第 43 卷第 3 期
2014 年 3 月

光 子 学 报
ACTA PHOTONICA SINICA

Vol. 43 No. 3
March 2014

非相干耦合热电光伏光折变空间孤子对

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摘　要：基于无偏压光伏光折变晶体，理论研究了热电效应和光伏效应共同作用下的稳态空间孤子对. 从非相干耦合波方程出发，推导了热电光伏孤子对的数值解；采用有限差分方法，数值模拟了各类孤子对的强度包络. 结果表明：在自散焦光伏光折变晶体中，当正的热电场远大于光伏场时，亮-亮孤子对可以存在；当暗-暗或者灰-灰孤子对存在时，负的热电场可以增强自散焦效应，进而减小孤子的半峰全宽. 各类孤子对的半峰全宽可以通过改变热电场的大小而加以操控. 此外，非相干耦合热电光伏孤子对可以在一定条件下分别退化成热电孤子对或者光伏孤子对. 借助晶体的热电效应，可以使光折变晶体 $LiNbO_3$ 从自散焦转变成自聚焦，从而灵活控制晶体中孤子对的特性.

关键词：非线性光学；光折变空间光孤子；热电效应；光伏效应；孤子对

中图分类号：O437　　　**文献标识码**：A　　　　　**文章编号**：1004-4213(2014)03-0319002-4

Incoherently Coupled Photorefractive Spatial Soliton Pairs Based on the Combination of Pyroelectric and Photovoltaic Effect

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Abstract：Steady-state spatial soliton pairs resulted from the combination of pyroelectric and photovoltaic effect were predicted for unbiased photovoltaic photorefractive crystals. The numerical solutions of pyroelectric photovoltaic soliton pairs were deduced theoretically based on the incoherent coupling wave equations. The intensity envelopes of soliton pairs were obtained by finite difference method. The results show that the bright-bright soliton pair can be established in self-defocusing photovoltaic crystals when the positive pyroelectric field is large enough compared with photovoltaic field. For the dark-dark and grey-grey soliton pairs，the negative pyroelectric field can enhance the self-defocusing nonlinear effect and reduce the full width at half maximum of soliton pairs. The full width at half maximum of soliton pairs can be controlled by adjusting the value of the pyroelectric field. Moreover，the pyroelectric photovoltaic soliton pairs can degenerate into pyroelectric soliton pairs or photovoltaic soliton pairs under a certain conditions. The characteristics of photorefractive crystal $LiNbO_3$ can transfer from self-defocusing to self-focusing by virtue of the pyroelectric effects，which can help to control the characteristics of soliton pairs.

Key words：Nonlinear optics；Photorefractive spatial soliton；Pyroelectric effect；Photovoltaic effect；Soliton pairs

OCIS Codes：190.0190；190.5330；190.6135

0　Introduction

Photorefractive spatial solitons can form when the diffraction effect is balanced by the nonlinear photorefractive effect. The nonlinear photorefractive effect represents refractive index change produced by

the space-charge-field and this field can be induced by three methods[1-2]: external electric field, photovoltaic field or pyroelectric field. In general, screening solitons[3-4] are possible in steady state when an appropriate external electric field is applied to a non-photovoltaic photorefractive crystal. Photovoltaic solitons[5-6] are possible in an unbiased photorefractive crystal with appreciable photovoltaic effect. Furthermore, screening photovoltaic spatial solitons[7-9] can form when an electric field is applied to a photovoltaic photorefractive crystal. Recently, the pyroliton, i. e. pyroelectric spatial soliton[10-12] due to the combination of pyroelectric field and photovoltaic field has been investigated in unbiased photovoltaic photorefractive crystals. The pyroelectric spatial soliton can be formed by simple adjustment of the photorefractive crystal temperature. The Ref. [13] pointed out that the pyroelectric field can persist for several weeks if the dark conductivity is small enough. Then, to a good approximation, pyroelectric field can be treated as an external electric field. In other words, the pyroelectric soliton based on the combination of pyroelectric field and photovoltaic field can also be called pyroelectric photovoltaic spatial soliton.

In this paper, we will prove that bright-bright, dark-dark and grey-grey pyroelectric photovoltaic spatial soliton pairs can exist in unbiased photovoltaic photorefractive crystals provided that two optical beams have the same polarization, wavelength, and are mutually incoherent. The properties of these soliton pairs are discussed in detail.

1 Theoretical model

Two optical beams propagate in an unbiased photovoltaic photorefractive crystal along the z-axis and are permitted to diffract only along the x-direction. The two incident beams have the same polarization, wavelength, and are mutually incoherent. The crystal is placed between an insulating plastic cover and a metallic plate whose temperature is accurately controlled. As usual, the optical field are expressed in terms of slowly varying envelopes φ and ψ, i. e. $E_1 = \hat{x}\varphi(x,z)\exp(\mathrm{i}kz)$ and $E_2 = \hat{x}\psi(x,z)\exp(\mathrm{i}kz)$, where k is the wavenumber given by $k = k_0 n_e = (2\pi/\lambda_0)n_e$, n_e is the unperturbed index of refraction, and λ_0 is the free-space wavelength. Under these conditions, two optical beams satisfy the following envelope evolution equations[14-15]

$$\mathrm{i}\frac{\partial\varphi}{\partial z}+\frac{1}{2k}\frac{\partial^2\varphi}{\partial x^2}-\frac{k_0 n_e^3 r_{33} E_{sc}}{2}\varphi=0 \tag{1}$$

$$\mathrm{i}\frac{\partial\psi}{\partial z}+\frac{1}{2k}\frac{\partial^2\psi}{\partial x^2}-\frac{k_0 n_e^3 r_{33} E_{sc}}{2}\psi=0 \tag{2}$$

r_{33} is the electro-optic coefficient, E_{sc} is the space-

charge field. In unbiased photovoltaic photorefractive, the space-charge field E_{sc} consists of two parts[2, 12]

$$E_{sc}=-E_p\frac{I}{I+I_d}-E_{py}\frac{\sigma I}{I_d} \tag{3}$$

Here, the former represents the space-charge field originated from the photovoltaic field E_p and the latter represents the space-charge field originated from the pyroelectric field E_{py} resulted from temperature change. $I(x,z)=(n_e/2\eta_0)(|\varphi|^2+|\psi|^2)$ is the total optical intensity. I_d is dark irradiation, σ is a parameter related to the character of crystal and satisfies conditions of $\sigma I/I_d<1$.

Substituting Eq. (3) into Eq. (1) and (2), and adopting the dimensionless coordinates and variables: $s=x/x_0$, $\xi=z/(kx_0^2)$, $\varphi=(2\eta_0 I_d/n_e)^{1/2}U$, $\psi=(2\eta_0 I_d/n_e)^{1/2}V$, x_0 is an arbitrary spatial width, the dynamical evolution equation of soliton pairs can be obtained as following

$$\mathrm{i}\frac{\partial U}{\partial\xi}+\frac{1}{2}\frac{\partial^2 U}{\partial s^2}+\beta(|U|^2+|V|^2)U+$$
$$\alpha\frac{|U|^2+|V|^2}{1+|U|^2+|V|^2}U=0 \tag{4}$$

$$\mathrm{i}\frac{\partial V}{\partial\xi}+\frac{1}{2}\frac{\partial^2 V}{\partial s^2}+\beta(|U|^2+|V|^2)V+$$
$$\alpha\frac{|U|^2+|V|^2}{1+|U|^2+|V|^2}V=0 \tag{5}$$

where $\beta=\sigma\tau E_{py}$, $\alpha=\tau E_p$, $\tau=(k_0 x_0)^2(n_e^4 r_{eff}/2)$. In the following sections, we will discuss the possible soliton pair solution of Eq. (4) and (5).

2 Bright-bright soliton pairs

The case of bright-bright soliton pairs is considered firstly. In this case, the bright solitary wave solution of Eq. (4) and (5) can be obtained by expressing the beam envelope U and V in the usual fashion: $U=r^{1/2}y(s)\cos(\theta)\exp(\mathrm{i}\nu\xi)$ and $V=r^{1/2}y(s)\sin(\theta)\exp(\mathrm{i}\nu\xi)$. ν represents a nonlinear shift of the propagation constant and $y(s)$ is a normalized real function bounded as $0\leqslant y(s)\leqslant 1$, and is required to satisfy the boundary conditions of $y(0)=1$, $y'(0)=0$ and $y(s\rightarrow\pm\infty)=0$. The positive quantity r is defined as $r=I(0)/I_d$. θ is an arbitrary projection angle. Direct substitution of these forms of U and V in Eq. (4) and (5) lead to the following differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}s^2}=2\nu y-2\beta r y^3-2\alpha\frac{r y^3}{1+r y^2} \tag{6}$$

By integrating Eq. (6) once and by employing the y-boundary conditions, it can be found

$$\nu=\frac{\beta r}{2}-\frac{\alpha}{r}[\ln(1+r)-r] \tag{7}$$

$$\left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)^2=\beta r(y^2-y^4)+\frac{2\alpha}{r}[\ln(1+ry^2)-y^2\ln(1+r)] \tag{8}$$

$y(s)$ can be easily obtained by use of simple numerical

integration procedures. To illustrate the results, we consider the following example: let $\lambda_0 = 0.532$ μm, $x_0 = 20$ μm, $\sigma = 0.5$, $\theta = 30°$, $r = 1$. The LiNbO$_3$ parameters are taken here to be $r_{33} = 30 \times 10^{-12}$ mV^{-1}, $E_p = -2 \times 10^6$ V/m, $E_{py} = 5 \times 10^6$ V/m, $n_e = 2.2$. For this set of values, we have $\alpha = -39.2$, $\beta = 49$. Fig. 1 depicts the intensity envelopes of bright-bright soliton pairs. Note that, all previous observations[6, 16] indicate that LiNbO$_3$ has a negative perturbation of refractive index because of the negative photovoltaic coefficient. Here we can obtain bright-bright soliton pairs by the parameters of LiNbO$_3$ through the large positive pyroelectric field resulted from temperature increase. If the value of the pyroelectric field is decreased, the self-focusing nonlinear effect will decrease consequently and the FWHM of solitons will increase as shown in Fig. 1 (b). In other words, the formation of bright-bright soliton pairs requires that the positive pyroelectric field is larger than negative photovoltaic field, and so the right value of Eq. (8) is positive.
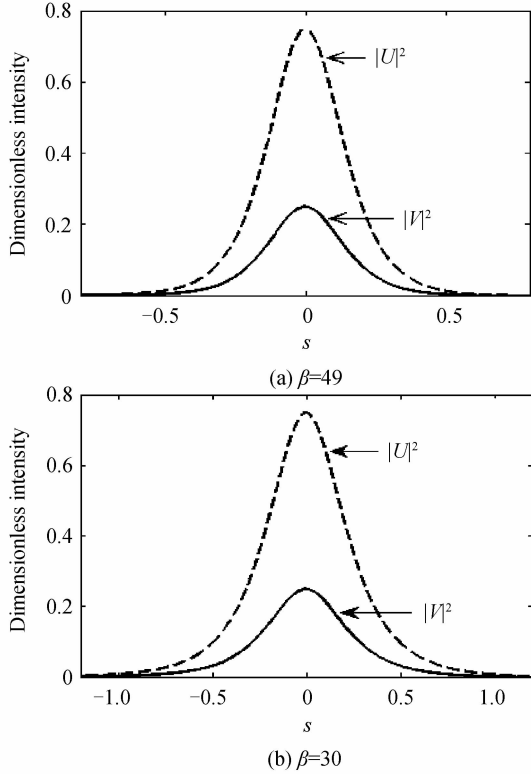

(a) $\beta = 49$

(b) $\beta = 30$

Fig. 1   The intensity envelopes of soliton pairs

3   Dark-dark soliton pairs

Similarly, dark-dark soliton pairs can also be analyzed. Let $U = \rho^{1/2} y(s) \cos(\theta) \exp(i\mu\xi)$ and $V = \rho^{1/2} y(s) \sin(\theta) \exp(i\mu\xi)$, where μ represents propagation constant and $y(s)$ is a normalized odd function of s, and is required to satisfy the boundary conditions of $y(0) = 0$, $y(s \to \pm\infty) = \pm 1$ and all the derivatives of y vanish at infinity. $\rho = I_\infty / I_d$, $I_\infty = I(s \to \pm\infty)$ indicates that

the intensity of the dark soliton beam attains asymptotically a constant value at infinity. From Eq. (4) and (5) the dark-dark soliton pair should satisfy

$$\left(\frac{dy}{ds}\right)^2 = -\beta\rho(y^2 - 1)^2 - 2\alpha\left[\frac{y^2 - 1}{1 + \rho} - \frac{1}{\rho}\ln\left(\frac{1 + \rho y^2}{1 + \rho}\right)\right] \quad (9)$$

Eq. (9) can be numerically solved and then the pair components can be simply obtained through a θ projection. Such a solitary-wave pair solution with or without pyroelectric field is shown in Fig. 2, where, $\alpha = -39.2$, $\rho = 1$, $\theta = 30°$. It shows that the negative pyroelectric field can enhance the self-defocusing nonlinear effect, and reduce the FWHM of dark-dark soliton pairs. Moreover, we can find that the formation of dark-dark soliton pairs requires that the right value of Eq. (9) is positive, which can be obtained by adjusting the value of α, β.
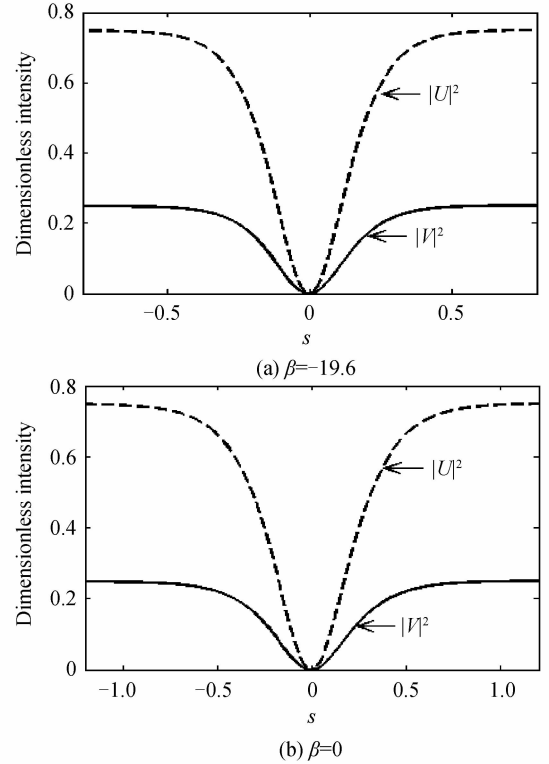

(a) $\beta = -19.6$

(b) $\beta = 0$

Fig. 2   The intensity envelopes of dark-dark soliton pairs

4   Grey-grey soliton pairs

Finally, grey-grey soliton pairs are also possible. The solution of grey-grey soliton pairs can be expressed as following

$$U(s, \xi) = \rho^{1/2} y(s) \cos(\theta) \exp\left[i\left(\mu\xi + \int \frac{Qds}{y^2(s)}\right)\right] \quad (10)$$

$$V(s, \xi) = \rho^{1/2} y(s) \sin(\theta) \exp\left[i\left(\mu\xi + \int \frac{Qds}{y^2(s)}\right)\right] \quad (11)$$

where Q is a real constant to be determined. $y(s)$ is a normalized even function and satisfies the boundary conditions: $y(s \to \pm\infty) = 1$, $y^2(0) = m(0 < m < 1)$, m is the grayness of the solitons, $y'(0) = 0$, $y^{(n)}(\infty) = 0$ $(n \geq 1)$. By substituting Eq. (10)$\sim$(11) into Eq. (4)

and （5）, we find that the normalized soliton envelope $y(s)$ satisfy

$$\left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)^2 = 2\mu(y^2-1) - \beta\rho(y^4-1) - \left(\frac{1}{y^2}-1\right)Q^2 - \frac{2\alpha}{\rho}\left[\rho(y^2-1) - \ln\left(\frac{1+\rho y^2}{1+\rho}\right)\right] \quad (12)$$

where

$$Q^2 = 2\beta\rho + 2\alpha\frac{\rho}{1+\rho} - 2\mu \quad (13)$$

$$\mu = \frac{1}{2(m-1)^2}\left\{m(m^2-1)\beta\rho + (1-m)\left(2\beta\rho + \frac{2\alpha\rho}{1+\rho}\right) + \frac{2m\alpha}{\rho}\left[\rho(m-1) - \ln\left(\frac{1+\rho m}{1+\rho}\right)\right]\right\} \quad (14)$$

Similarly, the grey-grey soliton pair is shown in Fig. 3 with pyroelectric field and without pyroelectric field, where, $\alpha=-39.2, m=0.3, \rho=1$. Note that the dark-dark soliton pair will be derived when the parameter m is zero.
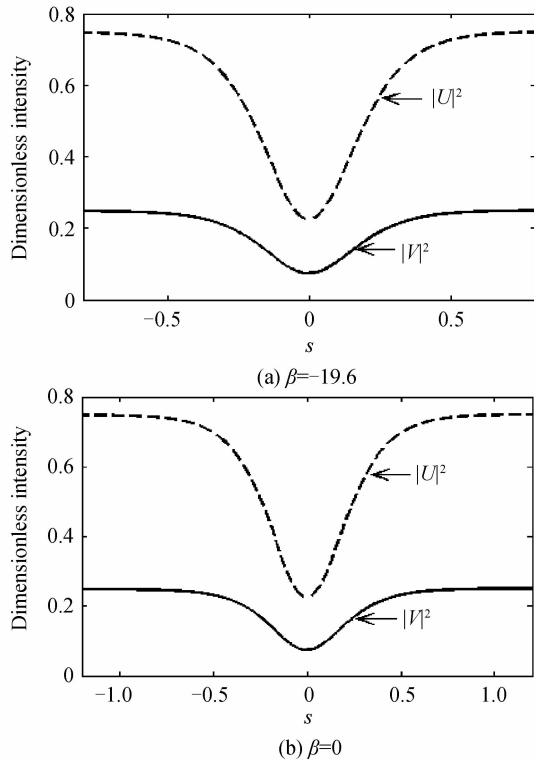

Fig. 3　The intensity envelopes of grey-grey soliton pairs

5　Conclusions

Our results show that bright-bright, dark-dark and grey-grey pyroelectric photovoltaic spatial soliton pairs can exist in unbiased photovoltaic photorefractive crystals. The properties of soliton pairs can be adjusted conveniently by controlling the crystal temperature. Due to the fact that soliton pairs resulted from the combination of photovoltaic and pyroelectric field, we can predict that the pure pyroliton pairs will be obtained when the photovoltaic effect is negligible such as in SBN crystal, and the photovoltaic soliton pair can be reduced when the pyroelectric field is negligible i. e. the crystal temperature is unchanged.

References

[1] CHEN Z G, SEGEV M, CHRISTODOULIDES D N. Optical spatial solitons: historical overview and recent advances[J]. *Reports on Progress in Physics*, 2012, **75**(8): 086401.

[2] BUSE K, RINGHOFER K H. Pyroelectric drive for light-induced charge transport in the photorefractive process[J]. *Applied Physics A*, 1993, **57**(2): 161-165.

[3] SEGEV M, VALLEY G C, CROSIGNANI B, et al. Steady-state spatial screening solitons in photorefractive materials with external applied field[J]. *Physics Review Letters*, 1994, **73**(24): 3211-3214.

[4] CHRISTODOULIDES D N, CARVALHO M I. Bright, dark and gray spatial soliton states in photorefractive media[J]. *Journal of the Optical Society of America B*, 1995, **12**(9): 1628-1633.

[5] VALLEY G C, SEGEV M, CROSIGHANI B. Dark and bright photovoltaic solitions[J]. *Physics Review A*, 1994, **50**(6): R4457-R4460.

[6] SHE Wei-Long, XU Chi-cheng, GUO Bin, et al. Formation of photovoltaic bright spatial soliton in photorefractive LiNbO3 crystal by a defocused laser beam induced by a background laser beam[J]. *Journal of the Optical Society of America B*, 2006, **23**(10): 2121-2126.

[7] LIU Jin-song, LU Ke-qing. Screening-photovoltaic spatial solitons in biased photovoltaic photorefractive crystals and their self-deflection[J]. *Journal of the Optical Society of America B*, 1999, **16**(4): 550-555.

[8] FAZIO E, RENZI F, RINALDI R, et al. Screening-photovoltaic bright solitons in lithium niobate and associated single-mode waveguides[J]. *Applied Physics Letters*, 2004, **85**(12): 2193-2195.

[9] JIANG Qi-chang, SU Yan-li, JI Xuan-mang. Bright-dark self-coupled vector spatial solitons in biased two-photon photovoltaic photorefractive crystals [J]. *Optics Communications*, 2010, **283**(18): 3512-3515.

[10] SAFIOUI J, DEVAUX F, CHAUVET M. Pyroliton : pyroelectric spatial soliton[J]. *Optics Express*, 2009, **17**(24): 22209-22216.

[11] SAFIOUI J, FAZIO E, DEVAUX F, et al. Surface-wave pyroelectric photorefractive solitons [J]. *Optics Letters*, 2010, **35**(8): 1254-1256.

[12] JIANG Qi-chang, SU Yan-li, JI Xuan-mang. Pyroelectric photovoltaic spatial solitons in unbiased photorefractive crystals[J]. *Physics Letters A*, 2012, **376**(45): 3085-3087.

[13] JOHANN F, SOERGEL E. Quantitative measurement of the surface charge density[J]. *Applied Physics Letters*, 2009, **95**(23): 232906.

[14] CHRISTODOULIDES D N, SINGH S R, CARVALHO M I. Incoherently coupled soliton pairs in biased photorefractive crystals[J]. *Applied Physics Letters*, 1996, **68**(13): 1763-1765.

[15] SU Yan-li, JIANG Qi-chang, JI Xuan-mang, et al. Incoherently coupled spatial soliton families in photorefractive polymers[J]. *Optics and Laser in Engineering*, 2011, **49**(4): 526-529.

[16] TAYA M, BASHAW M, FEJER M M, et al. Observation of dark photovoltaic spatial solitons[J]. *Physics Review A*, 1995, **52**(4): 3095-3100.