第44卷第4期　　光　子　学　报　　Vol. 44 No. 4
2015年4月　　ACTA PHOTONICA SINICA　　April 2015

doi:10.3788/gzxb20154404.0426002

少周期飞秒径向偏振脉冲聚焦特性仿真

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摘　要:建立聚焦数值模型,分析少周期径向偏振光聚焦光场的强度以及电场矢量方向周期性变化规律,对比无啁啾少周期径向偏振光脉冲及连续光束聚焦光场的时空电矢量及强度分布.结果表明,光强从聚焦光斑中心处向四周迅速减小且振动方向周期性变化;在相同能量下,相比于连续光,脉冲光的聚焦光斑更小,峰值功率更强;通过控制光程,无啁啾少周期径向偏振光束聚焦场的前向电场强于后向电场,适合于激光电子直接加速.该结果对指导利用少周期径向偏振光进行激光电子直接加速的方案设计及超快显微成像和超快探测具有参考意义.

关键词:径向偏振光;少周期;飞秒脉冲;聚焦场;电场
中图分类号:O436;O438　　**文献标识码**:A　　**文章编号**:1004-4213(2015)04-0426002-5

Numerical Modelling of the Focusing Features for Few-cycle Radially Polarized Femtosecond Pulses

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Abstract: In order to investigate the detail propertities of the focal spots for the few-cycle radially polarized femtosecond pulses, the numerical focusing modeling was established, and the intensity and vector distribution of the focal electric field were calculated and simulated. The temporal and spatial properties of the focal electric field are included in the modeling. According to the simulation results, the intensity decreases sharply from the focal center, and the oscillation direction changes periodically. Compared with the CW laser beam with the same power, the pulse possesses smaller focusing spot and higher peak power. Furthermore, by controlling the optical length, the synthetic electric fields in the forward direction are of greater ratio than the backward electric fields for a non-chirp few-cycle radially polarized femtosecond pulses beam, which will be favorable for direct laser-induced electron acceleration. The analysis is significant for designing scheme for electron acceleration with the few-cycle radially polarized femtosecond pulses, and also can be a valuable reference for the ultra-fast microscopic imaging and detection.

Key words: Radially polarized light, Few-cycle, Femtosecond pulse, Focusing fields, Electric vector
OCIS Codes: 260.1960 000.4430; 320.2250; 320.5550; 320.7090

0　Introduction

As a cylindrical vector light beam with original and form-unique polarized light pattern, the Radially Polarized light Beam (RPB) has different polarization mechanism compared with the linearly polarized and the circularly polarized light beams, i. e. with sharper and stronger symmetrical focal spot[1-4].

Foundation item: The National Natural Science Foundation of China (Nos. 61070040, 61108089, 61205087, 61107005 and 51302316), the Hunan Provincial Natural Science Foundation of China (No. 13JJ4003), and the Advanced Research Foundation of National University of Defense Technology (No. JC13-02-09)

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Received: Oct. 8, 2014; **Accepted**: Nov. 27, 2014

Recently, researches on the RPB are increasing boomingly and continuously. By tightly focused and pulsed radially-polarized laser beams, it has already made the jobs realized such as the direct acceleration of a free electron in infinite vacuum along the axis[5-6], the optical trapping of micrometer-sized dielectric particles[7-8], and calculating radiation forces on a Rayleigh particle[9] and so on. Besides, researchers also found that the source coherence length and the maximal numerical aperture angle have direct influence on the focal intensity, as well as the coherence and polarization properties which are important in optical micromanipulation and beam shaping[10]. Furthermore, the scintillation index of a partially coherent radially polarized beam can be smaller than that of a completely coherent beam[11], such that the RPB would make some sense in the information transmission.

In order to find out the focusing features in details for the few-cycle radially polarized pulses beam, modeling simulations are indispensable since most recent work done are based on the scalar focusing and only concern about the intensity amplitude, neglecting the change of electric vectors. Early in 1959, Richard and Wolf established the lens focusing theory in cylindrical coordinate based on the vector[12-13], which obtained some satisfied results and is more suitable for the study of the focusing of randomly polarized beams. Here, the focusing details of RPB, especially the spatial-temporal distribution of the electric vector around the focal spot will be analyzed.

1 Basic features of RPB

For the RPB, the oscillation direction of electrical vector in the cross profile possesses the advantage of axial symmetry, and it propagates along the radial direction all the time with the expression as following.

$$\boldsymbol{E}(r,\alpha)=E_0(r)[\cos\varphi\boldsymbol{X}+\sin\varphi\boldsymbol{Y}]=E_0(r)\boldsymbol{r} \tag{1}$$

In which, the $E_0(r)$ represents the amplitude of electric field and has the form of Bessel-Gauss function solved on the basis on the Maxwell′s equations. According to the expression, the beams only require the direction of oscillating be radial, while its spatial phase distribution is not mentioned. In fact, the RPB can be divided into two kinds, i. e., in-phase and out-phase, which usually referred to is the former.

Except for the features of the symmetry and the zero light intensity on the optical axis, this beam can also generate a tiny spot beyond the diffraction limit when focused by the lens of high numerical aperture. And just because of the geometric distribution of electric field, every two symmetrical oscillations are counteracted with no transversal components and the resultant longitudinal field $E_{\text{resultant}}$ (shown in Fig. 1) is consequently and considerably large. And for the few-cycle ones, they do not only have the excellent focusing features of RPB, but also enjoy the super-high peak power and super-narrow pulse width, which would be widely applied on the direct laser acceleration, super-density storage and etc. This character could be attributed to the analysis of the properties of the few-cycle RPB.

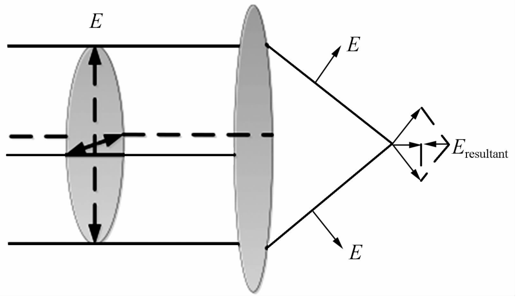


Fig. 1 The schematic diagram of focusing of RPB

Especially, for a time-harmonic plane wave with wave length λ_0, frequency ω, wave vector $\boldsymbol{k}_0=(0,0,k_{0z})$, and cross profile $f(\theta)=\exp\left(-\frac{\sigma^2}{\sin^2\alpha}\sin^2\theta\right)$ on the focusing lens along the z axis, where θ is deflection angle and α is the largest one. And the distribution of focusing fields at the point $P(r_p,\varphi_p,z=0)$ can be calculated[12-13] as below.

$$\boldsymbol{E}^r(r_P,\varphi_P)=2A\Gamma_1\cos(2\varphi_p)\boldsymbol{X}+2A\Gamma_1\sin(2\varphi_p)\boldsymbol{Y}+\mathrm{i}2A\Gamma_0\boldsymbol{Z} \tag{2}$$

where, A is a constant, and

$$\Gamma_0=\int_0^\alpha f(\theta)\mathrm{e}^{\mathrm{i}k_z z}\sqrt{\cos\theta}\sin^2\theta\mathrm{J}_0(kr_P\sin\theta)\mathrm{d}\theta \tag{3}$$

$$\Gamma_1=\int_0^\alpha f(\theta)\mathrm{e}^{\mathrm{i}k_z z}\sqrt{\cos^3\theta}\sin^2\theta\mathrm{J}_1(kr_P\sin\theta)\mathrm{d}\theta \tag{4}$$

Assumed that the expression of the pulse shape is as below

$$E(z,t)=A(z,t)\mathrm{e}^{\{\mathrm{i}[\tilde{\omega}_0 t-kz+\varphi_0(z,t)]\}} \tag{5}$$

where $A(z,t)$ representing the pulse Gaussian envelope expression as

$$A(z,t)=A_0\mathrm{e}^{\{-1.385(t/\tau_p)^2\}} \tag{6}$$

And $\varphi_0(z,t)$ is the Carrier-Envelope Phase (CEP).

So the focusing field distribution can be expressed as

$$F(r_p,\varphi_p,z,t)=E^r(r_p,\varphi_p)E(z,t) \tag{7}$$

According to the Fourier transform limited relation

$$\Delta\lambda=\frac{0.441\lambda^2}{c\tau_p} \tag{8}$$

For a Fourier limited femtosecond pulse, whose CEP $\varphi_0(z,t)=0$, pulse duration $\tau_p=30$ fs, and central wavelength of 800 nm, the broadband wavelength will be from 770 nm to 830 nm. According to the shape of the radially polarized beam, the on-axis parabolic mirror with high NA will be favorable for the pulse focusing.

It is known that the few-cycle femtosecond modelocked pulse is superposed by numerous oscillations of varied frequencies in transmission[14]. In order to achieve maximum go-ahead electric field at the focal area, the phase distribution of the oscillations of varied frequencies should be adjusted to oscillate at the maximum positive point at the focal plane.

2 Simulations and discussions

For the further figuring, the input Fourier transform limited pulse can be expressed as

$$E(z,t=0)=A_0\mathrm{e}^{-1.385\left(\frac{z}{c\tau_p}\right)^2}\mathrm{e}^{-\mathrm{i}kz} \tag{9}$$

With the above Eqs. (1)～(3) and assumed that $z=0$ (at focal plane), the features of focusing including the energy and the electric field distribution on the transversal and longitudinal direction can be as Eq. (10).

$$I=\left|E^r(r_P,\varphi_P)\right|^2 \tag{10}$$

when $t=0$, i. e., the oscillation with largest amplitude gets focused, the corresponding intensity distribution can be shown as Fig. 2, in which the NA is 0.9 and the focal length is 1 mm.

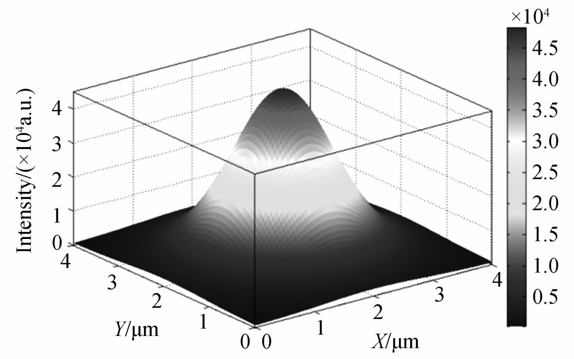


Fig. 2 Synthetic focused field at $t=0$ where the amplitude is the largest

When $r_p=0$, i. e., in the focusing central along the z axis, the relevance between the intensity I and z values (delay time relative to $t=0$) is shown in Fig. 3. From the fitting curve in Fig. 3, the FWHM around the focal plane is about $0.7\tau_p$, shorter than that of the incident pulse (τ_p), which indicates that after focusing the pulse will be compressed. The reason is that the intensity in focal spot increases with the original values when focused, and that the relations are nonlinear. And the symmetric intensity distributions before and after the focal plane ($t=0$) is derived from the radial symmetric polarization distribution of polarized beam, and the symmetric effect of the focusing device, which can be illustrated with the geometrical optics as Fig. 1.

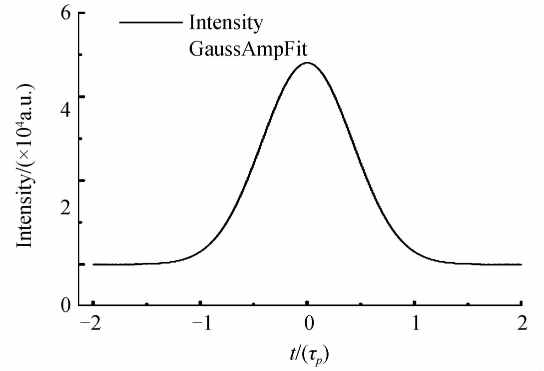
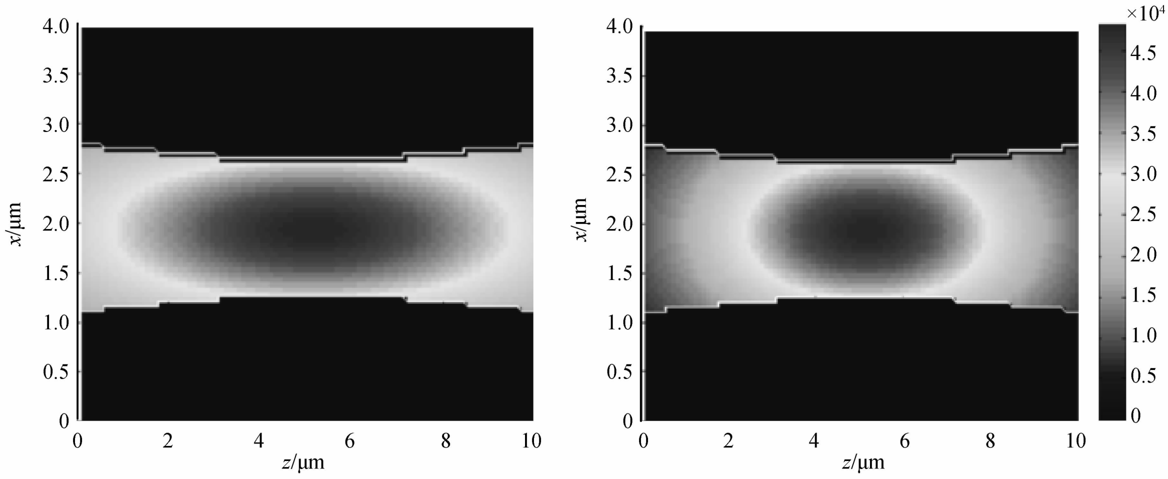


Fig. 3 The intensity at the central of focusing field at different time around focusing

When the value of z is not zero in the integrations of Eqs. (2) and (3), we can get the focusing distribution around a certain distance. Furthermore, thanks to the focusing symmetry of RPB, the result is reasonable judging from the cross profile.

According to the vector focus theory of Richards-Wolf, the intensity distribution along z axis can be obtained as shown in Fig. 4. Obviously, in comparison with the CW, the distribution on the focal plane of the pulse has a sharper decrease, which means more energy concentration. This is due to the fact that the pulse already has the intensity distribution before focusing of



(a) CW

(b) Few-cycle pulse

Fig. 4 Cross profile in focusing intensity field which the peak power would be much higher and the power of most other places are weaken , while the CW does not. And the FWHM of the pulse would be quite smaller in comparison.

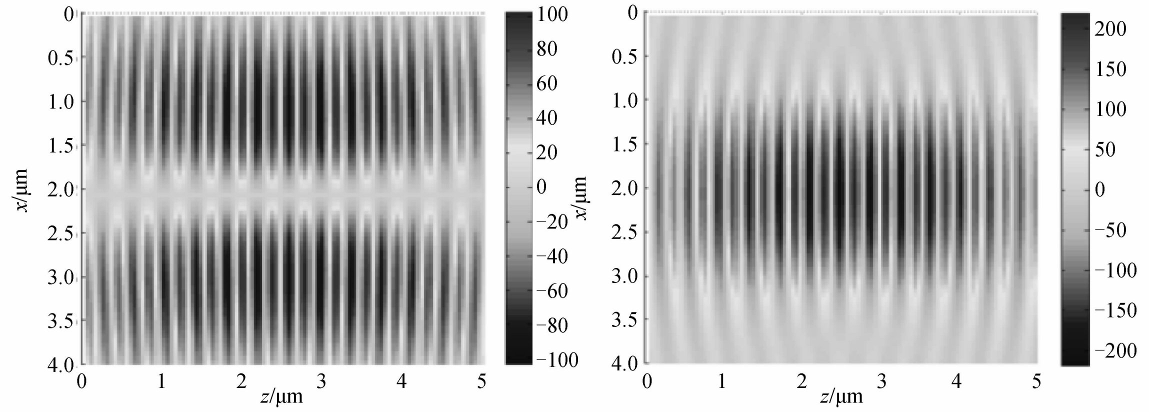
On the other hand, in order to analyze the electric field force amplitude and direction on the charged particles, the distributions of transversal and longitudinal electric vector is pretty essential, which directly determines the motion trace of the particles. Also, according to Eq. (2), the electric vector in the x-z plane can be calculated as

$$\boldsymbol{E}_x=\mathrm{real}(2A\Gamma_1\cos 2\varphi_p) \tag{11}$$

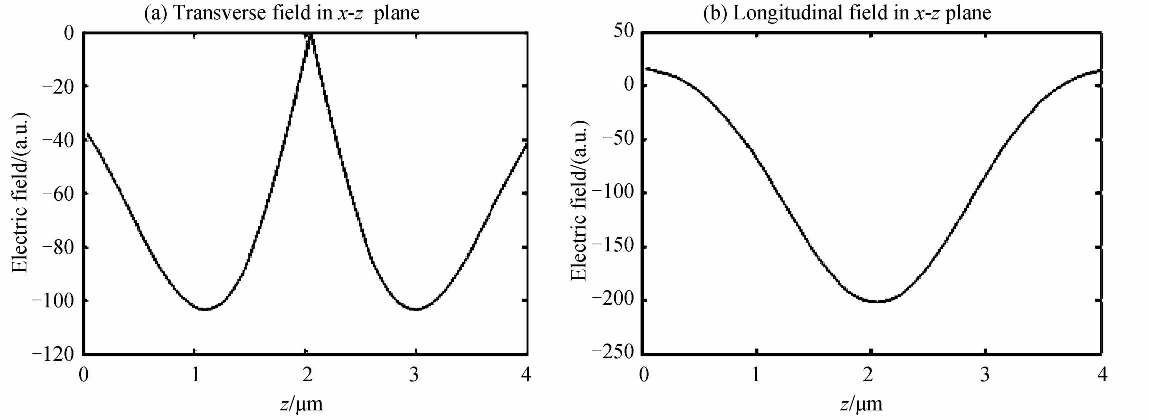
And

$$\boldsymbol{E}_z=\mathrm{real}(\mathrm{i}2A\Gamma_0) \tag{12}$$

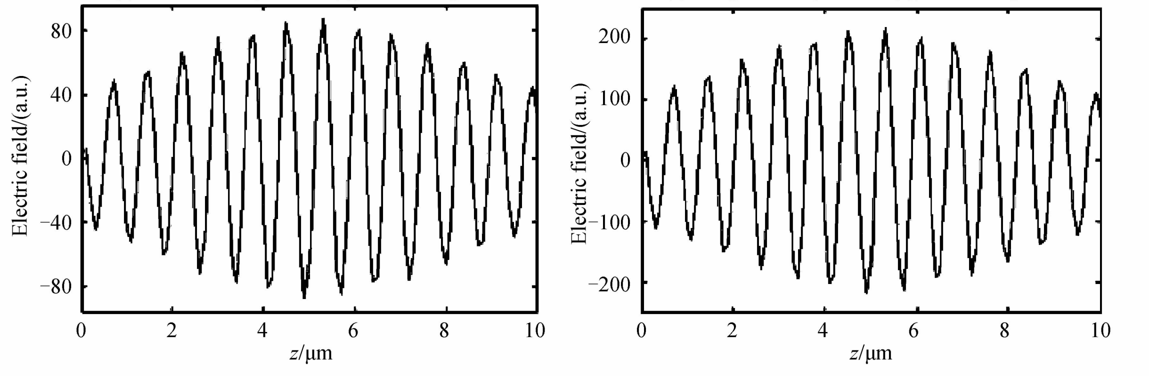
In Fig. 5, (a) and (b) give us the overviews of the transversal and longitudinal field around the focusing area, and the other four figures, i. e., (c)～(f), are the details of a certain line or column cross the focal point, all of which give the manifestation of the vector focusing.



(a) Transverse field in x-z plane (b) Longitudinal field in x-z plane



(c) Transeverse (z=2.2 in (a)) field along x axis (d) Longitudinal (z=2.1 in (b)) field along x axis



(e) Transeverse (x=1.5 in (a)) field along z axis (f) Longitudinal (x=2.1 in (b)) field along z axis

Fig. 5 Electric vector components

In Fig. 5(a), the transversal $\boldsymbol{E}$ component is zero at the central axis, which is the result of the fact that in an in-phase RPB, the transversal electric vector is cancelled out when compounded for every two centro-symmetric points, while the longitudinal one is strengthened, as shown in Fig. 5 (b). Furthermore, as the direction of electric field changes periodically, the resultant field also shows the phenomenon of ups and downs. The peak at $z=2.2$ in Fig. 5 (c) should be highlighted. Normally, according to the Eq. (11), the curve should be smooth and sine-like or cosine-like shape. However, because of the vectorial resultant, the curve is transversally zygomorphic from the zero point, and it causes such a sharp saltation. According to Fig. 5 (e) and Fig. 5 (f), both the transverse and longitudinal electric fields will fluctuate with the change of z value, and the amplitude will decrease from the central to the sides, and the difference is only about the amplitude value.

Besides, when the pulse gets shorter to contain only few-cycle, the number of oscillations in a pulse would be much more numerous, which can generate higher peak power and electric field beneficial to further applications.

3 Conclusions

The numerical modeling on the focusing features of the few-cycle RPFP is established. The regular distributions such as the obvious symmetry and the intensity periodically fluctuating from the axis along the focusing field of few-cycle RPFB are given, involving the absolute intensity and direction of the electric vector, which indicates that the radial symmetry of the beams and is the key to the force analysis for the particles in the accelerating areas. Since the value at maximum point changes so sharply and the point is so close to the negative maximum one that a dinky distance can cause fairly different results. Obviously, the synthetic electric fields in the forward direction are of greater ratio than the backward electric fields for a non-chirp few-cycle radially polarized femtosecond pulses beam, which will be favorable for direct laser-induced electron acceleration.

For a radially polarized femtosecond pulses beam, the ultra-fast oscillating light electric vectors in the focal spot will be symmetrical and will produce fine synthetic electric vector, which is determined by the temporal and spectral properties of the few-cycle ultra-short pulse, and will significantly determine the results of the laser- induced electrons acceleration. The analysis is significant for designing scheme for particle acceleration with the few-cycle radially polarized femtosecond pulses, and also can be a valuable reference for the ultra-fast microscopic imaging and the super-fast detection.

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