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单色光束的衍射度

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摘 要:受近期 Gawhary 和 Severini 在 Opt. Lett. 33,1360(2008)一文中引入的光束近轴度的启发,定义了一个独立于光束近轴度的衍射度参数来描述单色光束的衍射扩散程度,所定义的衍射度只依赖于光束的角谱分布,并可以定量比较不同光束的衍射扩散程度.

关键词:光物理;衍射度;理论分析;无衍射光束;近轴度
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Degree of Diffraction for Monochromatic Light Beams

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Abstract: Enlightened by the degree of paraxiality that was recently introduced by Gawhary and Severini in Opt. Lett. 33, 1360 (2008), a parameter, called the degree of diffraction, is defined to describe the diffractive spreading of a monochromatic light beam. This is a parameter that is independent of the degree of paraxiality. Nevertheless, the same as the degree of paraxiality, the degree of diffraction depends only on the angular spectrum of a beam. With this definition, it is ready to compare quantitatively the diffractive spreading of different beams.

Key words: Optical physics; Degree of diffraction; Theoretical analysis; Diffraction-free light beams; Degree of paraxiality
OCIS Codes: 260.1960; 140.3295; 070.3185

0 Introduction

Recently, Gawhary and Severini[1-2] defined a universal beam parameter, called the Degree Of Paraxiality (DOP), from the viewpoint of energy flow. Later on, we reexamined[3] the definition of this parameter from the viewpoint of energy. The same as defined in Refs. [1-2], the DOP that we defined depends only on the angular spectrum of a light beam. It has nothing to do with the detail of the intensity distribution at the cross section. In doing so, we realized that the same idea can be generalized to define another universal beam parameter, the Degree Of Diffraction (DOD), to describe the diffractive spreading of a light beam. This amounts to the purpose of present paper. To the best of our knowledge, this is the first time to define such a universal parameter. With its help, it will be easy to compare quantitatively the diffractive spreading of different beams.

In Section 1, it is shown that there does not exist a universal parameter to describe the diffractive spreading of light beams. The DOD that meets the need is shown in Section 2 to be conceptually independent of the DOP. The mechanism for the diffractive spreading is also analyzed. The DOD is defined in Section 3. Its applications are provided in Section 4. Section 5

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concludes the paper with remarks.

1 Rayleigh and diffraction-free ranges

One of propagation properties of light beams is their diffraction behavior. It is well known that for a fundamental Gaussian beam of field distribution

$$E_G(\rho,\varphi)\propto\exp\ (-\rho^2/w_0^2) \tag{1}$$

at waist plane $z=0$, a propagation distance, $L_R=\pi w_0^2/\lambda$, called the Rayleigh range[4], is used to characterize its diffraction behavior, where λ is the wavelength. For a truncated or pseudo diffraction-free beam[5-12], a propagation distance called the diffraction-free range is advanced. As an example, for a Bessel-Gaussian (BG) beam[7,12] of field distribution

$$E_{BG}(\rho,\varphi)\propto J_0(\beta\rho)\exp\ (-\rho^2/w_0^2) \tag{2}$$

at $z=0$, the diffraction-free range is[7] $L_G=2\pi w_0/(\beta\lambda)$, where condition

$$\beta w_0/2>1 \tag{3}$$

should be satisfied[7] for the BG beam to behave like a portion of the diffraction-free beam. The so-called Rayleigh range for a truncated Bessel beam[13] is in fact the diffraction-free range[5-6].

Although the diffraction-free range of the BG beam is much shorter[14] than the Rayleigh range of its component Gaussian beams (according to Gori et al[7]., a BG beam is produced by the superposition of Gaussian beams whose axes are uniformly distributed on a cone), $L_G=\dfrac{L_R}{\beta w_0/2}<L_R$, it is hard to quantitatively compare the diffractive spreading of the Gaussian beam Eq. (1) with that of the BG beam Eq. (2). This is because physically the Rayleigh range of the Gaussian beam is defined differently from the diffraction-free range of the pseudo diffraction-free beam. The Rayleigh range is defined[4] as the distance along the propagation direction of the Gaussian beam from its waist to the point where the area of the cross section is doubled. On the other hand, the diffraction-free range is defined[5-6] as the propagation distance over which the profile of the pseudo diffraction-free beam remains invariant. To the best of our knowledge, there does not exist a universal parameter to describe the diffractive spreading of light beams. Even the commonly used M^2 factor, the beam-quality factor, does not meet the need. As is well known, Gaussian beams with different waist radii w_0 have different Rayleigh ranges. But they all have the same M^2 factor, the unity. The purpose of this paper is to introduce such a parameter.

2 DOD is different from DOP

To this end, let us first make use of diffraction-free beams to show that the diffraction behavior of a light field is conceptually independent of its another propagation property, the paraxiality[1-3]. It is known that the wavevector of all the plane wave that composes a diffraction-free beam lies on a cone. In the scalar case[5], the angular spectrum can be written as[12]

$$f(\boldsymbol{w})=A(\varphi)\delta(k_\rho-\beta)$$

in circular cylindrical coordinates, where $\boldsymbol{w}=\boldsymbol{k}/k$ is the unit wavevector, δ is the Dirac delta function, $\beta=k\sin\vartheta_0$, and ϑ_0 denotes the apex half-angle of the wavevector cone. In the vectorial case[15-16], the angular spectrum can be factorized into a polarization vector $\boldsymbol{e}(\boldsymbol{w})$ and the above scalar angular spectrum,

$$\boldsymbol{f}(\boldsymbol{w})=\boldsymbol{e}(\boldsymbol{w})A(\varphi)\delta(k_\rho-\beta),$$

where the polarization vector is a unit vector, $|\boldsymbol{e}|=1$, and is constrained by the transversality condition $\boldsymbol{e}\cdot\boldsymbol{w}=0$. In both cases the paraxiality of the diffraction-free beam is completely determined[1,3] by the apex half-angle ϑ_0 of the wavevector cone. The smaller the angle ϑ_0 is, the larger the DOP is. Nevertheless, as the name suggests, all the diffraction-free beams with different ϑ_0 should have the same diffraction behavior. They are all free of diffraction. This means that the diffraction behavior of a beam is indeed conceptually independent of its paraxiality. Therefore it is essential to introduce a parameter that is different from the DOP to characterize the diffraction behavior. Such a parameter will be referred to as the DOD.

To introduce the DOD, it is instructive to analyze the mechanism for the diffractive spreading of a light beam. For this purpose, let us look at the difference between a diffractive beam and a diffraction-free beam in the angular spectrum. As mentioned before, the wavevector in a diffraction-free beam is only distributed on a cone, which means that a diffraction-free beam is an eigen state of the longitudinal component of the momentum[12]. So the standard deviation of k_z in this case is equal to zero,

$$\Delta k_z\equiv\left[\langle k_z^2\rangle-\langle k_z\rangle^2\right]^{1/2}=0 \tag{4}$$

where

$$\langle Q\rangle=\frac{\int|\boldsymbol{f}|^2Q\mathrm{d}\Omega}{\int|\boldsymbol{f}|^2\mathrm{d}\Omega}=\frac{\int|\boldsymbol{f}|^2Q\mathrm{d}\Omega}{\int|\boldsymbol{f}|^2\mathrm{d}\Omega}$$

is the expectation value[1-2] of quantity $\boldsymbol{Q}$, $\boldsymbol{f}$ is the vector angular spectrum, and $\mathrm{d}\Omega=\sin\vartheta\mathrm{d}\vartheta\mathrm{d}\varphi$ is the solid-angle element in wavevector space. For a diffractive beam, we take the BG beam as an example, which can be regarded as composing of Gaussian beams whose axes are uniformly distributed on a cone[7]. Because the angular spectrum of a Gaussian beam also takes a Gaussian form, it follows that the wavevector in a BG beam is distributed around the axis cone. Consequently, the standard deviation of k_z does not vanish. This shows

that whether a light beam is diffraction-free or not depends on whether the standard deviation of its k_z vanishes or not. Considering that any coherent light field can be expanded in terms of a particular complete set of diffraction-free beams[15-16], the diffractive spreading of a light beam propagating in free space lies in the interference between its diffraction-free components that have different wavevector cones.

3 Definition of DOD

The same as the DOP[1-3], a well-defined DOD should also range from 0 to 1. To see how to define such a DOD on the basis of the above analysis, we consider two extreme light fields. One is the least diffractive fields, the diffraction-free beams. Just as its name implies, the DOD for such beams should be equal to 0. In view of this, the expected DOD should be proportional to Δk_z in accordance with Eq. (4). The other extreme light field is described by angular spectrum of vector spherical harmonics,

$$\boldsymbol{f}(\boldsymbol{w}) = \boldsymbol{e}(\boldsymbol{w}) Y_{\lambda\mu}(\boldsymbol{w}),$$

where

$$Y_{\lambda\mu}(\boldsymbol{w}) = \left\{ \frac{2\lambda+1}{4\pi} \frac{(\lambda-\mu)!}{(\lambda+\mu)!} \right\}^{1/2} P_\lambda^\mu(\cos\vartheta) \mathrm{e}^{\mathrm{i}\mu\varphi}$$

is the spherical harmonics satisfying normalization condition $\int |Y_{\lambda\mu}|^2 \mathrm{d}\Omega = 1$. Due to the spherically symmetric distribution of the wavevector, such a field is the most "diffractive". The DOD for this field should be equal to 1. Because the expectation value of k_z in this field vanishes, $\langle k_z \rangle = 0$, as can be easily checked, we have $\Delta k_z = \langle k_z^2 \rangle^{1/2}$. Combining these two considerations together, it seems reasonable to define the DOD for any coherent light field as

$$D' = \frac{\Delta k_z}{\langle k_z^2 \rangle^{1/2}} = \left(1 - \frac{\langle k_z \rangle^2}{\langle k_z^2 \rangle} \right)^{1/2} \tag{5}$$

So defined DOD has the following properties:

1) The same as the DOP[1-3], it does not necessarily require the knowledge about the intensity distribution of the electric field in spatial space. Besides, it does not depend on the vector nature of the angular spectrum and therefore applies to scalar as well as vector fields.

2) As is required, it ranges from 0 to 1. For the least diffractive fields, the diffraction-free beams, it is equal to 0; for the most diffractive fields such as the spherical waves, it is equal to 1.

3) It describes the diffractive spreading of a beam in such a way that the more diffractive a beam is, the larger its value is.

It is noted that D' is proportional to the standard deviation of $\cos\vartheta$, $\sigma_P = (\langle \cos^2\vartheta \rangle - \langle \cos\vartheta \rangle^2)^{1/2}$, that was discussed in Ref. [2]. Indeed, with the help of $k_z = k\cos\vartheta$, Eq. (5) can be written as

$$D' = \sigma_P / \langle \cos^2\vartheta \rangle^{1/2}.$$

Because the DOP introduced in Ref. [2] is $\langle \cos\vartheta \rangle$, which is different from the denominator $\langle \cos^2\vartheta \rangle^{1/2}$, the ratio $\sigma_P / \langle \cos\vartheta \rangle$ discussed there is something like, but different from, D'. Let us examine the above mentioned third property of D' in the following by applying it to a uniformly distributed angular spectrum with respect to the polar angle.

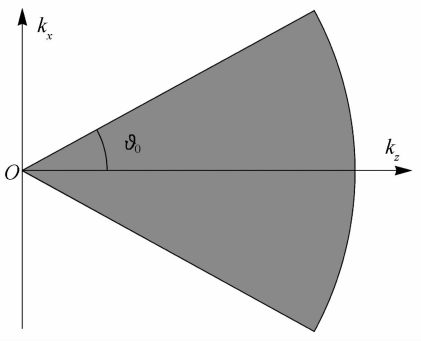

Fig. 1 Schematic illustration of the cone within which the wavevector is uniformly distributed

Since D' in Eq. (5) does not depend on the vector nature of the angular spectrum, the angular spectrum that we consider assumes the form,

$$f_u(\boldsymbol{w}) = \begin{cases} [2\pi(1-\cos\vartheta_0)]^{-1/2}, & 0 \leqslant \vartheta < \vartheta_0, \\ 0, & \vartheta_0 < \vartheta \leqslant \pi, \end{cases} \tag{6}$$

which satisfies normalization condition $\int |f_u|^2 \mathrm{d}\Omega = 1$. It describes a beam the wavevector of which is uniformly distributed within a cone of apex half-angle ϑ_0, as is illustrated graphically in Fig. 1. When $\vartheta_0 = 0$, it describes a plane wave propagating in the z direction. The DOD in this case should be equal to 0. With the increase of ϑ_0, the DOD should increase monotonously. When ϑ_0 approaches π, we will arrive at a beam the wavevector of which is uniformly distributed in the whole solid angle of 4π. This is one of the most diffractive beams and should have a DOD of unity. Such a feature is well reflected by the DOD (5). In fact, straightforward calculations with Eq. (5) give

$$D'_u = \frac{1}{2} \left(1 - \frac{3\cos\vartheta_0}{1+\cos\vartheta_0+\cos^2\vartheta_0} \right)^{1/2} \tag{7}$$

The dependence of D'_u on ϑ_0 is shown in Fig. 2. It properly reveals the above expectation.

As can be seen from Eq. (7), when $\vartheta_0 = \pi/2$, that is to say, when no component plane waves propagate in the negative z direction, one has $D'_u = 1/2$. If we further require that the DOD be applied only to beam-like light fields, that is to say, only to those light fields no component plane waves of which propagate in the backward direction, and that the DOD of the most diffractive beam in this case be still equal to unity, definition (5) should be replaced with

$$D=2D'=2\left(1-\frac{\langle k_z\rangle^2}{\langle k_z^2\rangle}\right)^{1/2} \tag{8}$$

Eq. (8) is the primary result of this paper. It should be emphasized that so defined DOD is a beam parameter, having nothing to do with the propagation distance. Let us make use of it to discuss two examples below.

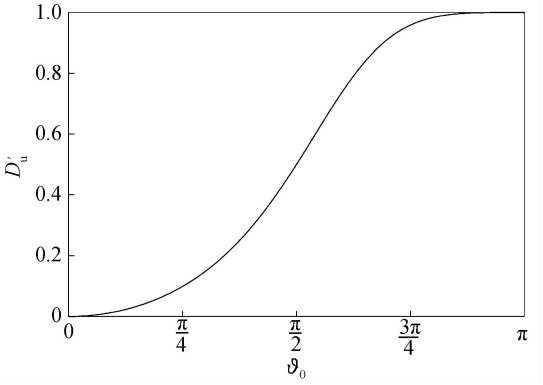

Fig. 2 Dependence of D_u' on beam parameter ϑ_0

4 Applications

4.1 Gaussian-like beams

In the first example, we consider a Gaussian-like beam[3] that has the following angular spectrum,

$$f_{GL}=A_0\exp\left(-\frac{k^2w_0^2}{4}\sin^2\vartheta\right),0\leqslant\vartheta\leqslant\frac{\pi}{2} \tag{9}$$

It is normalized in accordance with

$$\int|f_{GL}|^2\mathrm{d}\Omega=2\pi A_0^2\frac{F(s)}{s} \tag{10}$$

where $s=kw_0/\sqrt{2}$ and

$$F(x)=\exp(-x^2)\int_0^x\exp(t^2)\mathrm{d}t$$

is the Dawson function. With the help of Eq. (10), one easily obtains

$$\langle k_z\rangle=\frac{1-\exp(-s^2)}{2sF(s)}k$$

and

$$\langle k_z^2\rangle=\frac{s-F(s)}{2s^2F(s)}k^2.$$

Substituting them into Eq. (8) gives

$$D_{GL}=2\left\{1-\frac{[1-\exp(-s^2)]^2}{2F(s)[s-F(s)]}\right\}^{1/2} \tag{11}$$

When $s\to0$, angular spectrum Eq. (9) reduces to the uniformly distributed angular spectrum Eq. (6) with $\vartheta_0=\pi/2$. In this case, we have $D_{GL}\to1$, which is just what we expect. In obtaining this result, we have made use of the following Taylor series of the Dawson function,

$$F(x)=\sum_{n=0}^{\infty}\frac{(-1)^n2^n}{(2n+1)!!}x^{2n+1}=x-\frac{2}{3}x^3+\cdots$$

To the opposite, when $s\to\infty$, we find $D_{GL}\to0$, where the asymptotic behavior, $F(x)\sim1/(2x)$, of the Dawson function at large $|x|$ is used. This is also what we expect, because the beam in this case approaches a plane wave, a diffraction-free beam. With the increase of s, the DOD decreases monotonously from 1 to 0. The dependence of D_{GL} on the beam parameter s is schematically shown in Fig. 3.

It is worth noting[17] that when s is large enough (taking into account the difference between our Eq. (9) for the angular spectrum and Eq. (8) in [17], the beam parameter s here is exactly the reciprocal of the beam parameter f in [17]), for instance, when

$$s>2\pi \tag{12}$$

angular spectrum Eq. (9) describes a beam that has a negligible longitudinal component (only about 1% of the transverse component in intensity) and can be approximated by the scalar Gaussian beam Eq. (1).

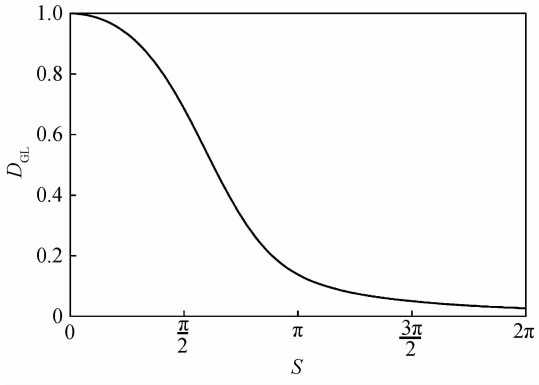

Fig. 3 Dependence of D_{GL} on the beam parameter s

4.2 BG beams

In the second example, we compare quantitatively the diffractive spreading of the BG beam with that of its component Gaussian beams. The angular spectrum of the BG beam Eq. (2) is given by[12]

$$f_{BG}=I_0\left(\frac{\beta w_0^2}{2}k_\rho\right)\exp\left[-\frac{w_0^2}{4}(\beta^2+k_\rho^2)\right]$$

in circular cylindrical coordinates, where I_0 is the modified Bessel function of the first kind and zeroth order. Because the evanescent components that correspond to $k_\rho>k$ do not contribute to the diffraction, we convert this angular spectrum into

$$f_{BG}=I_0\left(\frac{k\beta w_0^2}{2}\sin\vartheta\right)\exp\left[-\frac{w_0^2}{4}(\beta^2+k^2\sin^2\vartheta)\right] \tag{13}$$

in spherical coordinates, where $0\leqslant\vartheta\leqslant\pi/2$.

Although it is hard to obtain an analytical expression for the DOD, D_{BG} of the BG beam in Eq. (13), we can still calculate numerically the dependence of D_{BG} on the beam parameter s. Furthermore, when $\beta=0$, Eq. (13) reduces to Eq. (9). In this case, we arrive at the Gaussian beam at large s. This shows that definition Eq. (8) for the DOD allows us to compare quantitatively the diffractive spreading of the BG beam with that of its component Gaussian beams. In Fig. 4 is shown such a comparison, where the up curve is for the DOD of the BG beam, and the down curve is for the

DOD of the component Gaussian beams. The parameters for the BG beam are chosen as follows. Letting the radius ρ_0 of the central spot of the Bessel factor be about the wavelength, $\rho_0 = \lambda$, we have[6] $\beta = 1/\rho_0 = k/2\pi$. Moreover, in order that condition Eq. (3) be satisfied, we choose $s > 2\sqrt{2}\pi$, which also meets the requirement Eq. (12). Clearly, the BG beam is more diffractive than its component Gaussian beams. It should be pointed out that the comparison made here is not to be confused with the comparison that was made in the literature [5-6, 13] between a truncated Bessel beam and a Gaussian beam. After all, a truncated Bessel beam cannot be viewed as composing of Gaussian beams.

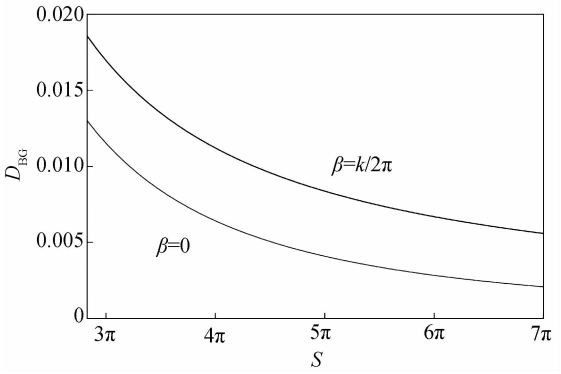

Fig. 4 Comparison of the DOD of the BG beam ($\beta = k/2\pi$) with that of its component Gaussian beams ($\beta = 0$)

5 Conclusions

In conclusion, we introduced a parameter called the DOD to describe the diffractive spreading of light beams. The same as the DOP[1-3], the DOD depends only on the angular spectrum of light beams. It ranges from 0 to 1. It describes the diffractive spreading of a light beam in such a way that the more diffractive a light beam is, the larger its value is. Specifically, it is equal to zero for the diffraction-free beams. Besides, we pointed out the relation of the DOD with the standard deviation[2] of the cosine of the angle that the wavevector makes with the beam axis. The parameter of DOD should be useful in areas in which the assessment of beam quality, especially its diffraction spreading, is of importance, such as in laser processing[18] and in free space optical communication[19].

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