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# 周期条状结构中长程表面等离子激元噪声特性分析

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**摘 要:** 基于表面等离子激元自发辐射放大噪声系数理论, 研究了长程表面等离子激元噪声系数变化规律, 设计了嵌有金属条的长程表面等离子结构. 数值模拟得到表面等离子及周期条状长程表面等离子噪声系数变化规律. 给出入射波长从  $0.5 \mu\text{m}$  到  $1.75 \mu\text{m}$ , 每隔  $0.25 \mu\text{m}$  的噪声特性, 发现随着波长的增加噪声系数趋近于常数. 仿真结果表明: 表面等离子及长程表面等离子自发辐射放大的噪声系数随着增益介质增大而明显增强; 周期条状长程表面等离子结构中, 随着金属膜厚度以及金属条高度及个数增加导致噪声系数增加, 其中金属膜厚度对于其作用较明显.

**关键词:** 表面等离子激元; 长程表面等离子激元; 自发辐射放大; 噪声系数; 增益介质

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## Characteristic Analysis of Noise Factor in the Periodic Stripe Long Range Surface Plasmon Polaritons Structure

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**Abstract:** Through the noise factor theory of amplified spontaneous emission of Surface Plasmon Polaritons (SPPs), the variations of noise factor of Long Range Surface Plasmon Polaritons (LRSPPs) were studied. And a waveguide structure with strips embedded in the metal film of LRSPPs structure is designed. Numerical simulation is used to get the noise factor variations of SPPs and LRSPPs. In the numerical simulation, incident wavelength ranges from  $0.5 \mu\text{m}$  to  $1.75 \mu\text{m}$ , and check the characteristics of noise factors for every  $0.25 \mu\text{m}$ , the results show that the noise factors approach to a constant value with the increase of wavelength. The simulation results show that the noise factor of the amplified spontaneous emission of the SPPs and the LRSPPs are enhanced apparently with the increase of grain mediums; noise factors of periodic stripe LRSPPs are increased as the thicknesses of metal film and the height and the number metal strip are increased, where the impact of the thickness of metal film is relatively evident.

**Key words:** Surface Plasmon Polaritons (SPP); Long Range Surface Plasmon Polaritons (LRSPPs); Amplified spontaneous emission; Noise factor; Gain medium

**OCIS Codes:** 190.4350; 240.6680; 250.4480; 310.6870

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## 0 Introduction

Surface Plasmon Polaritons (SPPs) [1] are TM surface waves which propagate along the metal-dielectric interface [2-3]. SPPs are applied in optical devices [4], such as beam splitter [5], the reflector [6], filters [7], sensor [8-9], but their high attenuation limits broader application. Long Range SPPs (LRSPPs) are excited by the structure that a thin metal film or stripe are bounded by the same dielectric on each side [2]. Long range means it propagates a longer distance than SPPs, for the attenuation of this structure is lower than SPPs. LRSPPs have shown great potential application in amplifiers [10-11] and lasers [12-13]. The study of SPPs exaction in the quantum level was reported by D. Ballester et. al, who measured single-photons and photon-number states. This is the theoretical basis for the study of LRSPPs amplification [14]. De Leon et. al were the first who measured directly the gain in the long-range surface plasmon polaritons and the measured mode power gain was  $8.55 \text{ dB} \cdot \text{mm}^{-1}$  [15]. Berini et. al presented the theory model of LRSPPs Amplified Spontaneous Emission (ASE) which was verified by a symmetric metallic waveguide incorporating a gain medium. And when the incident wavelength is 876 nm, the effective input noise power per unit bandwidth is  $7.47 \times 10^{-4} \text{ fW/Hz}$ , which is good agreement with experimental results [16]. It means the low spontaneous emission rate of long-range surface plasmon polaritons amplification is possible. De Leon and Berini measured the noise of ASE of LRSPPs at near-infrared, and the measured result showed that the photo-number per unit is 3.3. The recent studies show that the studies of LRSPPs amplifier and laser are based on noise properties. However, the recent studies about noise of LRSPPs depend on experiment measure, and the theoretical studies are not detailed.

The paper is based on the theoretical model of noise factor of ASE of SPPs. And it is applied to the periodic stripe LRSPPs structure. The numerical method is used to discuss the effect of high gain mediums and structural parameters.

## 1 Theoretical model and geometry

The structures of SPP and LRSPP are shown in the Fig. 1 (a) and (b), respectively. The relative permittivity of metal and gain medium are  $\epsilon_m = \epsilon'_m + i\epsilon''_m$  and  $\epsilon_g = \epsilon'_g + i\epsilon''_g$  respectively, where  $\epsilon'_m$  and  $\epsilon''_m$  are negative values,  $\epsilon'_g$  and  $\epsilon''_g$  are positive values, the thickness of metal is  $t$ .

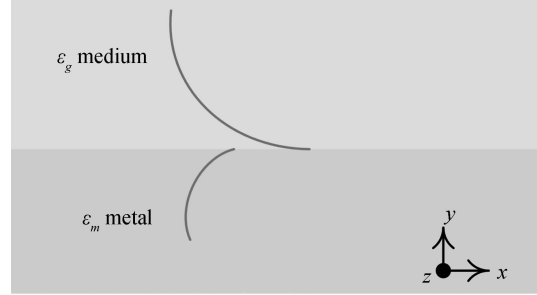
### 1.1 The theoretical model of noise factor of SPPs

Assume that SPP propagates along with  $+z$ , the electric field distribution of SPP amplification are

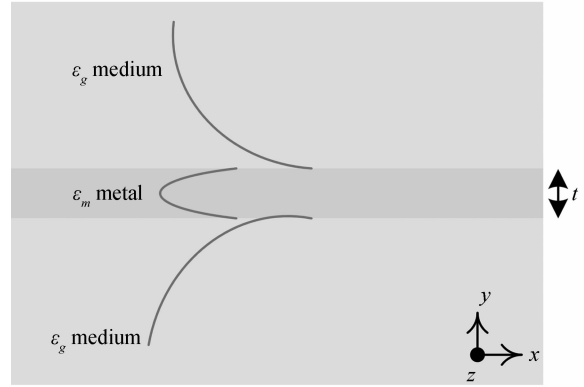
showed in Fig. 1.

$$E_y(x, y, z) = \frac{E_y(y)}{\bar{q}(z)} \exp \left\{ -i\kappa \left[ z + \frac{x^2}{2\bar{q}(z)} \right] \right\} \quad (1)$$

where  $\kappa = \beta + \frac{i\gamma}{2}$  is the complex propagation constant ;  $\beta$  is phase;  $\gamma$  is gain coefficient;  $\bar{q}(z) = z + iz_R$  is complex beam parameter,  $z_R = \beta\omega_0^2/2$  is Rayleigh range and  $\omega_0$  is the radius of light beam.



(a) Schematic diagram of SPPs in the x-y plane



(b) Schematic of LRSPPs in the x-y plane

Fig. 1 Schematic of SPPs and LRSPPs in the x-y plane

### 1.1.1 The noise factor of optical amplification

The Noise Factor (NF) of optical amplification is defined that the signal noise ratio of import point and export point [17-20], and the expression of NF is

$$\text{NF} = \frac{2n_N}{G} + \frac{n_N(n_N + 1)}{G^2 n_0} \quad (2)$$

where  $G$  is optical gain of amplification;  $n_N$  is the photo-number of ASE of output point,  $n_0$  is the photo-number of input point. For a high gain amplification, it must meet that  $G \gg 1$  and  $Gn_0 \gg n_N$ , and  $\frac{2n_N}{G}$  is main factor.

The export energy of ASE is  $P_N = Ah\nu_0 G\xi\gamma^{-1}$  where  $h$  is Planck constant,  $\nu_0$  is optical frequency,  $\xi$  is spontaneous emission ratio per unit,  $A$  is the cross section area  $n_N = P_N (B_v h\nu_0)^{-1}$ ,  $B_v$  is the optical direction wideband around  $\nu_0$ , NF can be approximately expressed as

$$\text{NF} \approx \frac{2\xi A}{\gamma B_v} \quad (3)$$

And the next task is to get the expression of  $A$ . Effective mode area is equal to the cross section area, and it is expressed that

$$A = \frac{\left| \int I(x, y) dx dy \right|^2}{\int |I(x, y)|^2 dx dy} \quad (4)$$

where  $I(x, y)$  is proportional to electric field intensity.

### 1.1.2 The spontaneous emission rate

Assume that the high gain medium is a set of isotropy dipoles. And the spontaneous emission rate  $\Gamma_0$  can be expressed

$$\Gamma(y_0) = \Gamma_0 \text{Re} \int_u du S(u, y_0) \quad (5)$$

where  $u$  is the wave numbers of dipoles. By solving the average value of Eq. (7), and get the effective spontaneous emission

$$\langle \Gamma \rangle = \frac{\Gamma_0}{\pi \delta} \int_{l_g} dy \text{Re} \int_u du S(u, y_0) \quad (6)$$

where  $l_g$  is the length of the structure, and  $\delta$  is penetration depth.

The dipoles densities of excited state depend on the location, and it is inversely proportional to the spontaneous emission rate. And a gain coefficient of model power is defined as

$$N = \frac{\gamma + \alpha}{\sigma_e(v_0)} = \frac{2\beta_g^2}{g(v_0)\Gamma_0\pi}(\gamma + \alpha) \quad (7)$$

where  $\alpha$  is attenuation of power model, and  $\sigma_e$  is the cross area of dipoles,  $g(v)$  is the line width of dipole's release and  $\beta_g = 2\pi\sqrt{\epsilon_g}/\lambda_0$ ,  $B_v g(v) \approx B_v g(v_0)$ , The spontaneous emission rate per unit of SPP can be expressed

$$\xi = g(v_0) B_v N \langle \Gamma \rangle F \quad (8)$$

where  $F = \theta/\pi$  is spontaneous emission factor,  $\theta = 2(\omega_0\beta)^{-1}$ .

### 1.1.3 The noise factor of ASE of SPPs

In order to achieve the noise factor of SPPs ASE, Eq. (2)-(6) are applied into Eq. (1), we get that

$$\text{NF} = 2 \frac{K}{\eta} \int_{l_g} dy \text{Re} \int_u du S(u, y) \quad (9)$$

We assume that  $K = \frac{4\beta_g^2}{\pi^{3/2}\beta}$  and  $\eta = \frac{\gamma}{(\gamma + \alpha)}$

## 1.2 Periodic stripe LRSPPs structure mode

Fig. 2 is the periodic stripe LRSPPs structure, and  $t$  is thickness of metal, his the height of metal stripe,  $N$  is the number of metal stripe, film and stripes are surrounded by high gain dielectric. The expression of electric field distribution is that



Fig. 2 Structure of periodic stripe LRSPPs in the  $x$ - $y$  plane

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) + \int G(\mathbf{r}, \mathbf{r}') k_0^2 (\epsilon(\mathbf{r}') - \epsilon_{\text{ref}}(\mathbf{r}')) \cdot \mathbf{E}(\mathbf{r}') d^2 \mathbf{r}' \quad (10)$$

where  $\mathbf{E}_0$  is the electric field propagating along metal

film and its expression

$$\mathbf{E}_0(\mathbf{r}) = \mathbf{E}_0 e^{i(k_x x + k_y y)} \begin{pmatrix} \hat{y} - \hat{x} \frac{k_y}{k_x} \end{pmatrix} \quad y > 0 \quad (11)$$

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{ik_x x} A \left[ \begin{pmatrix} \hat{y} - \hat{x} \frac{k_y}{k_x} \end{pmatrix} e^{ik_x y} + \begin{pmatrix} \hat{y} + \hat{x} \frac{k_y}{k_x} \end{pmatrix} e^{-k_x(y+d)} \right] \quad t < y < 0 \quad (12)$$

where  $k_x^2 + k_y^2 = k_0^2 \epsilon_g$  and  $k_x^2 + k_y^2 = k_0^2 \epsilon_m$ ,  $\hat{x}$  and  $\hat{y}$  are unit vector,  $G$  is Green's tensor,  $\mathbf{r}$  and  $\mathbf{r}'$  is position vector. By Eq. (11) and (12), and the noise factors of this structure with different heights and numbers of metal stripes is computed.

## 2 Characteristic analysis

### 2.1 Model validation of finite element analysis

The propagation loss of SPPs seriously restricts the development of its potential application. A thin metal film or a thin metal stripe is embedded in a dielectric which support LRSPPs. It is a much lower propagation waves, and may progress a few microns in the visual range and progress a few centimeters in the infrared range<sup>[2]</sup>. Finite element method can be used to accurately calculate SPPs electromagnetic field distribution. For the structure in the Fig. 2, we use finite element method to model the numerical computation, which may verify the validity of the structure by eigenmode solution. The results are shown in Fig. 3 and Fig. 4, which present the electromagnetic field distributions for the two LRSPPs structures. The results suggest that the electromagnetic field distribution of periodic stripe LRSPPs structure is not changed, and verifies the accuracy of this structure.

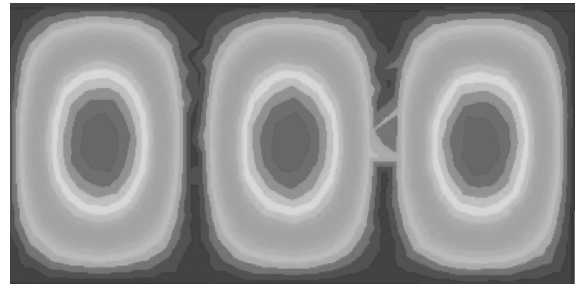


Fig. 3 Electromagnetic field distribution of LRSPPs with a thin metal film

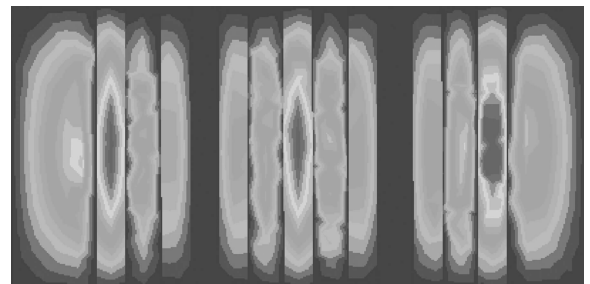


Fig. 4 Electromagnetic field distribution of periodic stripe LRSPPs

## 2.2 Character analysis of noise factor

Assume that gain mediums are air, SiO<sub>2</sub>, GaAs, respectively. And permittivity of them are 1, 3.4, 11.56. We calculate the noise factor in the range  $0.5 \mu\text{m} \leq \lambda_0 \leq 1.75 \mu\text{m}$

In the Fig. 5(a), NF<sub>1</sub> is the noise factor of SPP and NF<sub>2</sub> is the noise factor of periodic stripe LRSPs. As the Fig. 5(a) shows that the values of NF<sub>1</sub> approach to the minimum value in the infrared waves range. It is because that  $\epsilon'_m$  gets the maximum amplitude in the range. When  $|\epsilon'_m| \gg \epsilon'_g$ , noise figure increases as the  $\epsilon'_g$  increases. As the gain medium increase, NF<sub>1</sub> closely approaches to the energy asymptote (vertical dashed line), where the propagation loss of SPPs becomes the maximum. And it makes high gain SPPs more practical significance. In the Fig. 5(b), the thickness of gold  $t = 20 \text{ nm}$ , as it shows that NF<sub>2</sub> approach to the minimum value as wavelength increases; when  $\epsilon'_g$  is smaller, the wavelength has a weaker impact on NF<sub>2</sub>. And when the gain medium is GaAs, NF<sub>2</sub> approaches to the energy asymptote in the infrared region.

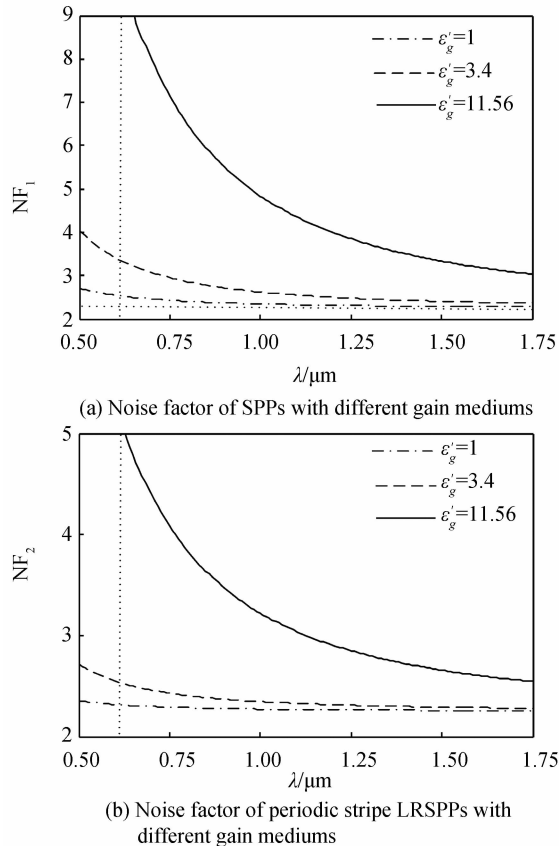


Fig. 5 Noise factor of SPPs and noise factor of periodic stripe LRSPs

Fig. 6 shows NF<sub>2</sub> changes with different thickness of gold film, and here permittivity and height and numbers of metal stripe of structure are that  $\epsilon'_g = 3.4$ ,  $h = 30 \text{ nm}$ ,  $N = 20$ . The locality of LRSPs increase as the thickness increases, which makes NF<sub>2</sub> increases.

When the thickness is enough thickness, the structure is SPPs model and NF<sub>2</sub> approach to NF<sub>1</sub>.

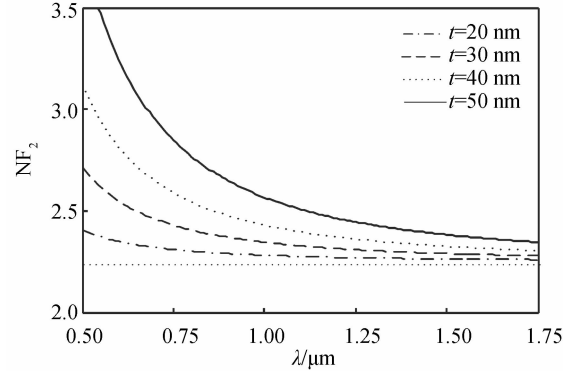


Fig. 6 Noise factor variation of periodic stripe LRSPs with different  $t$

The Fig. 7 shows that NF<sub>2</sub> changes with different  $h$ , and here  $\epsilon'_g = 3.4$ ,  $t_m = 30 \text{ nm}$ ,  $N = 20$ . As Fig. 7 shows that NF<sub>2</sub> increases with larger height, because the thickness of the structure are increased too.

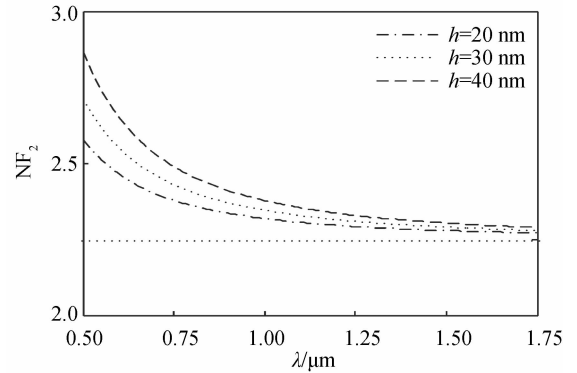


Fig. 7 Noise factor variation of periodic stripe LRSPs with different  $h$

The Fig. 8 shows that NF<sub>2</sub> changes with different  $N$ , and here  $\epsilon'_g = 3.4$ ,  $t_m = 30 \text{ nm}$ ,  $h = 30 \text{ nm}$ . As the Fig. 8 shows that NF<sub>2</sub> increases with larger  $N$ . And when the  $N$  is smaller, NF<sub>2</sub> weakly depends on the wavelength.

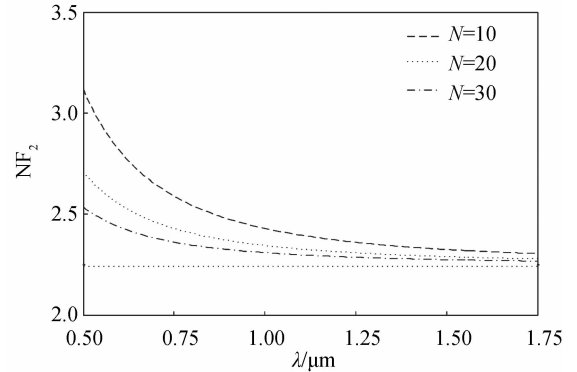


Fig. 8 The noise factor variation of periodic stripe LRSPs with different  $N$

## 3 Conclusion

Based on the theoretical model of the noise factor

of SPP amplification, the periodic stripe LRSPPs mode with different high gain mediums is studied, different thicknesses of metal film, different heights and numbers of mental stripe. The results suggest that noise factor of periodic stripe LRSPP amplification approach to a constant value in the visible range. Combined with theoretical analysis and simulation results, we can achieve that a high gain medium and the thickness of periodic stripe LRSPPs must be considered to describe the noise figure of LRSPPs ASE.

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