, P increases.

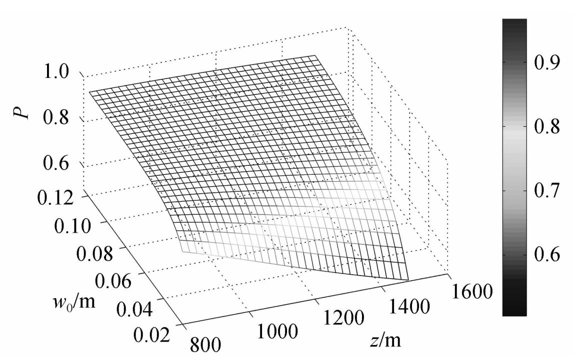

Fig. 5 The polarization degree of Gaussian-Schell model beams P as the functions of z and w_0

This result comes from, the longer source's transverse size w_0 makes a smaller beam spread which caused by vacuum diffraction and turbulence of the transmission beam, and it reduce the transformations of the beam pattern space distribution forms in transmission process.

Fig. 6 plots P as functions of λ and ρ_{s0}. The system parameters are given by $\varepsilon = 10^{-6} m^2 s^{-3}$, $n=40$,

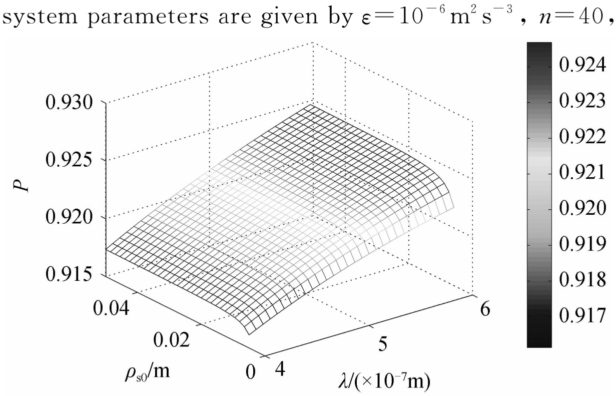

Fig. 6 The polarization degree of Gaussian-Schell model beams P as the functions of λ and ρ_{s0}

$z=1\,000$ m, $\bar{\omega}=-2$, $w_0=0.05$ m, $\rho=0.02$ m and $\chi_t = 10^{-8}$ $k^2 s^{-1}$. Fig. 5 shows that P increases with increasing λ and ρ_{s0}. These results show that increasing the spatial coherence length ρ_{s0} of partially coherent light and using longer seawater wavelength window is advantageous to the information transmission.

3 Conclusion

In this paper, we model the degree of the polarization for Gaussian Schell-model quantized fields propagating through the oceanic channel with eddy thermal diffusivity equaling to the diffusion of salt obtained based on developing of effective photon annihilation/creation operator of linearly quantized Gaussian Schell-beams in the turbulent ocean. The theoretical simulation and numerical experimental results show that the polarization degree P decreases with the increase of the refractive-index construction parameter C_m^2, the ratio of temperature and salinity contributions to the refractive index spectrum $\bar{\omega}$, the dissipation rate of temperature variance χ_T, the propagation distance z and the transverse coordinate of the photon in receiving plane ρ, but increases with the increase of the number of transmissive received photon n, the rate of dissipation of kinetic energy per unit mass of fluid ε, the wavelength λ, the source's transverse size w_0 and the source's transverse coherent width ρ_{s0}. We find that the effects of temperature-induced polarization decrease is surpassing the effects of salinity fluctuations. Increasing the launch photon number and the radius of the aperture is an effective way to mitigate the turbulent disturbance.

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