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# 双模光场与级联三能级原子在非旋波近似下的量子纠缠

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**摘要:**利用全量子化理论,在非旋波近似下对双模相干态光场与级联型三能级原子相互作用的量子纠缠进行了精确求解.讨论了初始时刻原子能级的叠加和平均光子数对量子纠缠演化的影响.数值计算结果表明:初始时刻原子处于单一能级时,量子纠缠演化曲线的周期随着平均光子数的增加逐渐变大;初始时刻原子能级的叠加导致初始阶段纠缠度显著降低,纠缠达到最大值的时间随着平均光子数的增大逐渐变长,且初始时刻原子能级的叠加使得量子纠缠的周期性消失;无论初始时刻原子能级处于哪种能级的叠加态,随着平均光子数的增大,由虚光子效应引起的小锯齿状的振荡逐渐增强.

**关键词:**量子光学;量子纠缠;相干态正交化展开;双模光场;三能级原子

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## Quantum Entanglement of a Two-Mode Field Interacting with a Cascade Three-level Atom without Rotating Wave Approximation

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**Abstract:** The quantum entanglement of the two mode coherent field interacting with a cascade three-level atom was calculated accurately with non-rotating wave approximation by using the complete quantum theory. The influences of the superposition between different atomic energy level at the initial time and the mean photon number on quantum entanglement evolution were considered. The results obtained by the numerical method show that the period of the quantum entanglement evolution becomes longer with the increase of the mean photon number when the atom initially at the single energy level, the quantum entanglement at the first few periods is reduced notably due to the fact of the atom is initially in the superposition state, it takes longer time for the entanglement to reach the maximum value with the increase of the mean photon number, moreover, because of the atom is initially in the superposition state, the periodic disappears. No matter which atom state, the quick oscillation due to the virtual photon process increases with the increasing of the mean photon number.

**Key words:** Quantum optics; Quantum entanglement; Coherent-state orthogonalization expansion; Two-mode field; Three-level atom

**OCIS Codes:** 270.0270; 270.5585; 270.6620

## 0 引言

量子纠缠是量子力学最显著的特性之一<sup>[1-2]</sup>,

Phoenix 和 Knight 等利用熵理论研究光场与二能级原子相互作用时的动力学特性<sup>[3-4]</sup>.熵自动包含了量子系统密度矩阵的全部统计矩,是解释量子系统动力学特

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性的重要工具,并且能够度量场与原子间的纠缠程度,因此有关熵的理论成为量子纠缠研究的热点<sup>[5-7]</sup>.近年来人们对单模光场在旋波近似下的场熵<sup>[8]</sup>、原子熵<sup>[9]</sup>、线性熵<sup>[10]</sup>、考虑双光子过程熵<sup>[11]</sup>以及克尔介质中的熵<sup>[12]</sup>等都做了相关研究,但是对双模光场在非旋波近似下与三能级原子相互作用下场熵的研究工作鲜有报道.本文利用全量子理论,采用相干态正交化展开方法<sup>[13-18]</sup>,对非旋波近似下双模光场与原子的量子纠缠进行了精确求解,讨论了初始时刻原子能级的叠加以及平均光子数对量子纠缠的影响.

## 1 模型与求解

考虑一个双模光场与一个级联型三能级原子的相互作用.原子的三个能级从下到上分别用 $|1\rangle$ 、 $|2\rangle$ 和 $|3\rangle$ 描述,其中 $|1\rangle$ 与 $|3\rangle$ 之间的跃迁是禁戒的,能级 $|1\rangle$ 和 $|2\rangle$ 之间以及能级 $|2\rangle$ 和 $|3\rangle$ 之间的跃迁分别与频率为 $\Omega_1$ 和 $\Omega_2$ 的腔场共振,系统的哈密顿量在非旋波近似下表示为(取 $\eta=1$ )

$$H' = H_f + H_a + H_1 \quad (1)$$

$$H_f = \Omega_1 a^+ a + \Omega_2 b^+ b \quad (2)$$

$$H_a = \omega_1 |1\rangle\langle 1| + \omega_2 |2\rangle\langle 2| + \omega_3 |3\rangle\langle 3| \quad (3)$$

$$H_1 = [g_1 \sin \theta (a^+ + a) + g_2 \sin \theta (b^+ + b)] \cdot (|1\rangle\langle 2| + |2\rangle\langle 1|) + [g_1 \cos \theta (a^+ + a) + g_2 \cos \theta (b^+ + b)] (|2\rangle\langle 3| + |3\rangle\langle 2|) \quad (4)$$

式中 $H_f$ 、 $H_a$ 和 $H_1$ 分别为场、原子及其相互作用的哈密顿量; $a^+$ ( $b^+$ ), $a$ ( $b$ )为光场的产生湮没算符; $\omega_1$ 、 $\omega_2$ 和 $\omega_3$ 为原子各个能级的本征频率; $g_1$ 、 $g_2$ 为光场与原子的耦合常量.作SO(3)旋转变换得到对角化的哈密顿量为

$$\mathbf{H} = \begin{bmatrix} h_{11} + P_1 & h_{12} & h_{13} \\ h_{21} & h_{22} + P_2 & h_{23} \\ h_{31} & h_{32} & h_{33} + P_3 \end{bmatrix} \quad (5)$$

其中, $P_1 = \Omega_1 a^+ a + \Omega_2 b^+ b - g_1 (a + a^+) - g_2 (b + b^+)$ , $P_2 = \Omega_1 a^+ a + \Omega_2 b^+ b + g_1 (a + a^+) + g_2 (b + b^+)$ , $P_3 = \Omega_1 a^+ a + \Omega_2 b^+ b$ , $h_{11} = h_{22} = \frac{\omega_3}{2} \sin^2 \theta + \frac{\omega_2}{2} + \frac{\omega_1}{2} \cos^2 \theta$ , $h_{12} = h_{21} = \frac{\omega_3}{2} \sin^2 \theta - \frac{\omega_2}{2} + \frac{\omega_1}{2} \cos^2 \theta$ , $h_{13} = h_{23} = h_{31} = h_{32} = -\frac{\sqrt{2}}{2} \sin \theta \cos \theta (\omega_3 - \omega_1)$ , $h_{33} = \omega_3 \cos^2 \theta + \omega_1 \sin^2 \theta$ .

令系统的波函数为

$$|\psi\rangle = \begin{bmatrix} \sum_{n=0}^N c_{nm} |n_{A-}\rangle |m_{B-}\rangle \\ \sum_{n=0}^N d_{nm} |n_{A+}\rangle |m_{B+}\rangle \\ \sum_{n=0}^N f_{nm} |n\rangle |m\rangle \end{bmatrix} =$$

$$\begin{bmatrix} \sum_{n=0}^N c_{nm} \frac{A_+^+ B_-^+ m}{\sqrt{n! m!}} e^{g_1/\Omega_1 a^+ + g_2/\Omega_2 b^+ - (g_1^2 \alpha_1^+ + g_2^2 \alpha_2^+)/2\alpha_1 \alpha_2} |0\rangle |0\rangle \\ \sum_{n=0}^N d_{nm} \frac{A_+^+ B_+^+ m}{\sqrt{n! m!}} e^{-g_1/\Omega_1 a^+ - g_2/\Omega_2 b^+ - (g_1^2 \alpha_1^+ + g_2^2 \alpha_2^+)/2\alpha_1 \alpha_2} |0\rangle |0\rangle \\ \sum_{n=0}^N f_{nm} \frac{a^+ b^+ m}{\sqrt{n! m!}} |0\rangle |0\rangle \end{bmatrix} \quad (6)$$

式中, $A_\pm^+ = a^+ - g_1/\Omega_1$ , $A_\pm^- = a^+ + g_1/\Omega_1$ , $B_\pm^+ = b^+ - g_2/\Omega_2$ , $B_\pm^- = b^+ + \frac{g_2}{\Omega_2}$ 是新的玻色算符; $|n_{A-}\rangle$ , $|m_{B-}\rangle$ , $|n_{A+}\rangle$ , $|m_{B+}\rangle$ 是新玻色算符的Fock态, $|n\rangle$ , $|m\rangle$ 为Fock态将式(5)和(6)代入薛定谔方程后分别左乘 $\langle k_{A-}|$ , $\langle l_{B-}|$ , $\langle k_{A+}|$ , $\langle l_{B+}|$ 和 $\langle k|$ , $\langle l|$ 得

$$h_{12} \sum_{nm=0}^N d_{nm} \langle k_{A-}|n_{A+}\rangle \langle l_{B-}|m_{B+}\rangle + (h_{11} + \Omega_1 k_{A-} - \frac{g_1^2}{\Omega_1} + \Omega_2 l_{B-} - \frac{g_2^2}{\Omega_2}) c_{kl} + h_{13} \sum_{nm=0}^N f_{nm} \langle k_{A-}|n\rangle \cdot \langle l_{B-}|m\rangle = E c_{kl} \quad (7)$$

$$h_{21} \sum_{nm=0}^N c_{nm} \langle k_{A+}|n_{A-}\rangle \langle l_{B+}|m_{B-}\rangle + (h_{22} + \Omega_1 k_{A+} - \frac{g_1^2}{\Omega_1} + \Omega_2 l_{B+} - \frac{g_2^2}{\Omega_2}) d_{kl} + h_{23} \sum_{nm=0}^N f_{nm} \langle k_{A+}|n\rangle \cdot \langle l_{B+}|m\rangle = E d_{kl} \quad (8)$$

$$h_{31} \sum_{nm=0}^N c_{nm} \langle k|n_{A-}\rangle \langle l|m_{B-}\rangle + h_{32} \sum_{nm=0}^N d_{nm} \langle k|n_{A+}\rangle \cdot \langle l|m_{B+}\rangle + (h_{33} + \Omega_1 k + \Omega_2 l) f_{kl} = E f_{kl} \quad (9)$$

令

$$D_{ij}(x) = \sum_{r=0}^{\min(i,j)} \frac{\sqrt{i! j!} (-1)^{-r} x^{i+j-2r}}{r! (i-r)! (j-r)!} e^{-x^2/2}, \text{有}$$

$$\langle k_{A-}|n_{A+}\rangle = (-1)^k D_{kn} \left( \frac{2g_1}{\Omega_1} \right),$$

$$\langle l_{B-}|m_{B+}\rangle = (-1)^l D_{lm} \left( \frac{2g_2}{\Omega_2} \right),$$

$$\langle k|n_{A+}\rangle = \langle k_{A-}|n\rangle = (-1)^k D_{kn} \left( \frac{g_1}{\Omega_1} \right),$$

$$\langle l|m_{B+}\rangle = \langle l_{B-}|m\rangle = (-1)^l D_{lm} \left( \frac{g_2}{\Omega_2} \right),$$

$$\langle k|n_{A-}\rangle = \langle k_{A+}|n\rangle = (-1)^n D_{kn} \left( \frac{g_1}{\Omega_1} \right),$$

$$\langle l|m_{B-}\rangle = \langle l_{B+}|m\rangle = (-1)^m D_{lm} \left( \frac{g_2}{\Omega_2} \right),$$

$$\langle k_{A+}|n_{A-}\rangle = (-1)^n D_{kn} \left( \frac{2g_1}{\Omega_1} \right),$$

$$\langle l_{B+}|m_{B-}\rangle = (-1)^m D_{lm} \left( \frac{2g_2}{\Omega_2} \right).$$

通过求解式(7)、(8)和(9),可得系统的本征波函数 $\{c_n^{(i)}\}$ 、 $\{d_n^{(i)}\}$ 和 $\{f_n^{(i)}\}$ .

假设原子初始时刻处于能级的叠加态 $C_a |1\rangle + C_b |2\rangle + C_c |3\rangle$ ( $C_a$ , $C_b$ 和 $C_c$ 分别是各能级的叠加系数),两个光场都处于相干态 $|\alpha\rangle = e^{(\alpha a^+ - a^2/2)} |0\rangle$ , $|\beta\rangle = e^{(\beta b^+ - \beta^2/2)} |0\rangle$ ,两腔场的平均光子数以及光场与原子的耦合强度始终相等( $\bar{n} = \alpha^2 = \beta^2$ , $g_1 = g_2$ ).如果原子

初始时刻处于  $|3\rangle$  能级 ( $C_a = C_b = 0, C_c = 1$ ), 系统初始时刻的波函数为

$$|\psi(0)\rangle = e^{a\alpha^+ - \alpha^2/2} |0\rangle e^{b\beta^+ - \beta^2/2} |0\rangle |3\rangle = \sum_{i=0}^{3N^2} \nu_i \left[ \sum_{n,m=0}^N c_{nm}^{(i)} \cdot |n_{A-}\rangle |m_{B-}\rangle |3'\rangle + \sum_{n,m=0}^N d_{nm}^{(i)} |n_{A+}\rangle |m_{B+}\rangle |2'\rangle + \sum_{n,m=0}^N f_{nm}^{(i)} |n\rangle |m\rangle |1'\rangle \right] \quad (10)$$

式中,  $\nu_i$  是叠加系数,  $|3'\rangle, |2'\rangle$  和  $|1'\rangle$  为 SO(3) 变换后的原子能级. 将式(10)分别左乘  $\langle l_{B-} | \langle k_{A-} |, \langle l_{B+} | \langle k_{A+} |$  和  $\langle l | \langle k |$  后得

$$\frac{U_{11} e^{-(g_1 + \Omega_1)^2/2\Omega_1^2} e^{-(g_2 + \Omega_2)^2/2\Omega_2^2}}{\sqrt{l_b! k_a!}} (\alpha - \frac{g_1}{\Omega_1})^{k_a} (\beta - \frac{g_2}{\Omega_2})^{l_b} = \sum_{i=0}^{3N^2} \nu_i c_{kl}^{(i)} \quad (11)$$

$$\frac{U_{21} e^{-(g_1 + \Omega_1)^2/2\Omega_1^2} e^{-(g_2 + \Omega_2)^2/2\Omega_2^2}}{\sqrt{l_b! k_a!}} (\alpha + \frac{g_1}{\Omega_1})^{k_a} (\beta + \frac{g_2}{\Omega_2})^{l_b} = \sum_{i=0}^{3N^2} \nu_i d_{kl}^{(i)} \quad (12)$$

$$\frac{U_{31} e^{-(g_1 + \Omega_1)^2/2\Omega_1^2} e^{-\beta^2 + \alpha^2/2}}{\sqrt{l_b! k_a!}} \alpha^k \beta^{l_b} = \sum_{i=0}^{3N^2} \nu_i f_{kl}^{(i)} \quad (13)$$

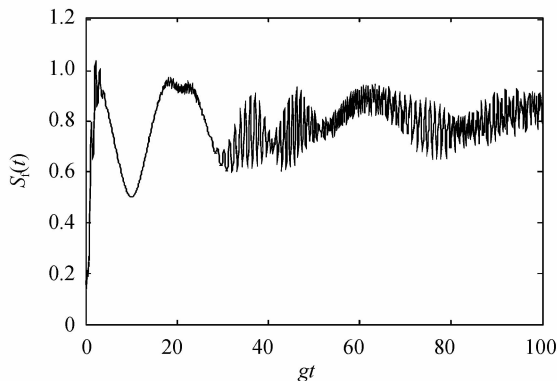
通过求解式(11)、(12)和(13)可得  $\{\nu_i\}$  任意时刻的波函数为

$$|\psi(t)\rangle = \sum_{i=0}^{3N^2} \nu_i e^{-iE_i t} \left[ \sum_{n,m=0}^N c_{nm}^{(i)} |n_{A-}\rangle |m_{B-}\rangle |3'\rangle + \sum_{n,m=0}^N d_{nm}^{(i)} |n_{A+}\rangle |m_{B+}\rangle |2'\rangle + \sum_{n,m=0}^N f_{nm}^{(i)} |n\rangle |m\rangle |1'\rangle \right] \quad (14)$$

## 2 量子纠缠

本文利用场(原子)熵来度量原子与光场相互作用时产生的量子纠缠. 冯诺依曼熵<sup>[7]</sup>为  $S = -\text{Tr} \rho \ln \rho$ , 其中  $\rho = |\psi(t)\rangle \langle \psi(t)|$  为系统的密度矩阵, 那么原子的约化密度矩阵为

$$\rho_a = \text{Tr}_f \rho = \sum_{i,j=0}^{3N^2} \nu_i \nu_j^* e^{-i(E_i - E_j)t} \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{bmatrix}$$



(a)  $\bar{n}=10$

其中,  $\rho_{11} = \sum_{kl=0}^N c_{kl}^{(i)} c_{kl}^{(j)*}$ ,  $\rho_{22} = \sum_{kl=0}^N d_{kl}^{(i)} d_{kl}^{(j)*}$ ,

$$\rho_{33} = \sum_{kl=0}^N f_{kl}^{(i)} f_{kl}^{(j)*},$$

$$\rho_{12} = \sum_{nmkl=0}^N c_{nm}^{(i)} d_{kl}^{(j)*} \langle k_{A+} | n_{A-} \rangle \langle l_{B+} | m_{B-} \rangle,$$

$$\rho_{13} = \sum_{nmkl=0}^N c_{nm}^{(i)} f_{kl}^{(j)*} \langle k | n_{A-} \rangle \langle l | m_{B-} \rangle,$$

$$\rho_{21} = \sum_{nmkl=0}^N d_{nm}^{(i)} c_{kl}^{(j)*} \langle k_{A-} | n_{A+} \rangle \langle l_{B-} | m_{B+} \rangle,$$

$$\rho_{31} = \sum_{nmkl=0}^N f_{nm}^{(i)} c_{kl}^{(j)*} \langle k_{A-} | n \rangle \langle l_{B-} | m \rangle,$$

$$\rho_{32} = \sum_{nmkl=0}^N f_{nm}^{(i)} d_{kl}^{(j)*} \langle k_{A+} | n \rangle \langle l_{B+} | m \rangle,$$

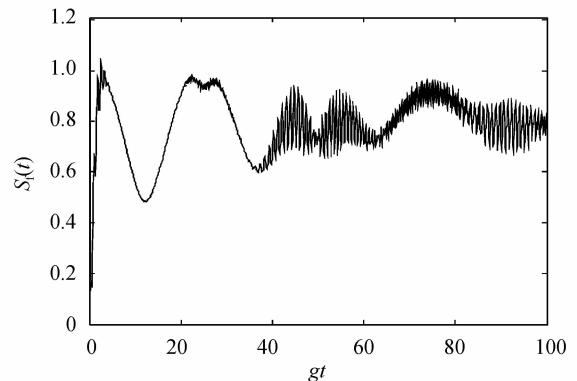
由于选择式(10)作为初态, 初始光场和原子均处于纯态, 整个系统的熵为零, 且不随时间变化. 根据 Araki-Lieb 不等式可知, 在  $t \geq 0$  的任意时刻, 光场与原子熵相等, 可利用原子约化密度矩阵得到量子系统的场(原子)熵, 即

$$S_f(t) = S_a(t) = -\text{Tr} \{\rho_a \ln \rho_a\} = -\sum_k \lambda_k \ln \lambda_k \quad (15)$$

式中  $\lambda_k$  ( $k=1, 2, 3$ ) 为原子约化密度矩阵的本征值.

## 3 结果与讨论

图 1 为原子初始时刻处于激发态 ( $C_a = C_b = 0, C_c = 1$ ),  $\Omega_1 = \Omega_2 = 100g_1 = 100g_2$ , 平均光子数  $\bar{n}$  取不同值时, 量子纠缠的演化. 从图 1 可知, 量子纠缠演化曲线总是先经过强烈的振荡, 之后达到并基本保持较为稳定的最大值 ( $\ln 3 \approx 1.1$ ). 随着平均光子数的增大, 纠缠演化曲线的周期逐渐增大, 并且曲线不再光滑且出现了小锯齿状的振荡, 这主要是非旋波项的贡献. 非旋波项跃迁所产生光子的寿命很短, 称为虚光子, 根据海森堡能量—时间不确定关系, 非旋波项对应的跃迁过程导致系统能量改变很大, 因此在光场与原子相互作用系统中, 虚光子对光场与原子相互作用系统具有很大的影响, 图中小锯齿状的振荡, 即为量子噪音, 说明平均光子数较大时非旋波项的贡献不可忽略.



(b)  $\bar{n}=15$

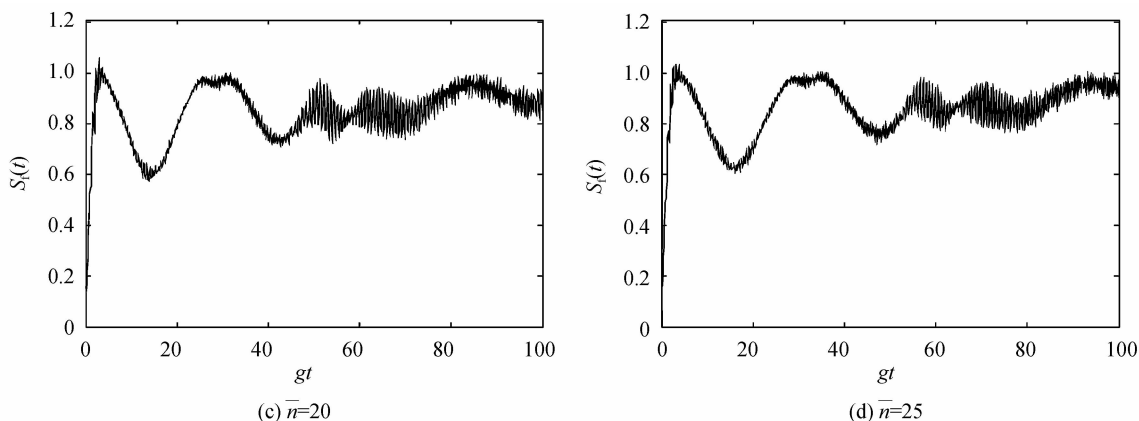


图 1 原子初始时刻处于激发态时量子纠缠的演化

Fig. 1 Entanglement evolution when the atom is initially in the excited state

图 2 为  $\Omega_1 = \Omega_2 = 100g_1 = 100g_2$ , 原子初始时刻处于  $|2\rangle$  和  $|3\rangle$  能级叠加态 ( $C_a = 0, C_b = C_c = \sqrt{2}/2$ ), 平均光子数  $\bar{n}$  取不同值时纠缠的演化. 从图 2 中看到, 纠缠演化曲线不具有周期性, 也没有原子初始时刻处在单一能级时(图 1)较强烈的振荡. 由于原子初始时刻能级的叠加使得量子纠缠在初始阶段的纠缠度降低. 图 1 中初始阶段的纠缠约为 1.1, 图 2 中初始阶段的纠缠

约为 0.65, 随着演化时间的增加, 纠缠会在达到最大值附近振荡, 说明在纠缠制备的时候, 由于原子能级的叠加, 初始阶段无法得到较大纠缠, 而此时纠缠稳定增长没有大的波动. 对比图 2 可以发现, 虽然原子能级初始时刻处在叠加态, 随着平均光子数的增加纠缠演化依然呈现出小锯齿状的振荡, 表明非旋波效应逐渐加强.

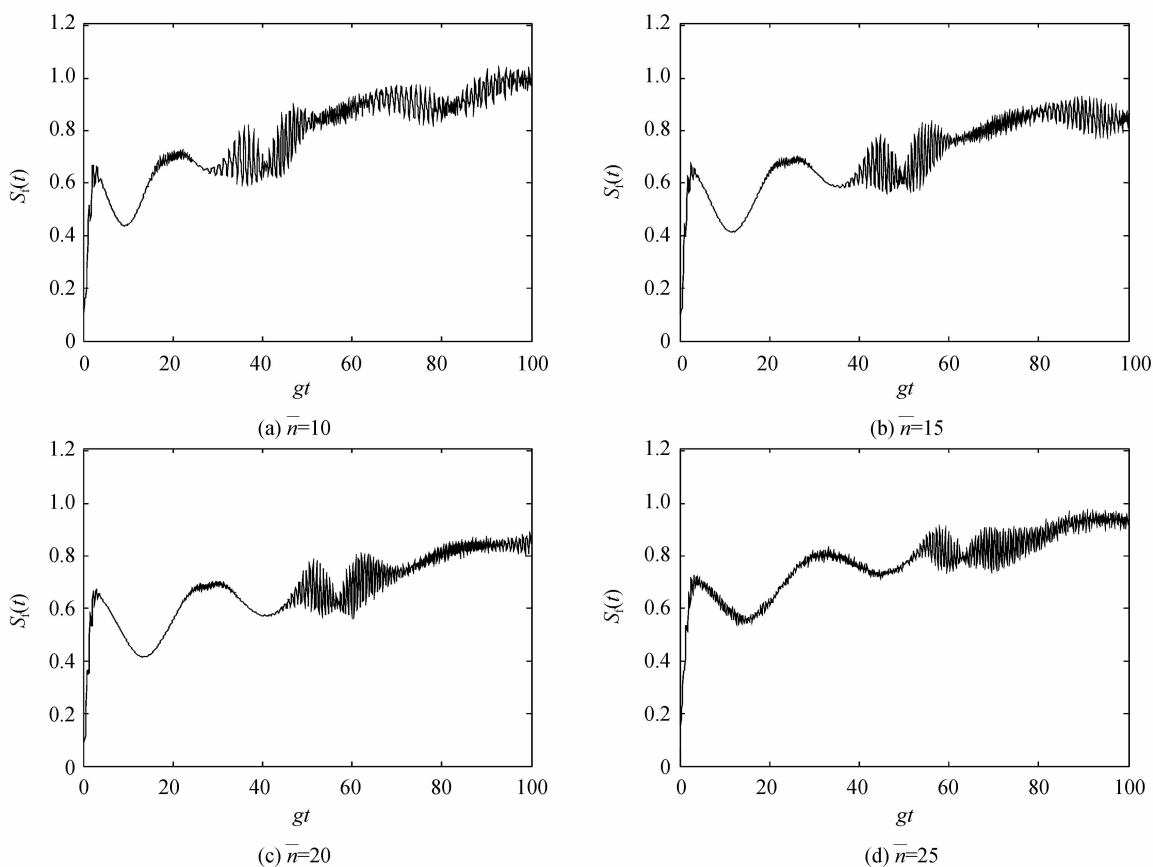


图 2 原子初始时刻处于  $|2\rangle$  和  $|3\rangle$  能级叠加态时量子纠缠的演化

Fig. 2 Entanglement evolution when the atom is initially in the superposition state of  $|2\rangle$  and  $|3\rangle$

图 3 为  $\Omega_1 = \Omega_2 = 100g_1 = 100g_2$ , 原子初始时刻处于三个能级的等权叠加态 ( $C_a = C_b = C_c = \sqrt{3}/3$ ), 平均

光子数  $\bar{n}$  取不同值时纠缠的演化. 从图 3 可以看到, 纠缠演化曲线不具有周期性, 也未见图 1 中出现的强烈

振荡. 初始时刻原子三个能级的叠加使得纠缠在初始阶段的纠缠度显著降低, 约为 0.2, 随着演化时间的增加纠缠仍然会在达到最大值附近平稳振荡. 随着平均光子数  $\bar{n}$  的逐渐增大, 如图 3(d), 光场与原子纠缠达到

稳定最大值的时间逐渐变长. 通过对比可以发现, 虽然原子能级初始时刻处在叠加态, 随着平均光子数的增加纠缠演化依然呈现出现小锯齿状的振荡, 说明非旋波效应逐渐加强.

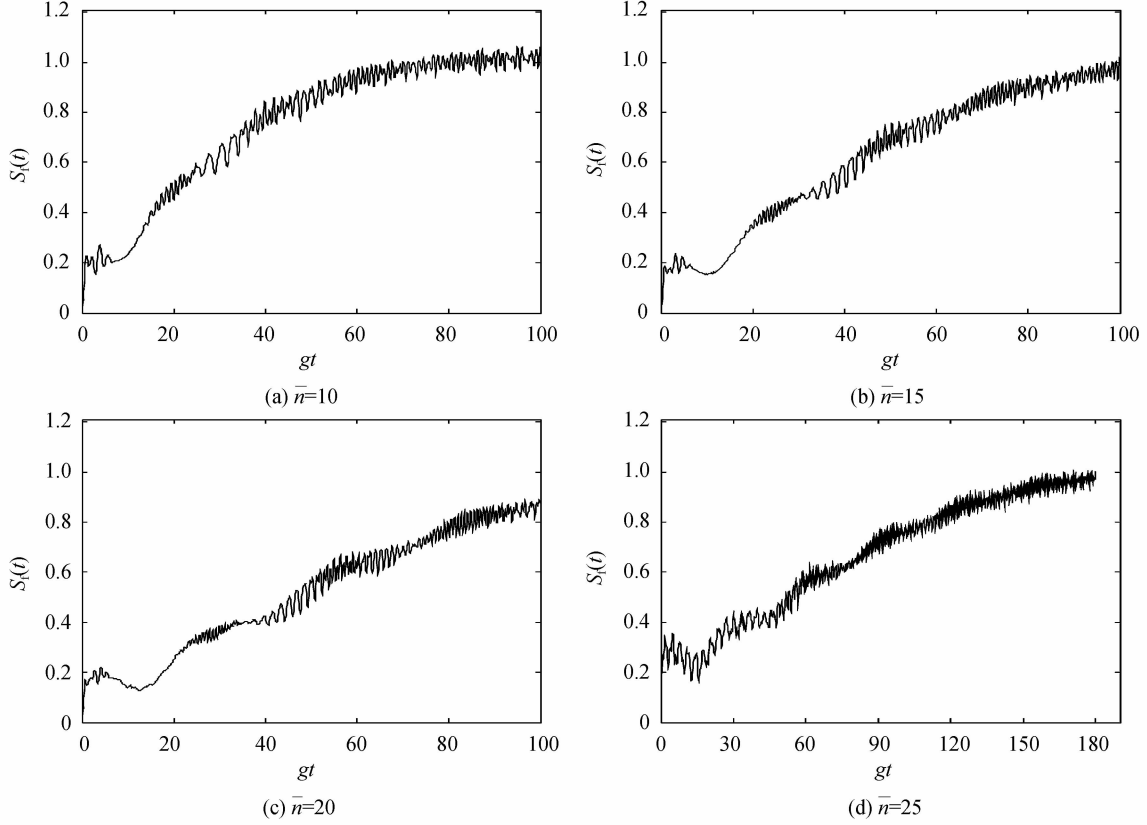


图 3 原子初始时刻处于三个能级等权叠加态时量子纠缠的演化

Fig. 3 Entanglement evolution when the atom is initially in the superposition state of all three levels

## 4 结论

本文利用相干态正交化展开方法, 在非旋波近似下对双模相干态光场与级联型三能级原子之间的量子纠缠进行了精确求解, 数值计算结果表明, 初始时刻原子处于激发态时, 随着平均光子数  $\bar{n}$  的增加, 量子纠缠演化周期逐渐增大. 初始时刻原子能级的叠加, 导致初始阶段纠缠度显著降低, 且纠缠达到最大值的时间逐渐变长. 无论初始时刻原子能级处于哪个状态, 随着平均光子数  $\bar{n}$  的增大, 非旋波效应逐渐增强.

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