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两 Jaynes-Cummings 原子间的纠缠动力学和转移特性

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摘　要:采用两种不同的纠缠度量方法(并发度和负值度),研究了两 Jaynes-Cummings 原子之间的纠缠演化以及各子系统之间的纠缠转移,分析了两原子之间初始纠缠度对纠缠的影响.结果表明纠缠的幅值依赖于初始纠缠度,而解纠缠时间长度与初始纠缠度无关.制备了两个腔场之间的最大纠缠态,数值分析显示两原子之间的初始纠缠流入了其它各个子系统,导致演化过程中的纠缠突然死亡和纠缠突然产生现象.

关键词:量子光学;量子信息;量子纠缠;纠缠突然死亡;纠缠突然产生;并发度;负值度
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Dynamics and Transfer of Entanglement between Two Jaynes-Cummings Atoms

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Abstract: Two different measures of entanglement, concurrence and negativity, were used to study the evolution of the two Jaynes-Cummings atoms and the transfer of the entanglement between the subsystems. The effect of the initial entanglement degree of two atoms on the time evolution of entanglement was analyzed. It is found that the amplitude of the entanglement depends on the initial entanglement degree while the length of time interval for the zero entanglement does not. The maximally entangled state between the two cavities was created. Moreover, the numerical analysis shows that the initial entanglement of the two atoms flows into the other subsystems and leads to entanglement sudden death and entanglement sudden birth in the evolution.

Key words: Quantum optics; Quantum information; Quantum entanglement; Entanglement sudden death; Entanglement sudden birth; Concurrence; Negativity
OCIS Codes: 270.0270; 270.5565; 270.5580; 270.5580

0 Introduction

Quantum entanglement is one of the most fantastic and important characters of quantum mechanics[1], which plays a very key role in quantum information like quantum cryptography[2], quantum dense coding[3] and quantum teleportation[4]. One of the most common used models on the interaction between radiation and matter is the Jaynes-Cummings Model (JCM)[5] that describes the interaction of a two-level atom with a single mode quantized electromagnetic field in the Rotating Wave Approximation (RWA). There are

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many extensions and applications based on the JCM like entangled atom-field state in JCM[6], deep strong coupling regime of JCM[7], two/multi-photon transitions[8-9], two cavity modes for three level atoms[10-12], squeezed states in JCM[13] and nonlinear JCM[14]. Entanglement evolution of atom interacting with quantized field without RWA has been investigated in Refs. [15-16] and shown that the periodic evolution of entanglement can be obtained. Zhou, *et al* have studied the generation of strongly coherent entanglement by manipulating the atom located outside cavity[17].

The evolution of the entanglement in JCM is always a hot topic so many years. Yu and Eberly found that the evolution of entanglement of a bipartite system could vanish abruptly in finite time in 2004[18]. This strange phenomenon was termed as Entanglement Sudden Death (ESD), which has attracted lots of attention from both theoretically and experimentally[19]. Yönac, *et al* studied the dynamics of ESD of two JC atoms[20]. They showed that the non-interacting and non-communicating atoms could abruptly lose their entanglement with each other. Then Li, *et al* discussed the case of an isolated atom and a JC atom[21]. They found that atom and cavity could entangle with each other even though there were no interactions between them. Isabel and Gunnar investigated the entanglement dynamics for a double JCM and found an entanglement invariant valid for a large class of so called X states in the closed system[22]. Han studied the entanglement transfer from initial pairwise entanglement to multipartite entanglement for the same model and showed that during this simultaneous ESD period various multipartite entanglements could be created[23].

In this paper, we consider the case of two Jaynes-Cummings atoms coupling with a phase state cavity[24] and a vacuum cavity, respectively. Both cavities considered in Refs. [22-23] are in the vacuum states, while one of the cavities under our consideration is prepared initially in the phase state $|\varphi(0)\rangle_{f_1} = (|0\rangle_1 + e^{i\varphi}|1\rangle_1)/\sqrt{2}$. The system includes not only qubit-qubit entanglement but also the qubit-qutrit entanglement. It is shown that the maximally entangled state between the two cavities can be created and the initial entanglement of the two atoms flows into the other subsystems and leads to ESD and Entanglement Sudden Birth (ESB) in the evolution.

1 Model and measure of entanglement

Consider a system consisting of two Jaynes-Cummings atoms. The atom a_1 interacts with a phase state cavity field via one-photon transition. The atom a_2 interacts with a vacuum state cavity field. Each model is completely isolated from the other atom and cavity. The two atoms are identical. For simplicity, we consider the exact resonance of the field with the atoms and the quality factors sufficiently high such that it allows us to neglect the dissipative effects [25-27]. High quality factor cavities may be prepared in the microwave regime, with factors from 10^7[28] to 10^{10}[29]. The physical model of our system under the consideration is presented in Fig. 1. The Hamiltonian describing the whole system can be written, in the RWA, as ($\hbar=1$)

$$H_I = g(a\sigma_1^+ + a^\dagger\sigma_1^-) + g(b\sigma_2^+ + b^\dagger\sigma_2^-) \tag{1}$$

where g is the constant of coupling between the atom and the field, $\sigma_+ = |e\rangle\langle g|$ and $\sigma_- = |g\rangle\langle e|$ are the atomic raising and lowering operators, a (b) and $a^\dagger$ ($b^\dagger$) are the annihilation and creation operators of the cavity field f_1 (f_2) respectively.

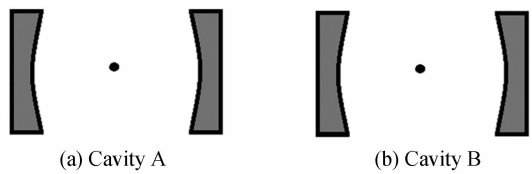

Fig. 1 The schematic diagram of the physical model

Assume that at time $t=0$ the cavity A is prepared in the phase state

$$|\varphi(0)\rangle_{f_1} = \frac{1}{\sqrt{2}}|0\rangle_1 + \frac{e^{i\varphi}}{\sqrt{2}}|1\rangle_1 = K_1|0\rangle_1 + K_2|1\rangle_1 \tag{2}$$

The phase state has been produced by two-photon absorption. The generation of such a state has been proposed in Ref. [24] by controlling the phase of the initial coherent state. The cavity B is in the vacuum state

$$|\varphi(0)\rangle_{f_2} = |0\rangle_2 \tag{3}$$

The two atoms are in pure entangled state

$$|\varphi(0)\rangle_{\text{atom}} = \cos\theta|e_1g_2\rangle + \sin\theta|g_1e_2\rangle \tag{4}$$

Then the initial state of the system has the following form

$$\begin{aligned}|\varphi(0)\rangle_s = {} & (\cos\theta|e_1g_2\rangle + \sin\theta|g_1e_2\rangle)(K_1|0_10_2\rangle + \\ & K_2|1_10_2\rangle) = \cos\theta K_1|e_1g_20_10_2\rangle + \cos\theta K_2 \cdot \\ & |e_1g_21_10_2\rangle + \sin\theta K_1|g_1e_20_10_2\rangle + \sin\theta K_2 \cdot \\ & |g_1e_21_10_2\rangle = a|e_1g_20_10_2\rangle b|e_1g_21_10_2\rangle + \\ & c|g_1e_20_10_2\rangle + d|g_1e_21_10_2\rangle\end{aligned} \tag{5}$$

where $a=\cos\theta K_1$, $b=\cos\theta K_2$, $c=\sin\theta K_1$ and $d=\sin\theta K_2$.

The wave function of the system under the action of the Hamiltonian given by Eq. (1) evolves to

$$\begin{aligned}|\varphi(t)\rangle_s = {} & M_1|e_1g_20_10_2\rangle + M_2|e_1g_21_10_2\rangle + \\ & M_3|g_1e_20_10_2\rangle + M_4|g_1e_21_10_2\rangle + M_5|g_1g_21_10_2\rangle + \\ & M_6|g_1g_22_10_2\rangle + M_7|g_1g_20_11_2\rangle + M_8|e_1e_20_10_2\rangle + \\ & M_9|g_1g_21_11_2\rangle + M_{10}|e_1g_20_11_2\rangle\end{aligned} \tag{6}$$

where

$$M_1 = a\cos(gt),\ M_2 = b\cos(\sqrt{2}gt),$$
$$M_3 = c\cos(gt),\ M_4 = d\cos^2(gt),$$
$$M_5 = -ia\sin(gt),\ M_6 = -ib\sin(\sqrt{2}gt),$$
$$M_7 = -ic\sin(gt),\ M_8 = -\frac{1}{2}id\sin(2gt),$$
$$M_9 = -\frac{1}{2}id\sin(2gt),\ M_{10} = -d\sin^2(gt).$$

We begin with the entanglement between the two atoms. The reduced density matrix of the two atoms can be calculated by taking a partial trace over the cavity over the degrees of freedom in the two-qubit standard basis $\{|e_1e_2\rangle, |e_1g_2\rangle, |g_1e_2\rangle, |g_1g_2\rangle\}$ as

$$\boldsymbol{\rho}_{a_1a_2}(t) = \mathrm{Tr}_{f_1f_2}[|\varphi(t)\rangle_s\langle\varphi(t)|] = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & 0 \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ 0 & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix} \tag{7}$$

where $\rho_{12} = M_8M_1^*$, $\rho_{13} = M_8M_3^*$, $\rho_{21} = M_1M_8^*$,

$$\rho_{22} = M_1M_1^* + M_2M_2^* + M_{10}M_{10}^*,$$
$$\rho_{23} = M_1M_3^* + M_2M_4^*,\ \rho_{24} = M_2M_5^* + M_{10}M_7^*,$$
$$\rho_{31} = \rho_{13}^*,\ \rho_{32} = \rho_{23}^*,\ \rho_{33} = M_3M_3^* + M_4M_4^*,$$
$$\rho_{34} = M_4M_5^*,\ \rho_{42} = \rho_{24}^*,\ \rho_{43} = \rho_{34}^*,$$
$$\rho_{44} = M_5M_5^* + M_6M_6^* + M_7M_7^* + M_9M_9^*.$$

We use Wootter's Concurrence[30] to calculate the degree of entanglement between the two atoms, which is defined as

$$C(\rho) = \max[0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4] \tag{8}$$

where λ is are the eigenvalues, in decreasing order, of the Hermitian matrix $R \equiv \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$, and $\tilde{\rho}$ is the time reversal matrix of ρ, which is expressed as

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y) \tag{9}$$

Here $\rho*$ denotes the complex conjugation of ρ and σ_y is the Pauli matrix. It's shown that Concurrence ranges from $C = 0$ for a separable state and $C = 1$ for a maximally entangled state.

Next we calculate the concurrence between the atom a_2 and the cavity f_2. The reduced density matrix for the atom a_2 and field f_2 is obtained as

$$\rho_{a_2f_2} = \mathrm{Tr}_{a_1f_1}[|\varphi(t)\rangle_s\langle\varphi(t)|] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & \rho_{24} \\ 0 & \rho_{32} & \rho_{33} & \rho_{34} \\ 0 & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix} \tag{10}$$

in the standard basis

$$\{|e_21_2\rangle, |e_20_2\rangle, |g_21_2\rangle, |g_20_2\rangle\},$$

where

$$\rho_{22} = M_3M_3^* + M_4M_4^* + M_8M_8^*,$$
$$\rho_{23} = M_3M_7^* + M_8M_{10}^* + M_4M_9^*,$$
$$\rho_{24} = M_4M_5^* + M_8M_1^*,\ \rho_{32} = \rho_{23}^*,$$
$$\rho_{33} = M_7M_7^* + M_9M_9^* + M_{10}M_{10}^*,$$
$$\rho_{34} = M_9M_5^* + M_{10}M_1^*,\ \rho_{42} = \rho_{24}^*,$$
$$\rho_{43} = \rho_{34}^*,$$
$$\rho_{44} = M_1M_1^* + M_2M_2^* + M_5M_5^* + M_6M_6^*$$

The Concurrence can be obtained more easily for such a matrix given by Eq. 10.

$$C(\rho) = 2\sqrt{(c^2\cos^2(gt) + d^2\cos^4(gt) + \frac{1}{4}d^2\sin^2(2gt))} \cdot \sqrt{\left(e^2\sin^2(gt) + \frac{1}{4}d^2\sin^2(2gt) + d^2\sin^4(gt)\right)} \tag{11}$$

It is observed that the atom a_1 and cavity f_1 become a qubit-qutrit system and the corresponding reduced density matrix is expressed as an explicit 6×6 matrix in the standard basis

$$\{|1\rangle = |2_1e_1\rangle, |2\rangle = |1_1e_1\rangle, |3\rangle = |0_1e_1\rangle,$$
$$|4\rangle = |2_1g_1\rangle,\ |5\rangle = |1_1g_1\rangle, |6\rangle = |0_1g_1\rangle\}.$$

$$\boldsymbol{\rho}_{a_1f_1}(t) = \mathrm{Tr}_{a_2f_2}[|\varphi(t)\rangle_s\langle\varphi(t)|] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & \rho_{24} & \rho_{25} & \rho_{26} \\ 0 & \rho_{32} & \rho_{33} & \rho_{34} & \rho_{35} & \rho_{36} \\ 0 & \rho_{42} & \rho_{43} & \rho_{44} & \rho_{45} & 0 \\ 0 & \rho_{52} & \rho_{53} & \rho_{54} & \rho_{55} & \rho_{56} \\ 0 & 0 & \rho_{63} & 0 & \rho_{65} & \rho_{66} \end{pmatrix} \tag{12}$$

where

$$\rho_{22} = M_2M_2^*,\ \rho_{23} = M_2M_1^*,\ \rho_{24} = M_2M_6^*,\ \rho_{25} = M_2M_5^*,$$
$$\rho_{32} = M_1M_2^*,\ \rho_{33} = M_1M_1^* + M_8M_8^* + M_{10}M_{10}^*,$$
$$\rho_{34} = M_1M_6^*,\ \rho_{35} = M_1M_5^* + M_8M_4^* + M_{10}M_9^*,$$
$$\rho_{36} = M_8M_3^* + M_{10}M_7^*,\ \rho_{42} = \rho_{24}^*,\ \rho_{43} = M_6M_1^*,$$
$$\rho_{44} = M_6M_6^*,\ \rho_{45} = M_6M_5^*,\ \rho_{52} = \rho_{25}^*,\ \rho_{53} = \rho_{35}^*,$$
$$\rho_{54} = M_5M_6^*,\ \rho_{55} = M_4M_4^* + M_5M_5^* + M_9M_9^*,$$
$$\rho_{56} = M_4M_3^* + M_9M_7^*,\ \rho_{63} = \rho_{36}^*,\ \rho_{65} = M_3M_4^* + M_7M_9^*,$$
$$\rho_{66} = M_3M_3^* + M_7M_7^*.$$

Concurrence introduced above is applied for any reduced density matrix of two qubits system not a qubit-qutrit system. Thus we use another measure of entanglement named Negativity[31-32] here. The negativity of entanglement is defined as the absolute value of the sum of negative eigenvalues of ρ^{T_X}

$$N(\boldsymbol{\rho}) = 2\sum_i |\mu_i| \tag{13}$$

Where, μ_i is the negative eigenvalue of ρ^{T_X}, T_X denotes partial transpose with respect to the system X. $N(\rho)$ also varies from 0 for a disentangled state to 1 for a maximum entangled state.

At last we discuss the case of the two cavities f_1 and f_2. Just like the situation of the entanglement between atom a_1 and field f_1, the reduced density matrix is also a 6×6 matrix which may be written as

$$\rho_{f_1f_2}(t) = \mathrm{Tr}_{a_1a_2}[|\varphi(t)\rangle_s\langle\varphi(t)|] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & \rho_{24} & \rho_{25} & 0 \\ 0 & \rho_{32} & \rho_{33} & \rho_{34} & \rho_{35} & \rho_{36} \\ 0 & \rho_{42} & \rho_{43} & \rho_{44} & \rho_{45} & 0 \\ 0 & \rho_{52} & \rho_{53} & \rho_{54} & \rho_{55} & \rho_{56} \\ 0 & 0 & \rho_{63} & 0 & \rho_{65} & \rho_{66} \end{pmatrix} \tag{14}$$

in the standard basis

$$\{|1\rangle=|2_1 1_2\rangle, |2\rangle=|1_1 1_2\rangle, |3\rangle=|0_1 1_2\rangle,$$
$$|4\rangle=|2_1 0_2\rangle, |5\rangle=|1_1 0_2\rangle, |6\rangle=|0_1 0_2\rangle\},$$

where,

$$\rho_{22}=M_9 M_9^*, \rho_{23}=M_9 M_7^*, \rho_{24}=M_9 M_6^*, \rho_{25}=M_9 M_5^*,$$
$$\rho_{32}=\rho_{23}^*, \rho_{33}=M_7 M_7^*+M_{10} M_{10}^*, \rho_{34}=M_7 M_6^*,$$
$$\rho_{35}=M_7 M_5^*+M_{10} M_2^*, \rho_{36}=M_{10} M_1^*, \rho_{42}=\rho_{24}^*,$$
$$\rho_{43}=\rho_{34}^*, \rho_{44}=M_6 M_6^*, \rho_{45}=M_6 M_5^*, \rho_{52}=\rho_{25}^*, \rho_{53}=\rho_{35}^*,$$
$$\rho_{54}=\rho_{45}^*, \rho_{55}=M_2 M_2^*+M_4 M_4^*+M_5 M_5^*,$$
$$\rho_{56}=M_2 M_1^*+M_4 M_3^*, \rho_{63}=\rho_{36}^*, \rho_{65}=\rho_{56}^*,$$
$$\rho_{66}=M_1 M_1^*+M_3 M_3^*+M_8 M_8^*.$$

We still apply the negativity as the measure of entanglement of the qubit-qutrit system given above.

2 Results and discussion

In this section, we give numerical analysis for the evolution of entanglement between the atoms, the atom and the cavity as well as the cavities.

Fig. 2 is a plot for the maximum entanglement evolution between two atoms for $\theta=\pi/4$. Obviously, the concurrence C between the two atoms is 1 initially for a maximal entangled state. After a finite time, C abruptly decreases to 0 and remains zero for a period of time before it recovers, and ESD happens. The ESD is mainly caused by the atom-cavity interaction, which leads to the exchange of the information and energy. Coming through a short time, C abruptly increases to peak value, and the ESB happens. With the time going on, the ESB and ESD appear repeatedly. When $\theta=\pi/4, \pi/6, \pi/12$, corresponding to some different initial partial entangled states between the atoms, as shown in Fig. 3, the peak values of C reduce and the length of time interval for the zero entanglement is not dependent on the initial degree of entanglement. This character is different from the result of the double Jaynes-Cummings model in Ref. [20] in which the smaller the initial degree of entanglement, the longer the state will stay in the disentanglement state.

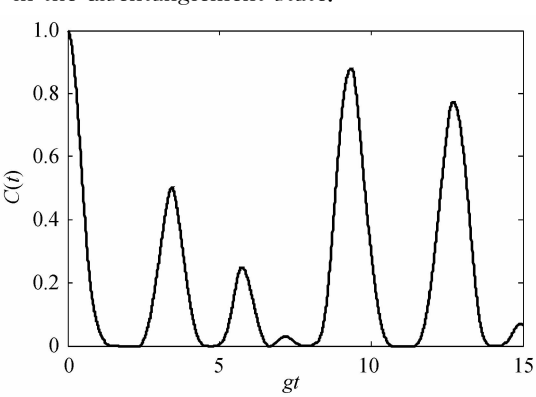

Fig. 2 Time evolution of Concurrence between the two atoms with the maximal entangled initial state $|\varphi(0)\rangle_{\text{atom}}=(|e_1 g_2\rangle+|g_1 e_2\rangle)/\sqrt{2}$

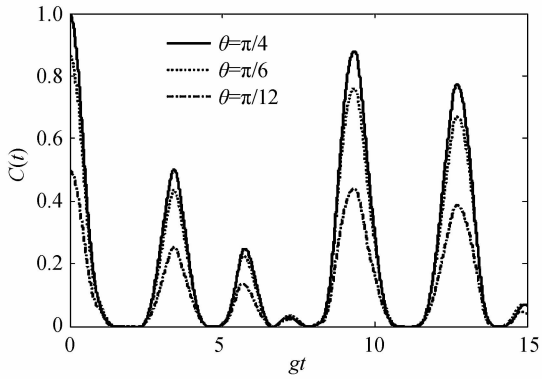

Fig. 3 Time evolution of Concurrence between the two atoms

The evolution of $N_{a_1 f_1}$ for $\theta=0$, $\theta=\pi/2$ and some other initial states are shown in Figs. 4, 5 and 6, respectively. When $\theta=0$, as shown in Fig. 4, the initial state of the atom a_1 is $|\varphi(0)\rangle_{\text{atom}}=|e_1 g_2\rangle$, which means that there is no initial entanglement between the two atoms. The evolution of the entanglement in the two cavities is totally uncorrelated and independent. This is due to the fact that each model is completely isolated from the other atom and cavity. We can see that after a short time, $N_{a_1 f_1}$ abruptly rises to peak value because of the interaction between the atom a_1 and the field f_1. It's remarkable that when it comes to the maximal entanglement at $gt=\pi$, $N_{a_1 f_1}$ remains 1 for a finite time interval. That is, the maximally entangled state between the atom a_1 and field f_1 can be long-lived. Moreover, the atom a_1 and field f_1 almost retain entanglement in the time evolution process.

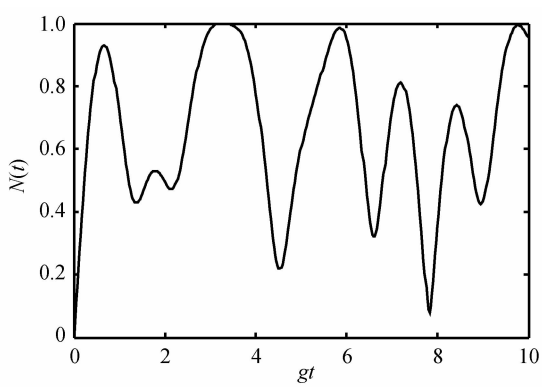

Fig. 4 Time evolution of the Negativity between the atom a_1 and the cavity field f_1 with an initial state $|\varphi(0)\rangle_{\text{atom}}=|e_1 g_2\rangle$

In Fig. 5, we can see that there is still no ESD appearing as the case in Figure 4. But there is a very important character that $N_{a_1 f_1}$ evolves periodically with a period of $\pi/2$. In Fig. 6, we try three different initial entangled states with $\theta=\pi/4$, $\theta=\pi/6$ and $\theta=\pi/12$. It is obvious from Fig. 6 that $N_{a_1 f_1}$ becomes larger with the decrease of θ while the trends of the entanglement evolution are almost the same, no matter how the value

of initial entanglement changes. That is, the maximum of the entanglement between the atom a_1 and the cavity field f_1 increases as the initial entanglement degree of two atoms decreases. In the experiment, the amount of the initial entanglement degree of two atoms can be adjusted, which modifies the entanglement between the atom and field.

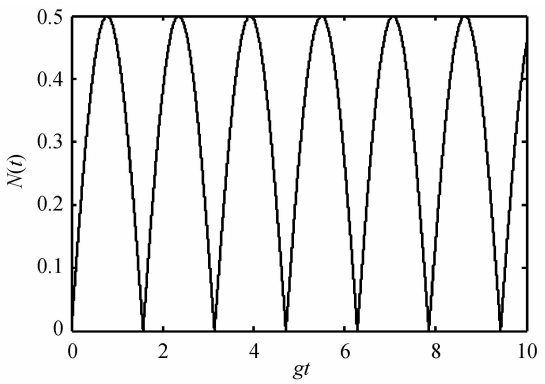

Fig. 5 Time evolution of the Negativity between the atom a_1 and the cavity field f_1 with an initial state $|\varphi(0)\rangle_{\text{atom}}=|g_1e_2\rangle$

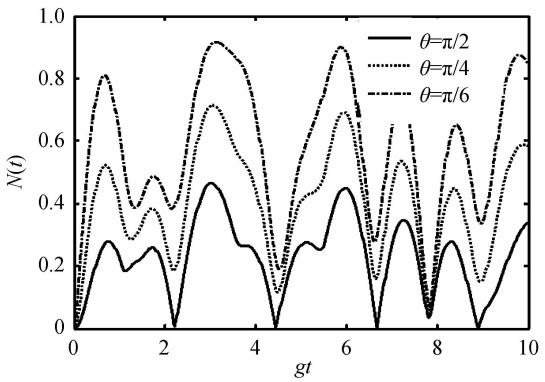

Fig. 6 Time evolution of the Negativity between the atom a_1 and the cavity field f_1

In Fig. 7, we plot the entanglement between the atom a_2 and cavity field f_2 with an initial state $|\varphi(0)\rangle_{\text{atom}}=|g_1e_2\rangle$. From the figure we find that $C_{a_2f_2}$ evolves periodically with a period of $\pi/2$ and disentanglement between the atom a_2 and cavity field

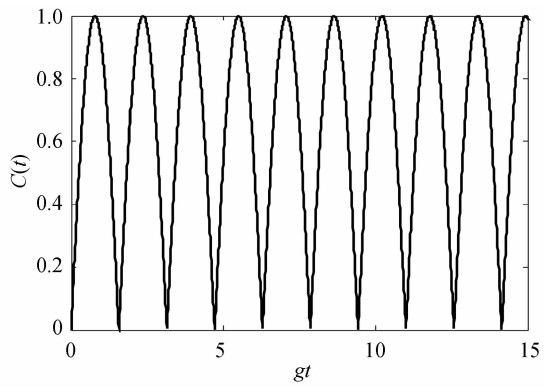

Fig. 7 Time evolution of the Concurrence between the atom a_2 and the cavity field f_2 with an initial state $|\varphi(0)\rangle_{\text{atom}}=|g_1e_2\rangle$

f_2 happens at the time $gt=\pi/2$. We can see that the maximal entanglement is achieved at the time $gt=(2n-1)\pi/4$ $(n=1,2,3,\cdots\cdots)$ and there is no ESD appearing through the evolution.

We also plot some different values of θ in Fig. 8. From Fig. 8 we find that $C_{a_2f_2}$ evolves periodically with a period of $\pi/2$ and achieves 0 at $gt=\pi/2$, which means that what the initial entanglement changes is only the peak value not the trends of the evolution. It's shown that $C_{a_2f_2}$ becomes larger with the value of θ increasing and also the trends of the evolution are still identical even with different initial entanglement. However, from Fig. 8, we see that the period of time evolution of the entanglement between the atom a_2 and cavity field f_2 is independent of the initial entanglement degree of two atoms.

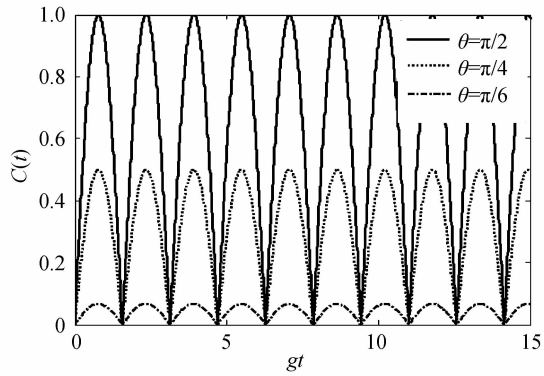

Fig. 8 Time evolution of the Concurrence between the atom a_2 and the cavity field f_2

In what follows, we consider the dynamics of the entanglement between the two cavities. The evolution of $N_{f_1f_2}$ are shown in Fig. 9. We can see that there is no entanglement at earlier times and suddenly at some finite time an entanglement starts to build up. This effect has been referred to as "entanglement sudden birth"[33]. The phenomenon of ESD happens as time develops and $N_{f_1f_2}$ achieves 1 at some time. That is, the maximally entangled state between the two cavities can be generated . Fig. 9 shows that the entanglement

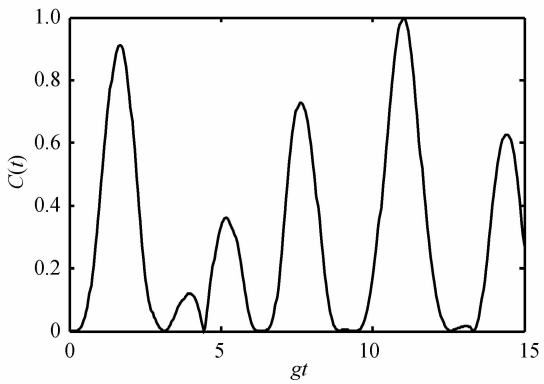

Fig. 9 Time evolution of the negativity between the two cavities for the initial atom state for $|\varphi(0)\rangle_{\text{atom}}=(|e_1g_2\rangle+|g_1e_2\rangle)/\sqrt{2}$

between the two cavities can be created, though the cavity A has no interaction with cavity B. Its origination may come from the entanglement of two atoms and the interaction between the atom and the cavity. Fig. 10 shows the time evolution of two cavities negativity for the different initial entanglement degree of two atoms. From Fig. 10 we can see that with the increasing of the initial entanglement of two atoms, the oscillating amplitude of entanglement between two cavities becomes larger.

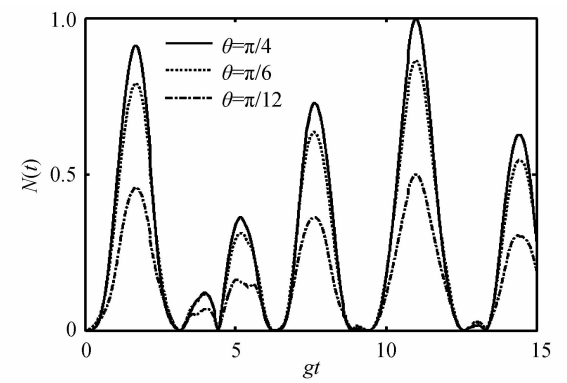

Fig. 10 Time evolution of the Negativity between the two cavities

After the discussion of the four bipartite subsystems, we notice that there is only entanglement between the two atoms initially while ESB happens in the other three subsystems after the interaction. We wonder where does the lost entanglement go and where does the new entanglement come from. We shall start considering the evolution of the entanglement of the four subsystems here.

Figs. 11 and 12 present the situation of $\theta=\pi/4$, which has been shown above that this is the maximal initial entanglement between the two atoms. It's worth emphasizing some significant properties of the entanglement dynamics of the whole system. Fig. 11 shows although the atoms only interact with their cavities, entanglement also flows to other noninteraction partitions . We can see $N_{a_1f_1}$, $C_{a_2f_2}$ and

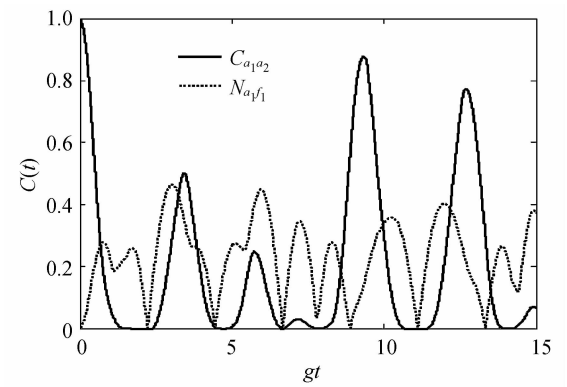

Fig. 11 Time evolution of $C_{a_1a_2}$ and $N_{a_1f_1}$ with the atom initial state for $|\varphi(0)\rangle_{atom}=(|e_1g_2\rangle+|g_1e_2\rangle)/\sqrt{2}$

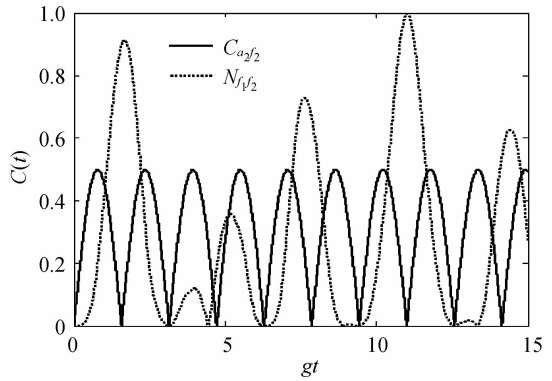

Fig. 12 Time evolution of $C_{a_2f_2}$ and $N_{f_1f_2}$ with the atom initial state for $|\varphi(0)\rangle_{atom}=(|e_1g_2\rangle+|g_1e_2\rangle)/\sqrt{2}$

$N_{f_1f_2}$ begin to arise exactly the time when $C_{a_1a_2}$ begins to decrease. The swap of the entanglement is especially obvious between $C_{a_1a_2}$ and $N_{f_1f_2}$. Every time when $C_{a_1a_2}$ ($N_{f_1f_2}$) reaches the peak value ESD or disentanglement happens in the evolution of $N_{f_1f_2}$ ($C_{a_1a_2}$). Look back to the evolution of $C_{a_2f_2}$ and $N_{f_1f_2}$, we find that the trends of the evolution of the two subsystems seem to be synchronous or near-synchronous and the peak values are nearly the same of $C_{a_2f_2}$ and $N_{a_1f_1}$.

We consider the different initial entanglement for $\theta=\pi/6$ and $\theta=\pi/12$ to observe the different characters for the four subsystems presented in Figs. 13 to 16.

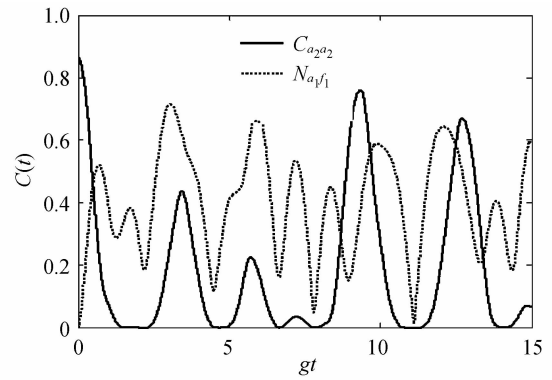

Fig. 13 Time evolution of $C_{a_1a_2}$ and $N_{a_1f_1}$ with the atom initial state for $|\varphi(0)\rangle_{atom}=\cos\theta|e_1g_2\rangle+\sin\theta|g_1e_2\rangle$ with $\theta=\pi/6$

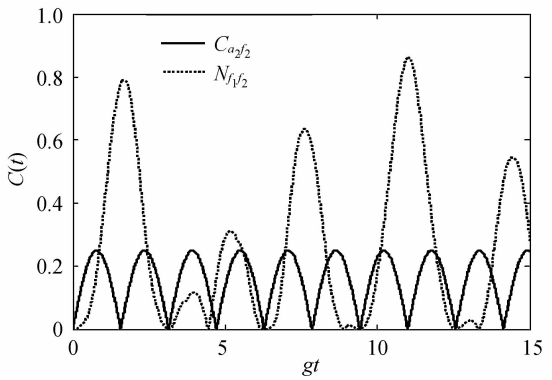

Fig. 14 Time evolution of $C_{a_2f_2}$ and $N_{f_1f_2}$ with the atom initial state for $|\varphi(0)\rangle_{atom}=\cos\theta|e_1g_2\rangle+\sin\theta|g_1e_2\rangle$ with $\theta=\pi/6$

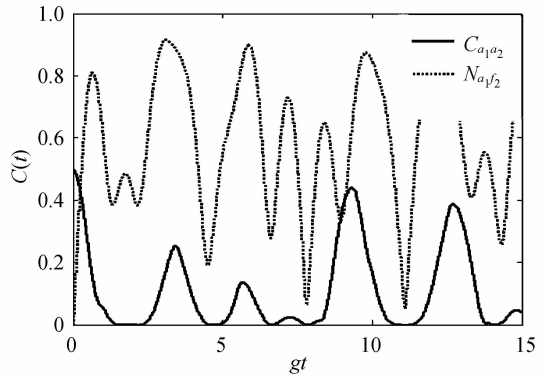

Fig. 15 Time evolution of $C_{a_1a_2}$ and $N_{a_1f_1}$ with the atom initial state for $|\varphi(0)\rangle_{\text{atom}}=\cos\theta|e_1g_2\rangle+\sin\theta|g_1e_2\rangle$ with $\theta=\pi/12$

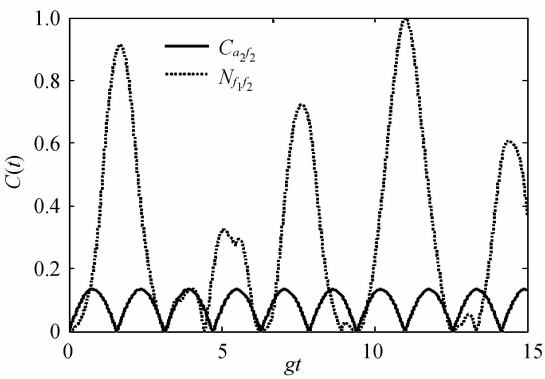

Fig. 16 Time evolution of $C_{a_2f_2}$ and $N_{f_1f_2}$ with the atom initial state for $|\varphi(0)\rangle_{\text{atom}}=\cos\theta|e_1g_2\rangle+\sin\theta|g_1e_2\rangle$ with $\theta=\pi/12$

One should note that the relation between $C_{a_1a_2}$ and $N_{f_1f_2}$ is still valid for different initial states. The most interesting consequence about $C_{a_2f_2}$ and $N_{a_1f_1}$ is shown by comparing the above three figures . Obviously the presence of the larger initial entanglement leads to the increase of the entanglement between the atom a_1 and the cavity f_1 and the decrease between the atom a_2 and the cavityf_2. The obtained results show that the evolution of $C_{a_2f_2}$ and $N_{a_1f_1}$ is correlated with the change of the initial entanglement, which may be due to the fact that the interaction between the atom and its cavity is local.

3 Conclusion

In summary, we have discussed the evolution of the entanglement of the two JCM where the atoms a_1 and a_2 interact with their cavity field by calculating the concurrence and negativity for the four bipartite systems respectively. We have investigated the influence of different initial entanglement of two atoms on the dynamics of the ESD and ESB phenomenon. The results show that there appears the ESD and ESB between the two atoms and the two cavities. We find that the change of the initial entanglement only leads to the change of the amplitude of the entanglement of all the subsystems instead of the trends of the evolution. At last we study the entanglement swapping among the bipartite systems. We can see that when $C_{a_1a_2}$ ($N_{f_1f_2}$) reaches the peak value the ESD or disentanglement happens in the evolution of $N_{f_1f_2}$ ($C_{a_1a_2}$). An interesting character is that the larger the initial entanglement is, the larger the entanglement between the atom a_1 and the cavity f_1 becomes and the smaller that between the atom a_2 and the cavity f_2 is. We believe that our study is of general property and we hope that what we presented in this paper will have a potential application in quantum communication in the near future.

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