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基于辛普森公式的对称分步傅里叶变换孤子串的传输特性

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摘　要:针对孤子串在传输过程中发生周期性碰撞,导致信息串扰的问题,提出采用准确度较高的辛普森公式近似地改进对称分步傅里叶变换,对孤子串的传输特性及传输过程进行数值模拟.实验结果表明,当初始半间距为 2.5 时,对于孤子对、三孤子、四孤子、五孤子和六孤子而言,只考虑自陡峭效应且自陡峭系数都为 0.02 时,或者只考虑自频移效应且自频移系数分别为 1、1、3、2、1.5 时,都能有效地减少孤子间的碰撞,增加孤子碰撞前的独立传输距离,当同时考虑自陡峭效应和自频移效应时,自频移效应占主导作用.

关键词:非线性光学;孤子传输;对称分步傅里叶法;自陡峭;自频移

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Characteristics of Transmission for the Solitons String Based on Simpson Algorithm of Modified Symmetric Split-Step Fourier Method

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Abstract: For the problem of information crosstalk that caused by periodic soliton string collision in the transmission, the modified symmetric split-step Fourier method that approximated by higher-precision Simpson algorithm was presented, and then the soliton string characteristics and transmitting procedure were simulated. It is impoint that the precision of method is more over than two orders of magnitude compared with the traditional method. The experimental results show that the solitons collision was decreased and the collision-free dependent transmission distance was increased, for the soliton pairs, three-soliton, four-soliton, five-soliton and six-soliton when only considered the self-steepening coefficient were 0.02 and only considered the self-frequency shift coefficient were 1, 1, 3, 2 and 1.5, respectively. Meanwhile, the self-steepening and the self-frequency shift effect were simultaneously considered, the self-frequency shift effect was predominant effect. The results indicated that the suitable parameters have a significant influence for the amount of information of soliton transmission.

Key words: Nonlinear optics; Soliton transmition; Symmetric split-step Fourier method; Self-steepenting; Self-frequency shift

OCIS Codes: 190.4370; 190.5530; 070.7345; 290.5910

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0 Introduction

As the soliton spacing became smaller, the nonlinear effects were increased. Therefore, it should be consider the self-steepening and self-frequency shift effect to build the higher-order nonlinear Schrödinger equation. The split-step Fourier transform method and its simulation process are presented in detail, the literature pointed out the soliton pair will strongly affect each other if they are very close, which make the solitons shape aberrant. While the solitons spacing was appropriately selected to decrease the collision and the EDFA was adopted to compensate the power of solitons [1-3]. The effect of high-order dispersion and high-order nonlinearity were studied in Ref. [4], the results show that the influence of third-order dispersion can lead to the ruleless split of the third-order solitons pairs, meanwhile the interactions and split between two neighboring third-order solitons may be well eliminated and improved the linear drift of the central position of solitons under appropriate parameter. Meanwhile, Ref. [5] reported that the symmetric split-step Fourier method was used to study the soliton characteristics. In this paper, the solitons string were researched by the modified symmetric split-step Fourier method which approximated by simpson algorithm, and then the self-steepening and self-frequency shift effect were considered for the solitons string.

1 Based on Simpson algorithm of modified symmetric split-step Fourier method

The properties of optical solitons considered so far are based on the nonlinear Schrödinger equations. When the input pulse is shorter than 100 fs, it is necessary to include the higher-order nonlinear and dispersion [4,6-7]. In terms of the soliton, it has the dimensionless

$$\frac{\partial U}{\partial \xi}+\frac{\alpha T_0^2}{2|\beta_2|}U+\mathrm{i}\,\frac{\mathrm{sgn}(\beta_2)}{2}\frac{\partial^2 U}{\partial \tau^2}-\frac{\beta_3}{6|\beta_2|T_0}\frac{\partial^3 U}{\partial \tau^3}+\frac{\mathrm{i}}{24}\frac{\beta_4}{|\beta_2|T_0^2}\frac{\partial^4 U}{\partial \tau^4}=N^2\Big[\mathrm{i}|U|^2U-\frac{2}{\omega_0 T_0}(\partial/\partial\tau)(|U|^2U)-\mathrm{i}\frac{T_R}{T_0}U\frac{\partial|U|^2}{\partial\tau}\Big] \tag{1}$$

There was the parameter T_R corresponded to the slope of the Raman gain, ω_0 presents the circular frequency, T_0 is initial pulse width. β_2, β_3 and β_4 were dispersion coefficient, P_0 was input power, L_D present the dispersion length, that is $L_D=\frac{T_0^2}{|\beta_2|}$. Meanwhile, the characteristic parameter N was provided, what's more, $N^2=\gamma P_0 L_D$, $u=UN$, γ presents nonlinear coefficient.

The parameters Γ, δ, ρ, s and τ_R govern, respectively, the fiber losses, effect of third-order dispersion, fourth-order dispersion, self-steepening, and self-frequency shift are defined as

$$\Gamma=\frac{\alpha T_0^2}{2|\beta_2|},\delta=\frac{\beta_3}{6|\beta_2|T_0},\rho=\frac{1}{24}\frac{\beta_4}{|\beta_2|T_0^2},\quad s=\frac{2}{\omega_0 T_0},\tau_R=\frac{T_R}{T_0} \tag{2}$$

$$\frac{\partial u}{\partial \xi}+\mathrm{i}\,\frac{\mathrm{sgn}(\beta_2)}{2}\frac{\partial^2 u}{\partial \tau^2}-\delta\frac{\partial^3 u}{\partial \tau^3}+\mathrm{i}\rho\frac{\partial^4 u}{\partial \tau^4}-\mathrm{i}|u|^2u+\Gamma u+s(\partial/\partial\tau)(|u|^2u)+\mathrm{i}\tau_R u\frac{\partial|u|^2}{\partial\tau}=0 \tag{3}$$

The modified symmetric split-step Fourier method was given, and the Eq. (3) was solved. The specific steps were indicated by the Fig. 1. Linear operator $\hat{D}$

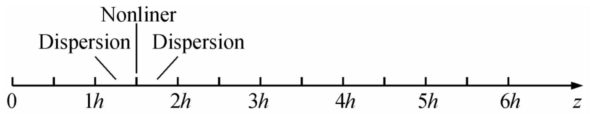


Fig. 1 Symmetric split-step Fourier method

and the nonlinear operator $\hat{N}$ were introduced, and the ordering $\frac{\partial u}{\partial \xi}=(\hat{D}+\hat{N})u$

$$\hat{D}=-\mathrm{i}\,\frac{\mathrm{sgn}(\beta_2)}{2}\frac{\partial^2}{\partial \tau^2}+\delta\frac{\partial^3}{\partial \tau^3}-\mathrm{i}\rho\frac{\partial^4}{\partial \tau^4}-\Gamma \tag{4}$$

$$\hat{N}=\mathrm{i}|u|^2-\frac{s}{u}(\partial/\partial\tau)(|u|^2u)-\mathrm{i}\tau_R\frac{\partial|u|^2}{\partial\tau} \tag{5}$$

The envelope field in the gauge form [8-11]

$$u(\xi+h,\tau)\approx\exp\Big(h\frac{\hat{D}}{2}\Big)\exp\Big(\int_{\xi}^{\xi+h}\hat{N}(\xi')d\xi'\Big)\cdot\exp\Big(h\frac{\hat{D}}{2}\Big)u(\xi,\tau) \tag{6}$$

The Simpson algorithm was introduced, and the modified symmetric split-step Fourier method was got

$$\int_{\xi}^{\xi+h}\hat{N}(\xi')\mathrm{d}\xi'\approx(h/6)[\hat{N}(\xi)+4\hat{N}(\xi+h/2)+\hat{N}(\xi+h)] \tag{7}$$

$$u(\xi+h,\tau)\approx\exp(h\hat{D}/2)\exp\{(h/6)[\hat{N}(\xi)+4\hat{N}(\xi+h/2)+\hat{N}(\xi+h)]\}\cdot\exp(h\hat{D}/2)u(\xi,\tau) \tag{8}$$

The precision of using Simpson algorithm is close to h^5 as the Eq. (8), while using the trapezoidal rule the precision is h^3.

The result can be computed by the following steps:

Step 1: Advance in the Fourier space to get the dispersion for the first half segment $h/2$

$$u(\xi+h/2,\tau)=F^{-1}\{\mathrm{e}^{\hat{D}h/2}F[u(\xi,\tau)]\} \tag{9}$$

Step 2: Nonlinear term for full segment h

$$u_N(\xi+h/2,\tau)=\mathrm{e}^{(h/6)[\hat{N}(\xi)+4\hat{N}(\xi+h/2)+\hat{N}(\xi+h)]}\cdot F^{-1}\{\mathrm{e}^{\hat{D}h/2}F[u(\xi,\tau)]\} \tag{10}$$

Step 3: Advance in the Fourier space to get the dispersion term for the second half segment $h/2$

$$u(\xi+h,\tau)=F^{-1}\{\mathrm{e}^{\hat{D}h/2}F[u_N(\xi+h/2,\tau)]\}=$$
$$F^{-1}\{\mathrm{e}^{\hat{D}h/2}F\{\mathrm{e}^{[\hat{N}(\xi)](h/6)}\mathrm{e}^{[\hat{N}(\xi+h/2)](2h/3)}\cdot$$
$$\mathrm{e}^{[\hat{N}(\xi+h)](h/6)}F^{-1}\{\mathrm{e}^{\hat{D}h/2}F[u(\xi,\tau)]\}\}\} \tag{11}$$

For the Eq. (3), the negative frequency componentw as selected, then $\frac{\partial^n}{\partial\tau^n}=(-\mathrm{i}\omega)^n$ and the Eq. (4) was got

$$\hat{D}=\frac{\mathrm{isgn}(\beta_2)}{2}\omega^2+\mathrm{i}\delta\omega^3-\mathrm{i}\rho\omega^4-\Gamma \tag{12}$$

The Eq. (5) and the Eq. (12) were substituted into the Eq. (11), the Eq. (13) was got

$$u(\xi+h,\tau)=F^{-1}\{\mathrm{e}^{[\mathrm{isgn}(\beta_2)\cdot\omega^2/2+\mathrm{i}\delta\omega^3-\mathrm{i}\rho\omega^4-\Gamma](h/2)}F\cdot\{\mathrm{e}^{[\mathrm{i}|u(\xi)|^2-(s/u(\xi))(\partial/\partial\tau)(|u(\xi)|^2u(\xi))-\mathrm{i}\tau_R(\partial|u(\xi)|^2/\partial\tau)](h/6)}\cdot$$
$$\mathrm{e}^{\{\mathrm{i}|u(\xi+h/2)|^2-s/[u(\xi+h/2)]\cdot(\partial/\partial\tau)[|u(\xi+h/2)|^2u(\xi+h/2)]-\mathrm{i}\tau_R(\partial|u(\xi+h/2)|^2/\partial\tau)\}(2h/3)}\cdot$$
$$\mathrm{e}^{\{\mathrm{i}|u(\xi+h)|^2-s/[u(\xi+h)]\cdot(\partial/\partial\tau)[|u(\xi+h)|^2u(\xi+h)]-\mathrm{i}\tau_R\partial|u(\xi+h)|^2/\partial\tau\}(h/6)}F^{-1}\{\mathrm{e}^{\{[\mathrm{isgn}(\beta_2)/2]\omega^2+\mathrm{i}\delta\omega^3-\mathrm{i}\rho\omega^4-\Gamma\}(h/2)}F[u(\xi,\tau)]\}\}\} \tag{13}$$

2 Discussion

Optical solitons include single soliton, soliton pairs and multi-solitons. A pulse width will be a small fraction of the bit slot, and a train of solitons can be represented as[6-7] 0111011010……. "01110" was three-soliton, "0110" was soliton pairs, "010" was single soliton. The special conditions for decreasing the interactions of solitons were discussed to ensure that the neighboring solitons are well separated. For the Eq. (13), the solution form were

Form 1:

$$u(0,\tau)=\mathrm{sech}(\tau-q_0)+\mathrm{sech}(\tau+q_0) \tag{14}$$

Form 2:

$$u(0,\tau)=\mathrm{sech}(\tau-2q_0)+\mathrm{sech}(\tau)+\mathrm{sech}(\tau+2q_0) \tag{15}$$

Form 3:

$$u(0,\tau)=\mathrm{sech}(\tau-3q_0)+\mathrm{sech}(\tau-q_0)+\mathrm{sech}(\tau+q_0)+\mathrm{sech}(\tau+3q_0) \tag{16}$$

Form 4:

$$u(0,\tau)=\mathrm{sech}(\tau-4q_0)+\mathrm{sech}(\tau-2q_0)+\mathrm{sech}(\tau)+\mathrm{sech}(\tau+2q_0)+\mathrm{sech}(\tau+4q_0) \tag{17}$$

Form 5:

$$u(0,\tau)=\mathrm{sech}(\tau-5q_0)+\mathrm{sech}(\tau-3q_0)+\mathrm{sech}(\tau-q_0)+\mathrm{sech}(\tau+q_0)+\mathrm{sech}(\tau+3q_0)+\mathrm{sech}(\tau+5q_0) \tag{18}$$

Where the parameter of q_0 was initial half of soliton spacing [6].

In terms of q_0, the small value of it can increase the number of solitons and the frequent collision was appeared; while, the big value of q_0 make the number of solitons and collision were decreased, so appropriate solitons spacing can ensure individual solitons well isolated. As the effect of higher-order dispersion can arouse soliton drifting and make the pulse energy decay. In this paper the effect of self-steepening and self-frequency shift were studied under the condition of the high-order dispersion to be ignored, then $\delta=0$, $\rho=0$.

2.1 Soliton pairs

The Eq. (14) was the form of soliton pairs, and the soliton pairs will appear periodic collision when transmission. While when the effect of self-steepening was considered, the results as shown in Fig. 2. In Fig. 2 (a), under the condition of $q_0=2.5$ and $s=0.005$, the collision of soliton pairs were happened when the transmission distance was 25 L_D and then the collision were not happened. As seen in Fig. 2 (c), when the self-steepening coefficient was increased to $s=0.02$, the soliton will appear the great disturbance. Therefore, under the condition of $q_0=2.5$, the collision can be effectively avoided, while it can not ensure the neighboring solitons were well separated when increasing the self-steepening coefficient. Meanwhile, when $q_0=4.5$ was considered, if the self-steepening coefficient was $s=0.01$ (In Fig. 2 (b)), the soliton was

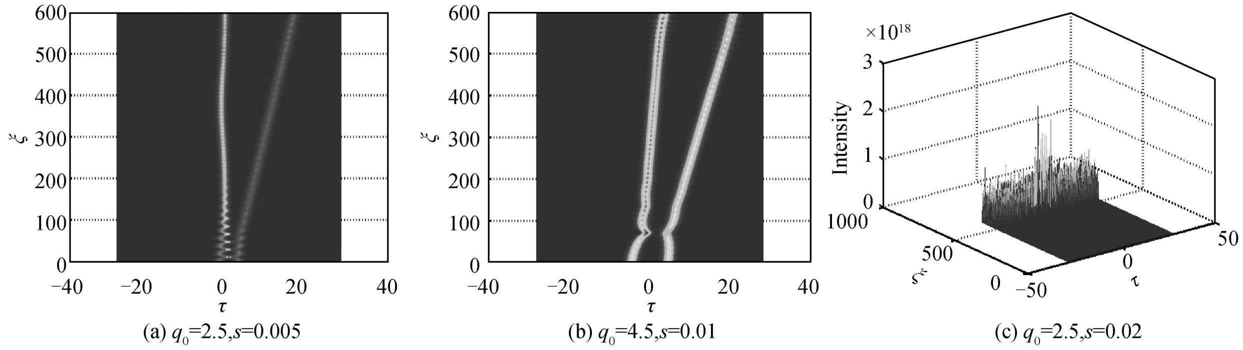


Fig. 2 The influence of self-steepening effect for soliton pairs

completely collision-free in transmission. But the weak attract was existed at the 50 L_D and then it appears exclusive phenomena. In conclusion, appropriate initial half of soliton spacing and self-steepening were selected can make the solitons transmit longer distance without collision.

From the Fig. 3, we can observe the influence of self-frequency shift effect. Now the initial half spacing of solitons was selected, that is $q_0 = 2.5$. In the Fig. 3 (a), the soliton pairs appear dispersed transmission under the condition of $\tau_R = 0.25$. As seen in Fig. 3 (b), the dependent transmission without collision was got. As the same time, when the self-steepening and self-frequency shift effect were considered, the soliton pairs will still maintain the dependent transmission, so the self-frequency shift was prominent effect. Therefore suitable self-frequency shift coefficient can ensure the neighboring solitons were well separated.

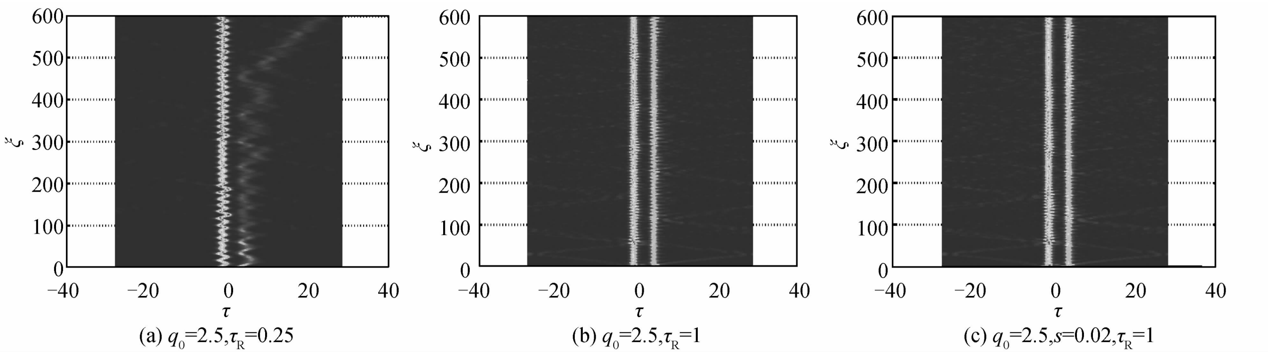


Fig. 3 The influence of self-steepening and self-frequency shift effect for soliton pairs

2.2 Multi-solitons

Firstly, the three-soliton was studied, as the Fig. 4. In the Fig. 4 (a), under the condition of $q_0 = 1.5$, the three-soliton were non-periodic collision, and then the solitons were separated when the transmission distance was close to 300 L_D; In the Fig. 3 (b), the intermittent attraction repulsion was appeared in the process of transmission. In addition, when the distance arrived to 400 L_D, the strong attraction was existed while there had no collision in 600 L_D. From Fig. 4 (b) and Fig. 4 (c), the tree-soliton and soliton pairs were different in the characteristic of transmission, furthermore, the three-soliton more beneficial for solitons transmission.

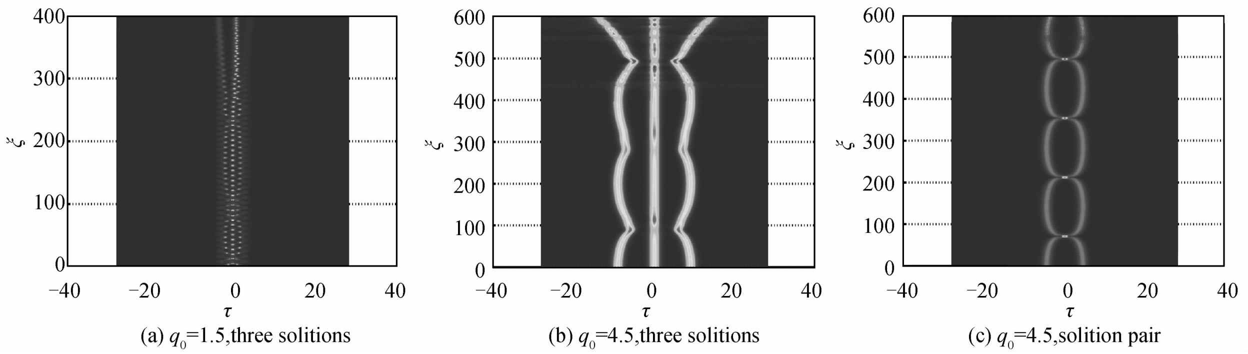


Fig. 4 The characteristic of three solitons and it compared to soliton pairs

Secondly, the collision was non-periodic for the four-soliton, five-soliton, and six-soliton. When q_0 was increased to $q_0 = 4.5$, from the Fig. 5, the characteristic of solitons was simulated . The results show that the

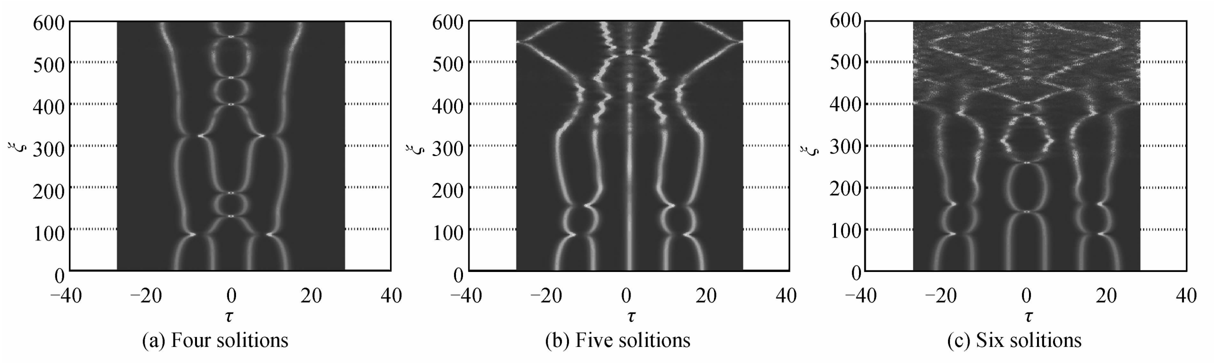
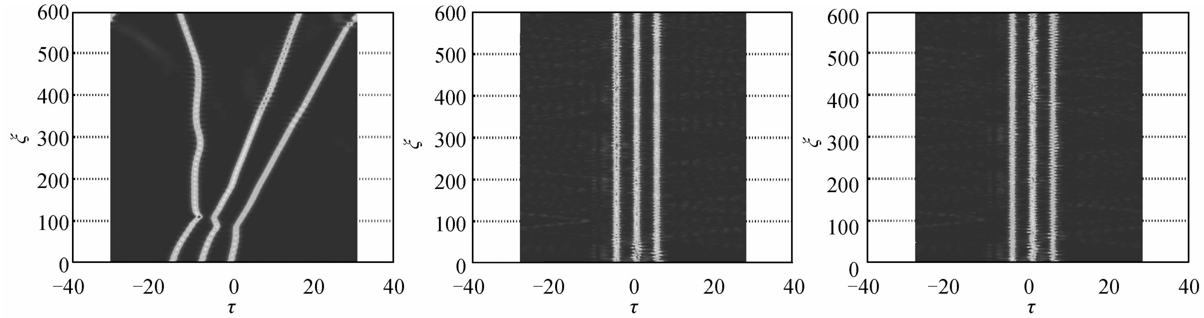


Fig. 5 Under the condition of $q_0 = 4.5$, the characteristics of four solitons, five solitons and six solitons

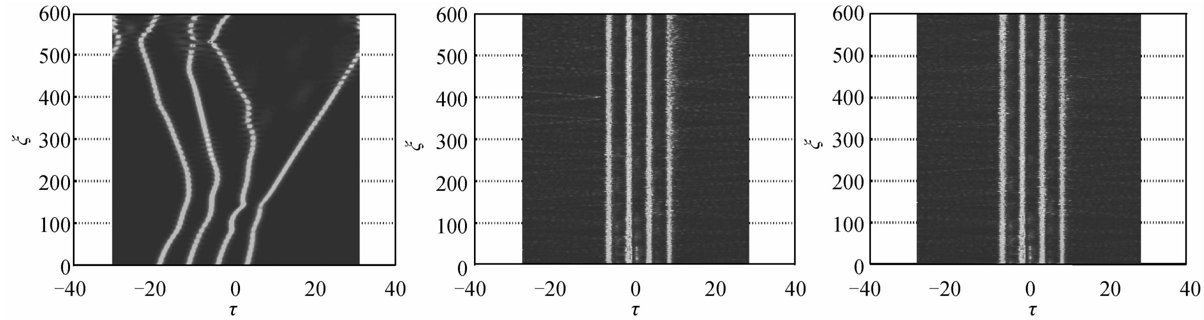
dependent transmission distance were 129 L_D, 514 L_D, and 143 L_D, respectively for the four-soliton, five-soliton, and six-soliton. Therefore, the five-soliton was suitable for the solitons transmission.

Finally, the coefficients of self-steepening and self-frequency shift were appropriately selected and it is conductive to solitons transmission without collision. As seen in Fig. 6 (a), Fig. 7 (a), Fig. 8 (a) and Fig. 9 (a), under the condition of $q_0 = 4.5$ and $s = 0.02$, the collision frequency was decreased. Meanwhile, three-soliton and four-soliton in transmission without collision, while the dependent distance of five-soliton and six-soliton were 186 L_D and 400 L_D, respectively. In Fig. 6 (b), Fig. 7 (b), Fig. 8 (b) and Fig. 9 (b), under the condition of $q_0 = 2.5$, the transmission was stable for three-soliton, four-soliton, five-soliton and six-soliton, when the self-frequency shift were 1, 3, 2 and 1.5, respectively. Moreover, from the Fig. 6(c), Fig. 7



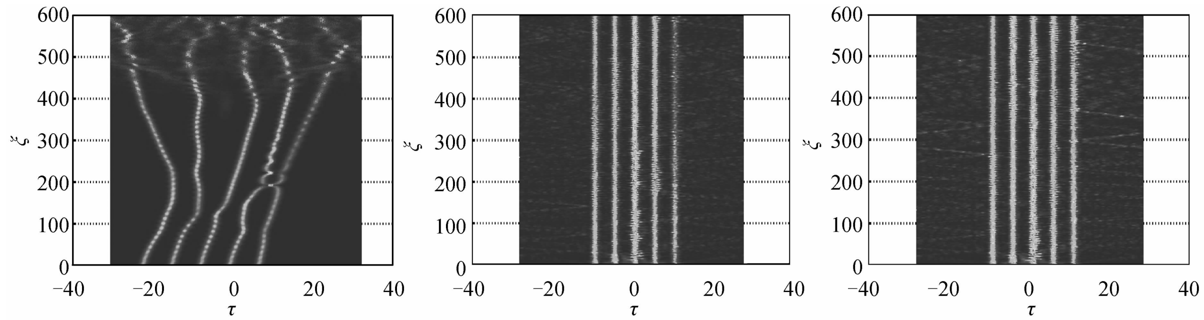
(a) q_0=4.5, s=0.02 (b) q_0=2.5, τ_R=1 (c) q_0=2.5, τ_R=1, s=0.02

Fig. 6 The influence of self-steepening and self-frequency shift effect for three solitons



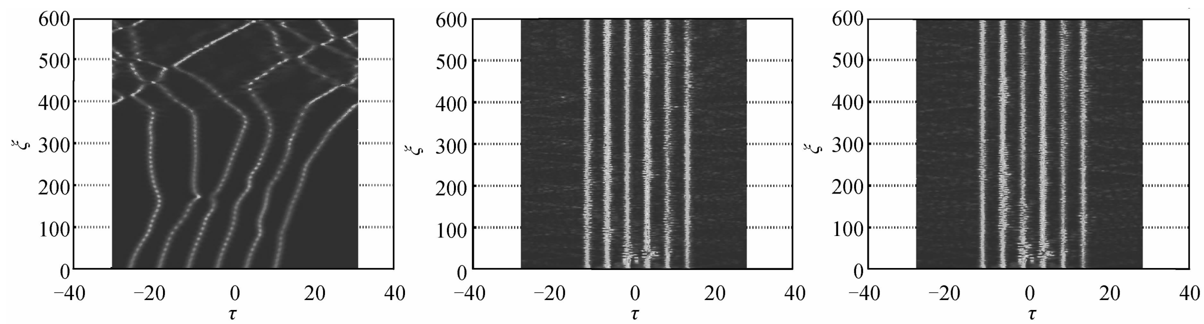
(a) q_0=4.5, s=0.02 (b) q_0=2.5, τ_R=3 (c) q_0=2.5, τ_R=3, s=0.02

Fig. 7 The influence of self-steepening and self-frequency shift effect for four solitons



(a) q_0=4.5, s=0.02 (b) q_0=2.5, τ_R=2 (c) q_0=2.5, τ_R=2, s=0.02

Fig. 8 The influence of self-steepening and self-frequency shift effect for five solitons



(a) q_0=4.5, s=0.02 (b) q_0=2.5, τ_R=1.5 (c) q_0=2.5, τ_R=1.5, s=0.02

Fig. 9 The influence of self-steepening and self-frequency shift effect for six solitons (c), Fig. 8 (c) and Fig. 9 (c), we can observe that the self-frequency shift effect plays an important role when the self-steepening and self-frequency shift effect were coinstantaneous considered.

3 Conclusion

The modified symmetric split-step Fourier method approximated by high-precision Simpson algorithm is a simple and effective method to solve the nonlinear Schrödinger equations. The results show that the collision was periodic for soliton pairs, while the collision was non-periodic for the three-soliton, four-soliton, five-soliton and six-soliton. Meanwhile, the appropriated coefficient of self-steepening and self-frequency shift can effectively decrease the collision of solitons and ensure that the neighboring solitons are well separated, so that the dependent transmission distance of solitons was increased. In addition, the self-frequency shift effect plays a prominent role when the self-steepening and self-frequency shift effect were coinstantaneous considered. The above research conclusion play an important role for decreasing the information crosstalk and it make the foundation for the solitons communication in the optical fiber.

Reference

[1] NAKAZAWA M, SUZUKI K, YAMADA E, *et al*. Straight-line soliton data transmissions over 2000km at 20Gbit/s and 1000km at 40Gbit/s using erbium doped fiber amplifier [J]. *Electronics Letters*, 1993, **29**(16): 1474-1476.

[2] SULEM C, SULEM P L. The nonlinear Schrödinger equation: self-focusing and wave collapse [M]. New York: Springer. 1991, 23-24.

[3] THEODOROS P H, DIMITRIOS J F. Dark solitons in the presence of higher-order effects [J]. *Optics Letters*, 2013, **38** (23): 5098-5010.

[4] SONG Li-jun, LI Lu, ZHOU Guo-sheng. Effect of third-order dispersion on breathing localized solutions in the quintic complex Ginzburg-Landau equation [J]. *Chinese Optics Letters*, 2005, **3**(5): 299-300.

[5] KUMAR D R, RAO B P, Soliton interaction in birefringent optical fibers: Effects of nonlinear gain devices [J]. *Optik*, 2012, **123**: 117-124.

[6] KIVSHAR Y S, AGRAWAL G P. Optical solitons: from fibers to photonic crystals [M]. San Diego: Academic Press, 2003, 5-15.

[7] ROBERT W B. Nonlinear optics [M]. New Rork: Associated Press 2007, 473-479.

[8] TAO Qing, LUO Feng-guang, HU Jian, *et al*. Solving nonlinear Schrödinger equation based on discrete split-step multi-wavelet method [J]. *Optik*, 2011, **122**: 1329-1331.

[9] YANG Jie, YU Song, LI Min-liang, *et al*. An integral split-step Fourier method for digital back propagation to compensate fiber nonlinearity [J]. *Optics Communications*, 2014, **312**: 80-84.

[10] ASIF Z R, LIN Chien yu, Holtmannspoetter M, *et al*. Optimized digital backward propagation for phase modulated signal in mixed-optical fiber transmission link [J]. *Optics Express*, 2010, **18**(22): 22796-22807.

[11] YAN Zhen-ya. Constructive theory and applications of complex nonlinear waves [M]. Beijing: Science Press, 2007.

[12] UTHAYAKUMAR T, RAJA R V J, PORSEZIAN K. Realization of all-optical logic gates through three core photonic crystal fiber [J]. *Optics Communication*, 2013, **296**: 124-131.