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扩散过程中弱相干光场的退相干

邱昌东, 卢道明

(武夷学院 机电工程学院, 福建 武夷山 354300)

摘 要:研究了扩散过程中弱相干光场量子特性的演化. 利用正规乘积、反正规乘积和 Weyl 编序算符内的积分技术, 采用热纠缠态表象求解密度矩阵主方程, 利用 Kraus 算符给出扩散过程中密度算符解的表达式, 导出初态为弱相干态的量子态密度算符演化规律. 讨论了扩散对光场压缩效应和反聚束效应的影响. 结果表明: 随着扩散过程的进行, 弱相干场压缩深度和压缩范围均在减小; 扩散初期光场呈反聚束效应, 扩散时间大于一定值后反聚束效应消失.

关键词:量子光学; 热纠缠态表象; 扩散过程; 弱相干场; 退相干

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Decoherence of Weak Coherent State in the Diffusion Process

QIU Chang-dong, LU Dao-ming

(College of Mechanic and Electronic Engineering, Wuyi University, Wuyishan, Fujian 354300, China)

Abstract: The quantum properties of weak coherent state in terms of squeezing effect and antibunching effect were investigated in the diffusion. Master equation of density operator in the diffusion process can be concisely solved by virtue of thermo-entangled state representation and the technique of integration within an ordered product of operators, which contains normal ordering, antinormal ordering and Weyl ordering. The solution of master equation of density operator in diffusion process was given by Kraus operator. The evolution formula of the field density operator which is in weak coherent state initially was got. Its squeezing effect and antibunching are investigated by the numerical method. The effect of decoherence on its nonclassical property is discussed. Numerical results show that compression depth is weakened and the compression range decreases with diffusion time increasing. On the other hand, the light field displays antibunching effect in the early diffusion. When the diffusion time is greater than a certain value, its antibunching effect disappears.

Key words: Quantum optics; Thermo entangled state representation; Diffusion process; Weak coherent state; Decoherence

OCIS Codes: 270.0270 ; 270.5585 ; 030.0030 ; 030.5290

0 引言

光场的非经典性质, 如压缩效应、反聚束效应和亚泊松分布等, 在量子计算、量子通讯、量子密码学和高准确度测量等领域发挥着重要的作用^[1-3]. 研究人员对不同光场的非经典特性开展了广泛的研究^[4-12]. Gu 等讨论了光子增加对双模压缩相干态非经典性质的影响^[4]. Zhou 等构建了光子减双模压缩热态, 并计算了

它的光子数分布^[5]. 文献[6]构建了三参量双模压缩粒子数态, 并研究了该量子态的压缩效应、反聚束效应对 Cauchy-Schwartz 不等式的违背. 然而, 量子系统不可能完全独立于外界环境, 不可避免会受到环境噪音的影响. 在与外界环境发生相互作用时, 系统不可避免地要失去它原有的量子特征, 造成退相干. 退相干现象是量子信息和量子计算中不可避免的问题, 也是当今量子物理中的一大难题和研究热点. 其中, 对耗散

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第一作者:邱昌东(1973-), 男, 副教授, 硕士, 主要研究方向为量子光学. Email: qiucd_321@126.com

通讯作者:卢道明(1963-), 男, 教授, 硕士, 主要研究方向为量子光学. Email: daominglu79@hotmail.com

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环境下量子系统的退相干问题已有不少报道^[13-17]. 王长春等研究了热真空态振幅衰减模型中的演化^[13]. 张浩亮等研究了光子增减叠加相干态在热环境中的退相干问题^[14].

在处理量子系统受环境影响的退相干问题中, 需要求解密度矩阵主方程. 传统求解主方程的途径是将主方程转换成 Fokker-Planck 方程或 Langevin 方程. 这些求解方法都不能方便地应用于任意初态的情况. Fan 等提出了在热纠缠态表象中利用有序(包括正规乘积、反正规乘积和 Weyl 编序)算符内的积分(Integration Within an Ordered Product, IWOP)技术求解密度矩阵主方程的方法^[18]. 本文利用 IWOP 技术, 采用热纠缠表象来处理主方程, 给出利用 Kraus 算符计算扩散过程密度算符演化的公式, 并进一步计算扩散过程中弱相干光场密度算符的演化, 讨论扩散对其压缩效应和反聚束效应的影响.

1 扩散过程中弱相干场密度算符的演化

在粒子数表象中, 相干态表示为

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} |n\rangle \quad (1)$$

式中 $\alpha = |\alpha|e^{i\varphi}$, $|\alpha|^2 = \bar{n}$, \bar{n} 为平均光子数, φ 为相位角. 弱相干态是对应相干态 $|\alpha|^2 \ll 1$ 的情况, 这时忽略式(1)中 $n \geq 2$ 的项. 那么, 弱相干态表示为

$$|\alpha\rangle = \frac{1}{\sqrt{1+|\alpha|^2}} (|0\rangle + \alpha|1\rangle) \quad (2)$$

密度算符为

$$\rho_0 = \frac{1}{1+|\alpha|^2} (|0\rangle\langle 0| + \alpha^*|0\rangle\langle 1| + \alpha|1\rangle\langle 0| + |\alpha|^2|1\rangle\langle 1|) \quad (3)$$

描述密度算符扩散过程演化的主方程为

$$\frac{d\rho}{dt} = -\kappa(a^+ a \rho - a^+ \rho a - a \rho a^+ + \rho a a^+) \quad (4)$$

式中 κ 为衰减率, a^+ 和 a 分别表示光场的产生和湮没算符. 为求解式(4), 首先引入热纠缠态表象

$$|\eta\rangle = \exp\left(-\frac{1}{2}|\eta|^2 + \eta a^+ - \eta^* \tilde{a}^+ + a^+ \tilde{a}^+\right) |0\tilde{0}\rangle \quad (5)$$

式中对应实模 a^+ 和 a , 引入虚模 \tilde{a}^+ 和 \tilde{a} , $|0\tilde{0}\rangle$ 表示虚模真空态. 虚模 \tilde{a}^+ 和 \tilde{a} 满足实模相同的对易关系 $[\tilde{a}, \tilde{a}^+] = 1$. 由式(5), 令 $\eta = 0$ 得到

$$|\eta=0\rangle = \exp(a^+ \tilde{a}^+) |0\tilde{0}\rangle = |I\rangle \quad (6)$$

态 $|I\rangle$ 具有的性质为

$$\begin{aligned} a|I\rangle &= \tilde{a}^+|I\rangle, \quad a^+|I\rangle = \tilde{a}|I\rangle \\ (a^+ a)^n |I\rangle &= (\tilde{a}^+ \tilde{a})^n |I\rangle \\ (a a^+)^n |I\rangle &= (\tilde{a} \tilde{a}^+)^n |I\rangle \end{aligned} \quad (7)$$

由式(7)可见, 实模 a^+ (a) 和虚模 \tilde{a}^+ (\tilde{a}) 作用在态 $|I\rangle$ 上具有互换关系, 即

$$\begin{aligned} a &\leftrightarrow \tilde{a}^+, \quad a^+ \leftrightarrow \tilde{a}, \quad a^+ a \leftrightarrow \tilde{a}^+ \tilde{a}, \\ a a^+ &\leftrightarrow \tilde{a} \tilde{a}^+ \end{aligned} \quad (8)$$

引入 $|\rho\rangle = \rho|I\rangle$, 将式(4)两边作用在 $|I\rangle$ 上, 那么, 扩散过程密度算符演化的主方程转化为

$$\begin{aligned} \frac{d}{dt} |\rho\rangle &= -\kappa(a^+ a - a^+ \tilde{a}^+ - a \tilde{a} + \tilde{a} \tilde{a}^+) |\rho\rangle = \\ &= -\kappa(a^+ - \tilde{a})(a - \tilde{a}^+) |\rho\rangle \end{aligned} \quad (9)$$

解式(9)得出

$$|\rho\rangle = \exp[-\kappa(a^+ - \tilde{a})(a - \tilde{a}^+)] |\rho_0\rangle \quad (10)$$

式中 $|\rho_0\rangle = \rho_0|I\rangle$, ρ_0 为系统的初始密度算符. 对于式(5)表示的热纠缠态, 利用式 $[a, : f(a, a^+) :] = :$

$\frac{\partial}{\partial a^+} f(a, a^+)$, $:$ 表示算符的正规排序, 导出

$$\begin{cases} \langle \eta | (a^+ - \tilde{a}) = \eta^* \langle \eta | \\ \langle \eta | (a - \tilde{a}^+) = \eta \langle \eta | \end{cases} \quad (11)$$

将 $\langle \eta |$ 作用到式(10)的两边, 得到

$$\begin{aligned} \langle \eta | \rho \rangle &= \langle \eta | \exp[-\kappa(a^+ - \tilde{a})(a - \tilde{a}^+)] |\rho_0\rangle = \\ &= \exp(-\kappa t |\eta|^2) \langle \eta | \rho_0 \rangle \end{aligned} \quad (12)$$

利用纠缠态的完备性关系 $\int \frac{d^2\eta}{\pi} |\eta\rangle\langle \eta| = 1$ 和 IWOP 技术, 导出

$$\begin{aligned} |\rho\rangle &= \int \frac{d^2\eta}{\pi} |\eta\rangle\langle \eta | \rho \rangle = \int \frac{d^2\eta}{\pi} \exp(-\kappa t |\eta|^2) \cdot \\ &= \int \frac{d^2\eta}{\pi} \exp[-(1+\kappa t)|\eta|^2 + \eta(a^+ - \tilde{a}) + \eta^*(a - \tilde{a}^+) + a^+ \tilde{a}^+ + \tilde{a} a - a^+ a - \tilde{a}^+ a] : |\rho_0\rangle = \\ &= \frac{1}{1+\kappa t} \exp\left(\frac{\kappa t}{1+\kappa t} a^+ \tilde{a}^+\right) \times \\ &= \exp\left[\frac{\kappa t}{1+\kappa t} (-\tilde{a}^+ \tilde{a} - a^+ a)\right] \times \\ &= \exp\left(\frac{\kappa t}{1+\kappa t} a \tilde{a}\right) |\rho_0\rangle \end{aligned} \quad (13)$$

在式(13)推导过程中, 利用公式

$$\begin{aligned} \exp[x(a^+ a + \tilde{a}^+ \tilde{a})] &= \\ &= \exp[(e^x - 1)(a^+ a + \tilde{a}^+ \tilde{a})] : \end{aligned}$$

和 $\int \frac{d^2\eta}{\pi} \exp(-h|\eta|^2 + s\eta + t\eta^*) = \frac{1}{h} \exp\left(\frac{st}{h}\right)$. 式

(13)可改写为

$$\begin{aligned} |\rho\rangle &= \frac{1}{1+\kappa t} \exp\left(\frac{\kappa t}{1+\kappa t} a^+ \tilde{a}^+\right) \left(\frac{1}{1+\kappa t}\right)^{a^+ \tilde{a}^+ a} \times \\ &= \exp\left(\frac{\kappa t}{1+\kappa t} a \tilde{a}\right) |\rho_0\rangle \end{aligned} \quad (14)$$

将式(14)中指数项展开, 并利用虚模算符与 ρ_0 对易的性质, 可导出

$$\begin{cases} \exp\left(\frac{\kappa t}{1+\kappa t} a \tilde{a}\right) |\rho_0\rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\kappa t}{1+\kappa t} a\right)^n \tilde{a}^n \rho_0 |I\rangle = \\ \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\kappa t}{1+\kappa t} a\right)^n \rho_0 a^{+n} |I\rangle \\ |\rho\rangle = \sum_{m,n=0}^{\infty} \frac{1}{m!n!} \frac{(\kappa t)^{m+n}}{(1+\kappa t)^{m+n+1}} a^{+m} \left(\frac{1}{1+\kappa t}\right)^{a^+ a} \times \\ a^n \rho_0 a^{+n} \left(\frac{1}{1+\kappa t}\right)^{a^+ a} a^m |I\rangle \end{cases} \quad (15)$$

因此

$$\rho(t) = \sum_{m,n=0}^{\infty} \frac{1}{m!n!} \frac{(\kappa t)^{m+n}}{(1+\kappa t)^{m+n+1}} a^{+m} \left(\frac{1}{1+\kappa t}\right)^{a^+ a} \times a^n \rho_0 a^{+n} \left(\frac{1}{1+\kappa t}\right)^{a^+ a} a^m = \sum_{m,n=0}^{\infty} M_{m,n} \rho_0 M_{m,n}^+ \quad (16)$$

式中

$$M_{m,n} = \sqrt{\frac{(\kappa t)^{m+n}}{m!n!(1+\kappa t)^{m+n+1}}} a^{+m} \left(\frac{1}{1+\kappa t}\right)^{a^+ a} a^n \quad \text{为}$$

Kraus 算符,它满足

$$\sum_{m,n} M_{m,n} M_{m,n}^+ = 1 \quad (17)$$

将式(3)代入式(16),导出

$$\rho(t) = \frac{1}{1+|\beta|^2} \left\{ \frac{1}{1+\kappa t} \exp\left(a^+ a \ln \frac{\kappa t}{1+\kappa t}\right) + \frac{\alpha^*}{(1+\kappa t)^2} \exp\left(a^+ a \ln \frac{\kappa t}{1+\kappa t}\right) \times a + \frac{\alpha}{(1+\kappa t)^2} a^+ \cdot \exp\left(a^+ a \ln \frac{\kappa t}{1+\kappa t}\right) + |\alpha|^2 \times \left[\frac{1}{(1+\kappa t)^3} a^+ \cdot \exp\left(a^+ a \ln \frac{\kappa t}{1+\kappa t}\right) \times a + \frac{\kappa t}{(1+\kappa t)^2} \cdot \exp\left(a^+ a \ln \frac{\kappa t}{1+\kappa t}\right) \right] \right\} \quad (18)$$

2 光场的压缩效应

为了讨论光场的压缩效应,定义光场的两个正交分量

$$\begin{cases} F_1 = \frac{1}{2}(a+a^+) \\ F_2 = \frac{1}{2i}(a-a^+) \end{cases} \quad (19)$$

它们满足对易关系

$$[F_1, F_2] = \frac{i}{2} \quad (20)$$

定义

$$X_j = \langle F_j^2 \rangle - \langle F_j \rangle^2 - \frac{1}{4}, \quad (j=1,2) \quad (21)$$

那么,光场 $F_j (j=1,2)$ 分量被压缩的条件为 $X_j < 0$. 将式(19)代入式(21),可得出

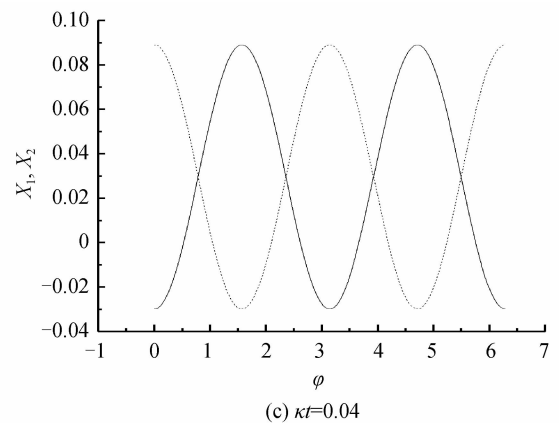
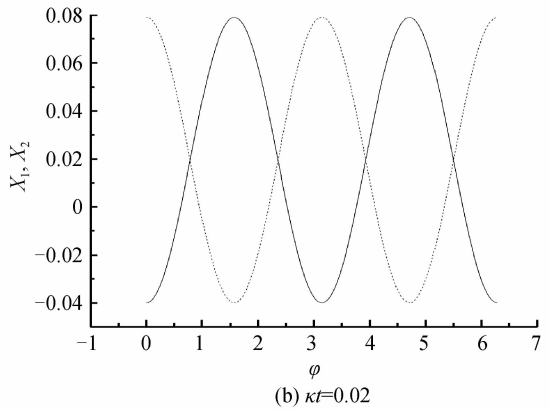
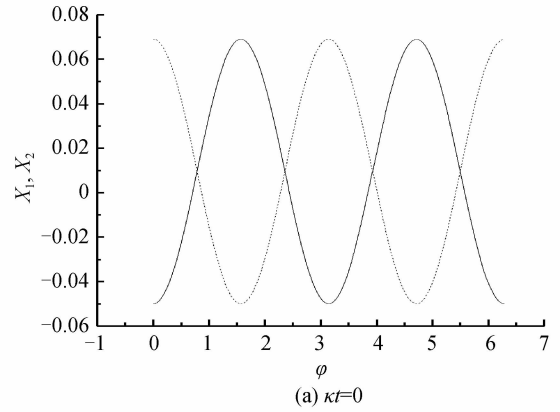
$$\begin{cases} X_1 = \frac{1}{4} (\langle a^2 + a^{+2} \rangle + 2\langle a^+ a \rangle - \langle a + a^+ \rangle^2) \\ X_2 = \frac{1}{4} (-\langle a^2 + a^{+2} \rangle + 2\langle a^+ a \rangle + \langle a - a^+ \rangle^2) \end{cases} \quad (22)$$

利用密度算符的演化规律式(18),求得

$$\begin{cases} \langle a \rangle = \frac{\alpha}{1+|\alpha|^2} \\ \langle a^+ \rangle = \frac{\alpha^*}{1+|\alpha|^2} \\ \langle a^2 \rangle = \langle a^{+2} \rangle = 0 \\ \langle a^+ a \rangle = \frac{1}{1+|\alpha|^2} [\kappa t + |\alpha|^2 (1+\kappa t)] \\ X_1 = \frac{1}{2} \left\{ \frac{1}{1+|\alpha|^2} [\kappa t + |\alpha|^2 (1+\kappa t)] - \right. \end{cases}$$

$$\begin{cases} \frac{2|\alpha|^2}{(1+|\alpha|^2)^2} \cos^2 \varphi \} \\ X_2 = \frac{1}{2} \left\{ \frac{1}{1+|\alpha|^2} [\kappa t + |\alpha|^2 (1+\kappa t)] - \right. \\ \left. \frac{2|\alpha|^2}{(1+|\alpha|^2)^2} \sin^2 \varphi \right\} \end{cases} \quad (23)$$

利用式(23),可计算出 X_1 和 X_2 分量随弱相干场相位角 φ 的演化. 当弱相干场强度 $|\alpha|=0.4$, 扩散时间 κt 分别取 0, 0.02, 0.04 和 0.1 时, X_1 和 X_2 的演化曲线如图 1. 图中实线和虚线分别对应 X_1 和 X_2 的演化曲线. 从图中可见: 随扩散时间的延长, 曲线重心上移, X_1 和 X_2 分量的压缩深度和压缩范围均在减小. 图 1(d) 显示压缩效应已消失, 这表明当耗散时间 κt 大于一定值后, 压缩效应将消失. 这一结果表明光场的压缩效应随扩散时间 κt 的延长而减弱, 直至消失.



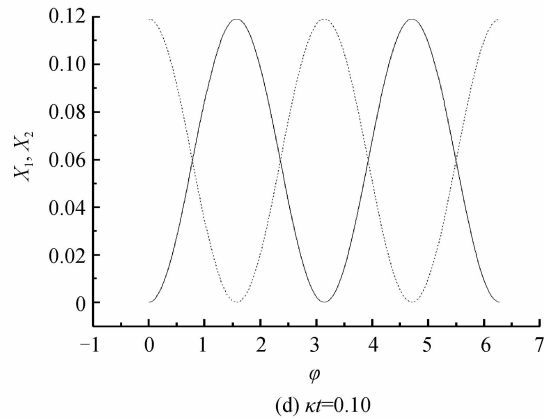


图1 压缩参量 X_1 和 X_2 随弱相干场相位角 φ 的演化
Fig. 1 The evolution of X_1 and X_2 with phase angle φ

3 光场的反聚束效应

光场的二阶关联函数是描述光场统计性质的一个重要物理量,定义为

$$g_a^2 = \frac{\langle a^{+2} a^2 \rangle}{\langle a^+ a \rangle^2} \quad (24)$$

$g_a^2 < 1$ 和 $g_a^2 > 1$ 分别表示光场呈现出反聚束和聚束效应. 为了描述光场呈现反聚束效应的程度,定义

$$G = g_a^2 - 1$$

那么, $G < 0$ 表示光场呈现反聚束效应. 利用式(18),得

$$\begin{cases} \langle a^{+2} a^2 \rangle = \frac{2\kappa t}{1 + |\alpha|^2} [\kappa t + |\alpha|^2 (\kappa t + 2)] \\ \langle a^+ a \rangle = \frac{1}{1 + |\alpha|^2} [\kappa t + |\alpha|^2 (1 + \kappa t)] \\ G = \frac{2\kappa t (1 + |\alpha|^2)}{[\kappa t + |\alpha|^2 (\kappa t + 1)]^2} [\kappa t + |\alpha|^2 (\kappa t + 2)] - 1 \end{cases} \quad (25)$$

从式(25)可见二阶关联函数以弱相干场的相位角无关. 当弱相干场强度 $|\alpha|$ 分别取 0.3 和 0.4 时,二阶关联函数随扩散时间的演化数值计算结果如图 2. 从图 2 可见:随扩散过程的延续,反聚束效应逐渐减弱,扩散时间大于一定值后光场的反聚束效应消失.

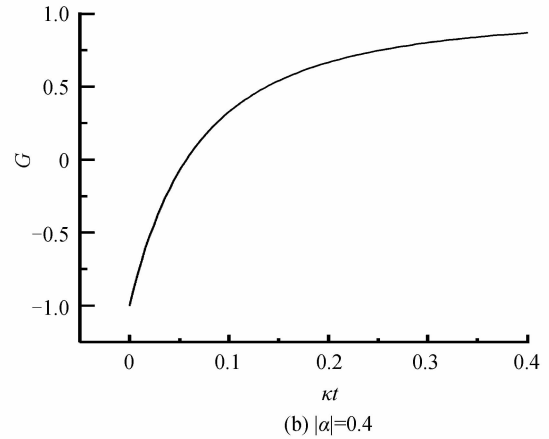
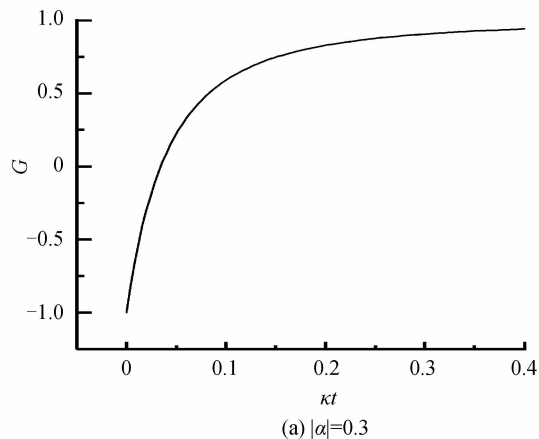


图2 二阶关联函数 G 随扩散时间 κt 的演化
Fig. 2 Time evolution of parameter G with κt

4 结论

利用范洪义建立的 IWOP 技术和热纠缠态表象求解密度矩阵主方程的方法,求解了扩散过程密度算符的主方程,给出了密度算符无限和表示,即 Kraus 算符. 导出了初态处于弱相干态的光场密度算符的演化规律,利用数值计算方法描绘了压缩效应和反聚束效应的演化曲线,讨论了扩散对量子特性的影响. 数值计算结果表明:随着扩散过程的延长,弱相干场的压缩深度和随相位角 φ 变化的压缩范围都在减小. 二阶关联函数 G 随扩散时间 κt 的演化曲线表明随扩散过程的延续,初始呈现反聚束效应的光场逐渐转变为呈现聚束效应,扩散时间大于一定值后光场的压缩效应和反聚束效应消失.

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