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Ladder 型原子辅助光力学系统的多稳响应

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摘 要:通过解析和数值模拟系统的海森堡朗之万方程的稳态解,研究了 Ladder 型三能级原子辅助光力学系统的多稳响应.结果表明振动镜子和原子系综的稳态行为与弹簧的劲度系数和经典泵浦场的拉比频率有关.随着劲度系数的减小和拉比频率的增大,腔内原子系综和整个光力学系统将呈现多稳现象,并且不同频段对应稳态解的个数有所不同.因此,可以通过改变泵浦场拉比频率和弹簧劲度系数来控制系统的多稳响应.研究结果在量子信息处理和精密测量等领域具有潜在的应用价值.

关键词:光力学系统;微腔;多稳;光学非线性;量子相干

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Multistable Optical Response of a Ladder-type Atom-assisted Optomechanical System

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Abstract: The optical response of a hybrid system, where a Ladder-type three-level atomic ensemble was confined in a optomechanical microcavity with an oscillating mirror in one end, was investigated by solving the Heisenberg-Langevin equations analytically and numerically. The results show that the steady-state behaviors of the oscillating mirror and the atomic ensemble relate to the elastic coefficient of the spring and the Rabi frequency of the classical pump field. As the elastic coefficient decreases and pump Rabi frequency increases, the atomic ensemble and entire optomechanical system will present multiple steady-state solutions with different steady-state numbers in different frequency domains. Therefore, the steady-state optical response of the system could be controlled by manipulating the Rabi frequency of the classical pump field and the spring elastic coefficient. These results may have potential applications in the area of quantum information processing and high-precision quantum measurement.

Key words: Optomechanical system; Microcavity; Multistability; Optical nonlinear; Quantum coherence

OCIS Codes: 270.0270; 270.1670; 270.5580; 230.3990

0 Introduction

Recently, optomechanical systems has become an attractive area^[1-3] exploring interactions between mechanical object and light. This interaction originates

from the mechanical effect of light, such as, optical force. Radiation pressure force^[4] is a typical category of optical forces^[5], and has attracted extensive attention in recent years. Atom-assisted optomechanical system^[6] as one of the most favorite hybrid optomechanical

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systems, has shown many interesting phenomena. For example, optomechanical storage^[7], strong optomechanical coupling between a micron-sized membrane and a single trapped atom^[8], and the optical feedback cooling of mechanical motion based on active feedback and precise measurement^[9,10] has been investigated. It has also been found that the atomic ensemble can enhance the radiation pressure effectively to induce a cavity-atom-mirror entanglement^[11]. Besides, bistable behavior of the steady-state position has been studied for a two-level cold atomic ensemble embedded in an optomechanical cavity^[12]. Recently, a hybrid system filled with three-level atomic ensemble has been widely investigated, say for example, the multistability phenomena of a hybrid optomechanical system with Lamda-type atoms^[13-14]. These observations promote the development of the nonlinear optical field, and can be used as optical bistable and multistable flip-flops since optical bistable and multistable systems have information storage capacity in optical computing and communications^[15-16]. Therefore, further researches on such nonlinear phenomena are important in areas such as quantum information and communication.

We investigate the optical response of a hybrid optomechanical system with a Ladder-type three-level atomic ensemble. We find that the multiple steady-state behaviors of the oscillating mirror and the confined atoms relate to the elastic coefficient and the Rabi frequency of the classical pump field. Compared with the Lamda-type system, the Ladder-type configuration has more advantage in meeting the weak-cavity-field assumption condition.

1 Model and equations

A hybrid optomechanical system, where an ensemble of N identical ladder-type three-level atoms is confined in a microcavity with an oscillating mirror in one end, as shown in Fig. 1. Level $|1\rangle_i$ corresponds to

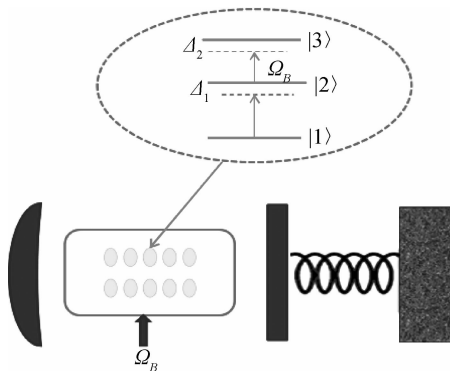


Fig. 1 Schematic diagram for the atom-assisted optomechanical system

the ground state, while levels $|2\rangle_i$ and $|3\rangle_i$ correspond to the excited states of the i th atom. A quantum cavity field probes the transition $|1\rangle_i \rightarrow |2\rangle_i$, while a classical pump field with Rabi frequency Ω_B (frequency ω_B) couples $|2\rangle_i$ and $|3\rangle_i$. The movable mirror, on the other hand, is described as a quantum harmonic oscillator with frequency ω and mass m .

The total Hamiltonian for the hybrid system is given as

$$\begin{aligned}
 H = & H_c + H_m + H_a + H_{a-l} + H_{m-c} = \\
 & \omega_0 a^\dagger a + \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2 + \\
 & \sum_{i=1}^N (\omega_3 \sigma_{33}^{(i)} + \omega_2 \sigma_{22}^{(i)}) + \\
 & \sum_{i=1}^N (\Omega_B e^{-i\omega_B t} \sigma_{32}^{(i)} + g a \sigma_{21}^{(i)} + h. c.) - \\
 & \frac{\omega_0}{l} x a^\dagger a
 \end{aligned} \tag{1}$$

where $\hbar = 1$, H_c , H_m , H_a , H_{a-l} , H_{m-c} represent Hamiltonian describe the Hamiltonian of free optical cavity, free oscillating mirror, free atomic sample, the atom-light interaction, and the mirror-cavity interaction, respectively. ω_0 is the effective cavity frequency when the oscillating mirror is fixed; a^\dagger (a) is the creation (annihilation) operator of the single-mode cavity field; x is the displacement of the oscillating mirror; p is the momentum of the oscillating mirror; ω_α ($\alpha=2,3$) is the transition frequency between level $|\alpha\rangle_i$ and level $|1\rangle_i$; $\sigma_{\alpha\alpha}^{(i)} = |\alpha\rangle_i \langle\alpha|$ ($\alpha=2,3$) is the projection operator of the i th three-level atom; l is the effective cavity length when the oscillating mirror stay at the equilibrium position; $\sigma_{\alpha\beta}^{(i)} = |\alpha\rangle_i \langle\beta|$ ($\alpha, \beta=1,2,3$) is the transition operator of the i th three-level atom; and Ω_B is the Rabi frequency of the classical pump field. $g = -\mu \sqrt{\omega_0/2V\epsilon_0}$ denotes the coupling constant of the atoms and the cavity field, while μ , ϵ_0 , and V denote the dipole moment, the vacuum permittivity, and the cavity volume, respectively. ω_B is assumed to satisfy the following condition

$$\omega_B = \omega_3 - \omega_2 - \Delta_2 \tag{2}$$

where Δ_2 is the frequency detuning of the pump field from the transition $|2\rangle_i \leftrightarrow |3\rangle_i$. For convenience in writing, we define $\Delta_0 = \omega_2 - \omega_0$, $\Delta_1 = \omega_2 - \omega_L$ and $\kappa = m\omega^2 l^2$, where $\omega_L = \omega_0 - \frac{\langle x \rangle}{l} \omega_0$ denotes the effective cavity frequency, $m\omega^2$ denotes the elastic coefficient, and l is a constant. So κ is proportional to the elastic coefficient $m\omega^2$.

The couplings between the light fields and the atoms can be homogeneous when linear size of the atomic ensemble is much smaller than the wavelengths of the light fields. Then we can define the following collective operators

$$\begin{cases} A = \sum_{i=1}^N \sigma_{22}^{(i)}, \\ B^+ = \frac{1}{\sqrt{N}} \sum_{i=1}^N \sigma_{21}^{(i)}, \\ D^+ = \sum_{i=1}^N \sigma_{32}^{(i)}, \\ E^+ = \frac{1}{\sqrt{N}} \sum_{i=1}^N \sigma_{31}^{(i)} \end{cases} \quad (3)$$

Assume the atomic system is in very low excitation and the atom number N is large enough, we can obtain the following dynamic equations according to the Heisenberg-Langevin equations

$$\begin{cases} \partial_t x = \frac{p}{m} \\ \partial_t p = -m\omega^2 x + \frac{\omega_0}{l} a^+ + a - \frac{\gamma_m}{2} p - \sqrt{\gamma_m} \epsilon_{\text{in}}(t) \\ \partial_t a = -i\omega_0 a \left(1 - \frac{x}{l}\right) - ig \sqrt{N} B - \frac{\gamma_0}{2} a + \sqrt{\gamma_0} a_{\text{in}}(t) \\ \partial_t B = -(i\omega_2 + \gamma_1) B - i\Omega_B e^{i\omega_2 t} E - ig \sqrt{N} a + f_1(t) \\ \partial_t E = -(i\omega_3 + \gamma_2) E - i\Omega_B e^{-i\omega_3 t} B + f_2(t) \end{cases} \quad (4)$$

where the mirror damping rate $\gamma_0, \gamma_1 (\gamma_2), \gamma_m$ denoting respectively the cavity decay rate, the coherence dephasing rate between level $|1\rangle_i$ and $|2\rangle_i (|3\rangle_i)$, and the mirror damping rate. Here we have also introduced the quantum fluctuation terms of the dressed atoms, the oscillating mirror, and the cavity field meantime, which satisfy the following conditions

$$\begin{cases} \langle \epsilon_{\text{in}}(t) \rangle = \langle f_1(t) \rangle = \langle f_2(t) \rangle = 0 \\ \langle a_{\text{in}}(t) \rangle = \alpha_{\text{in}}(t) \end{cases} \quad (5)$$

where $\alpha_{\text{in}}(t) \neq 0$, and $\alpha_{\text{in}}(t)$ have been considered as the external driving field. In addition, we have implicitly set the frequency of $\alpha_{\text{in}}(t)$ as ω_0 . Therefore, it can be rewritten as the following structure

$$\begin{cases} a_{\text{in}}(t) = \alpha_{\text{in}}(t) + \delta a_{\text{in}}(t) \\ \langle \delta a_{\text{in}}(t) \rangle = 0. \end{cases} \quad (6)$$

In order to get the steady-state solutions, we need to simplify the Heisenberg-Langevin equations by the following rotating transformations

$$\begin{cases} a = \tilde{a} e^{-i\omega_0 t} \\ B = \tilde{B} e^{-i\omega_2 t} \\ E = \tilde{E} e^{i(\Delta_1 + \Delta_2 - \omega_3) t} \\ f_1(t) = \tilde{f}_1(t) e^{-i\omega_2 t} \\ f_2(t) = \tilde{f}_2(t) e^{i(\Delta_1 + \Delta_2 - \omega_3) t} \\ a_{\text{in}}(t) = \tilde{a}_{\text{in}}(t) e^{-i\omega_0 t} \end{cases} \quad (7)$$

Replacing $\langle x \tilde{a} \rangle_s$ and $\langle \tilde{a}^+ \tilde{a} \rangle_s$ with $\langle x \rangle_s \langle \tilde{a} \rangle_s$ and $\langle \tilde{a}^+ \rangle_s \langle \tilde{a} \rangle_s$ indicates the adoption of a quasiclassical approximation valid, when the quantum-fluctuation correlation is much smaller than the mean-value product.

Then, we can get mean values in the steady state

$$\langle x \rangle_s = \frac{\omega_0 \langle \tilde{a}^+ \rangle_s \langle \tilde{a} \rangle_s}{m\omega^2 l} \quad (8)$$

$$\langle \tilde{B} \rangle_s = \frac{-ig \sqrt{N} \gamma_0 \tilde{\alpha}_{\text{in}} (\gamma_2 + i \tilde{\Omega}_1)}{(\gamma_2 + i \tilde{\Omega}_1) G + \frac{\gamma_0}{2} \Omega_B^2} \quad (9)$$

$$\langle \tilde{a} \rangle_s = \frac{2 \tilde{\alpha}_{\text{in}}}{\sqrt{\gamma_0}} \left[1 - \frac{g^2 N (\gamma_2 + i \tilde{\Omega}_1)}{(\gamma_2 + i \tilde{\Omega}_1) G + \frac{\gamma_0}{2} \Omega_B^2} \right] \quad (10)$$

where $\tilde{\Omega}_1 = \Delta_{1,s} + \Delta_2$ and $G = g^2 N + \frac{\gamma_0}{2} (i\Delta_{1,s} + \gamma_1)$. The parameter $\Delta_{1,s}$ denotes detuning between ω_2 and ω_L in the steady state. We also know that the probe cavity field can be written as following

$$E(t) = \sqrt{\frac{\omega_L}{2V\epsilon_0}} a e^{-i\omega_L t} + H. c. = \epsilon e^{-i\omega_L t} + H. c. \quad (11)$$

From the macroscopic polarization $\langle P \rangle = \frac{\mu}{V} \sum_{i=1}^N \langle \sigma_{12}^{(i)} \rangle = \chi \epsilon_0 \langle \epsilon \rangle$, we can further get the atomic susceptibility,

$$\chi = \frac{\mu^2 N}{V\epsilon_0} \cdot \frac{\gamma_2 \Xi - \tilde{\Omega}_1 \Theta}{\Xi^2 + \Theta^2} + i \frac{\mu^2 N}{V\epsilon_0} \cdot \frac{\tilde{\Omega}_1 \Xi + \gamma_2 \Theta}{\Xi^2 + \Theta^2} \quad (12)$$

where $\Theta = \gamma_1 \gamma_2 - \Delta_{1,s} \tilde{\Omega}_1 + \Omega_B^2$, $\Xi = \gamma_2 \Delta_{1,s} + \gamma_1 \tilde{\Omega}_1$.

2 Numerical results and discussion

With Eqs. (8) and (10), we can get the following nonlinear equation

$$\left| 1 - \frac{g^2 N (\gamma_2 + i \tilde{\Omega}_1)}{(\gamma_2 + i \tilde{\Omega}_1) G + \frac{\gamma_0}{2} \Omega_B^2} \right|^2 = \frac{\kappa \gamma_0 (\Delta_{1,s} - \Delta_0)}{4 \alpha_{\text{in}}^2 \omega_0^2} \quad (13)$$

Since it is a quintic equation of $\Delta_{1,s}$, it may have five steady-state solutions at most, which means that the whole hybrid system exhibits multistability. However, as the radiation pressure on the mirror is small, the stable positions of the mirror $\langle x \rangle_s$ can't deviate from equilibrium position far away. This means that it is just significant to study the solution near the atom-cavity detuning Δ_0 , and the solution far beyond Δ_0 (corresponding a large displacement of the mirror $\langle x \rangle_s$) is unstable^[13]. The underlying physics of the multistable phenomenon is the effective feedback formed by the hybrid optomechanical system. The oscillating mirror is driven by the radiation pressure, while the motion of the mirror may change the eigenfrequency of the cavity, and its variation will change the radiation pressure in return.

Without loss of generality, we can choose the following parameters on the basis of Ref. [13] as $\omega_0/\gamma_1 = 10^5$, $\gamma_0/\gamma_1 = 10^{-6}$, $g \sqrt{N}/\gamma_1 = 10^2$, $\tilde{\alpha}_{\text{in}}/\gamma_1 = 10$, and we set $\gamma_2/\gamma_1 = 1$, $\Delta_2/\gamma_1 = 0$.

Now using Eqs. (13) and (12) we numerically study how parameters κ and Ω_B affect the steady-state characters of the ladder-type three-level atom-assisted optomechanical system. In Fig. 2 (a) we show the mirror's steady-state position $\langle x \rangle_s$ as a function of Δ_0/γ_1 under the condition of $\kappa/\gamma_1 = 2 \times 10^6$ and

$\Omega_B/\gamma_1=8$. There are two steady-state solutions $\langle x \rangle_s^{(i)}$, ($i=1,2$). However, $\langle x \rangle_s^{(1)}$ corresponds to a very large displacement of the mirror (see inset of Fig. 2(a)). Thus, it's an unstable solution, and the movable mirror is monostable. Besides, $\langle x \rangle_s^{(2)}$ is very small and nearly approximates to zero. This means that when the elastic coefficient is large enough compared with cavity length given, it is difficult to make the mirror move. Fig. 2(b) and (c) show the imaginary and real part of the susceptibility of the ladder-type three-level atomic ensemble with and without the cavity. These imply that, in the limit $\kappa \rightarrow \infty$, the oscillating mirror doesn't affect the optomechanical system, and all physical characters revert to those of the original EIT system.

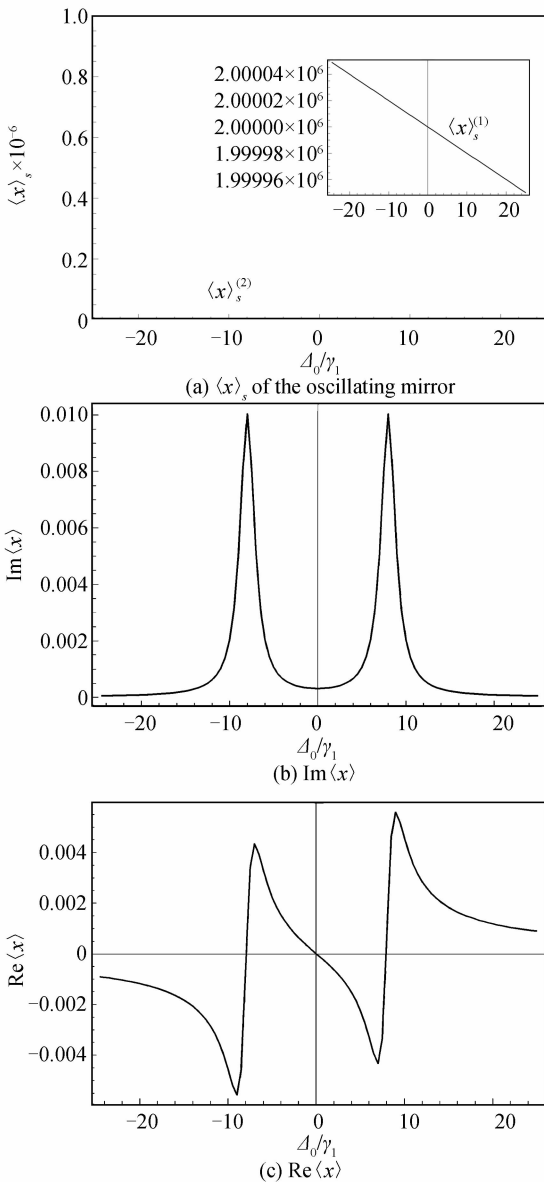


Fig. 2 $\langle x \rangle_s$ of the oscillating mirror and the susceptibility of the atomic ensemble versus Δ_0/γ_1

In Fig. 3, we show the mirror's steady-state position $\langle x \rangle_s$ as a function of Δ_0/γ_1 with $\kappa/\gamma_1=2 \times 10^1$

(dotted), 2×10^2 (solid), 2×10^6 (dashed), and $\Omega_B/\gamma_1=8$. As we can see, as the value κ decreases, the number of steady-state solutions increases, the mirror's steady-state displacement increases, and there are at most four solutions $\langle x \rangle_s^{(i)}$, ($i=1,2,3,4$). We also investigate effects of the pump Rabi frequency Ω_B on the mirror's steady-state solutions $\langle x \rangle_s$ in Fig. 4 with $\Omega_B/\gamma_1=8$ (solid), 5 (dotted), 1 (dashed), and $\kappa/\gamma_1=2 \times 10^2$. As we can see in Fig. 4, when the value Ω_B increases, the number of steady-state solutions increases, and there are at most four solutions $\langle x \rangle_s^{(i)}$, ($i=1,2,3,4$). However, since stable positions of the mirror can't deviate from equilibrium position far away, there exists at most three steady-state solutions $\langle x \rangle_s^{(i)}$, ($i=2,3,4$).

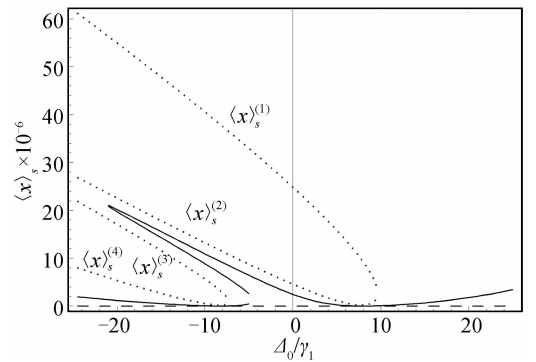


Fig. 3 Steady-state solutions $\langle x \rangle_s$ of the oscillating mirror versus Δ_0/γ_1

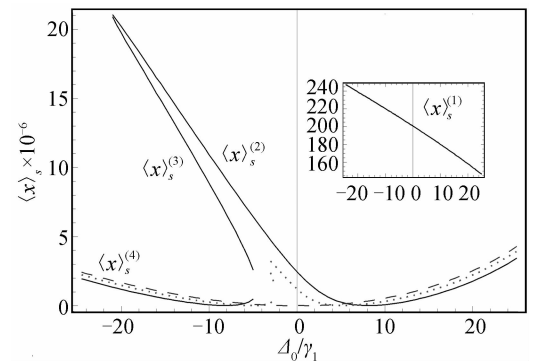


Fig. 4 Steady-state solutions $\langle x \rangle_s$ of the oscillating mirror versus Δ_0/γ_1

Here, in Fig. 5, as an example of multistable probe response, we plot the imaginary and real part of the probe susceptibility in two situations plotted in Fig. 4. For the case of $\Omega_B/\gamma_1=8$ (solid), due to the steady position $\langle x \rangle_s^{(2)}$ is ready similar to $\langle x \rangle_s^{(3)}$, we only need to plot the susceptibilities corresponding to $\langle x \rangle_s^{(2)}$ and $\langle x \rangle_s^{(4)}$ (solid), which are according to two steady-state solutions, respectively. For another case of $\Omega_B/\gamma_1=1$ (dashed), the probe susceptibility is according to a one steady-state solution.

In addition, the model could be implemented in practical system of cold atoms. For instance, we could consider cold rubidium atoms with a ladder-type

transition $5S_{1/2} \rightarrow 5P_{5/2} \rightarrow 5D_{5/2}$ as the medium, where levels $|1\rangle_i, |2\rangle_i, |3\rangle_i$ correspond to state $5S_{1/2}, 5P_{5/2}, 5D_{5/2}$, respectively. In particular, compared with the Λ -type system, the Ladder-type configuration has more advantage in meeting the weak-cavity-field assumption condition. This is because in Λ configuration there are two lower levels, between which the dephasing is none zero, while in the ladder-type configuration there is only one ground state and all the atoms are populated on it. Thus the model is more suitable for practical realization with cold atoms.

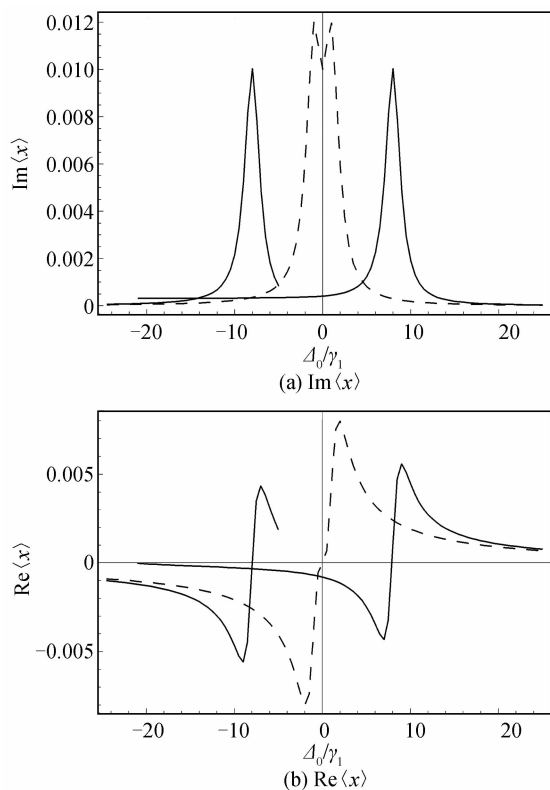


Fig. 5 The susceptibility of the atomic ensemble confined in the cavity versus Δ_0/γ_1

3 Conclusion

Steady-state displacement of the oscillating mirror and the probe susceptibility have been investigated. We have found that the optomechanical system could have multistable phenomenon for the oscillating mirror, and the number of steady-state solutions varied at different frequency domains. We have also studied the effects of the elastic coefficient and pump Rabi frequency on the steady-state properties of the atom-assisted optomechanical system. Numerical results showed that the number of steady-state solutions increased, as the elastic coefficient decreased, or as the pump Rabi frequency increased. If the elastic coefficient was large enough, the oscillating mirror would not affect the

optomechanical system, and all physical characters reverted to those of the original EIT system. These results may have potential applications in the area of high-precision quantum measurement and quantum information processing.

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