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Compton 散射对等离子体零色散附近调制不稳定性的影响

郝东山,朱江波
(郑州华信学院 信息工程系,河南 新郑 451150)

摘　要:应用 Compton 散射模型和非线性 Schrödinger 方程,研究了 Compton 散射对等离子体零色散附近调制不稳定性的影响,提出了将 Compton 散射作为形成调制不稳定性的新机制,给出了等离子体修正色散方程,并进行了数值计算.结果表明:二阶色散较大时,远离零色散附近调制不稳定性增益谱由入射和散射光产生的二阶色散决定,随二阶色散增大,四阶色散作用迅速减小,这是因散射效应有效削弱了四阶效应的缘故.零色散附近调制不稳定性增益谱由入射和散射光产生的四阶色散决定,二阶色散被湮灭,微扰频率最大值向临界微扰频率较快靠近,这是因散射效应有效增强了四阶效应的缘故.增益谱宽随等离子体损耗增大而迅速增大,这是因功率增大使散射效应增强,导致带电粒子辐射阻尼增强的缘故.

关键词:等离子体;非线性光学;数值计算;调制不稳定性;色散;耦合;多光子非线性 Compton 散射
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Influence of Compton Scattering on Modulation Instability Near Zero Dispersion in Plasma

HAO Dong-shan, ZHU Jiang-bo
(*Department of Information Engineering, Zhengzhou Huaxin University, Xi'zheng, Henan* 451100, *China*)

Abstract: The influence of Compton scattering on the modulation instability near zero dispersion in plasma is studied by using the model of multi-photon nonlinear Compton scattering and nonlinear Schrödinger′s equation, a new mechanism on the modulation instability is formed by Compton scattering has been raised, a amended dispersion equation on the light pulse propagation has been given out, and it is simulated with the numbers. The results show that when the second-step dispersion is bigger, the gain score of the modulation instability of the wander from the nearby zero dispersion is decided by the second-step dispersion produced by incident and scattered lights. The fourth-step dispersion effect is quickly decreased along with the increasing of the second-step dispersion, and the cause is that the fourth-step dispersion effect is affectively weakened by Compton scattering. The gain score of the modulation instability near the zero dispersion is decided by the fourth-step dispersion produced by incident light and scattered light, the second-step dispersion is annihilated, the maximum number of the perturbation frequency is quickly approached to critical perturbation frequency, and the cause is that the fourth-step dispersion effect is affectively increased by Compton scattering. The width of the gain score is quickly increased along with the increasing of the plasma loss, the cause is that the scattered effect is increased along with the power increasing, and the effect causes that the radiation damping of the charged particle is increased.

Key words: Plasma; Nonlinear optics; Numerical computing; Modulation instability; Dispersion; Coupling; Multi-photon nonlinear Compton scattering
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第一作者:郝东山(1949-),男,教授,学士,主要研究方向为激光物理与光纤通信. Email:haodongshan1948@126.com
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0 引言

等离子体调制不稳定性(Modulation Instability, MI)是色散和非线性导致连续信号变为幅度和相位被调制信号,形成对稳态调制,如连续波分裂成一系列超短脉冲[1]、核聚变快点火[2]、波长变换[3]、粒子加速[4]等,引起人们深入研究[5~7]. 张书敏等[8]指出,光脉冲在最小群速色散附近时,等离子体波长不在最小损耗处,四阶色散和损耗对 MI 起决定作用. 阿不都热苏力等[9]指出,电磁 MI 激发的强电磁场使电子束在极短距离内沉积能量,并对电子热流产生抑制作用. 汤伟等[10]指出,弱相对论效应引起电子质量变化的二级非线性对 MI 影响不大. 李明[11]等指出,激光诱导水等离子体屏蔽效应随光能量增强呈增强趋势,并首次观测到 ns 激光脉冲作用下水中线型击穿现象. 姚汝贤等[12-13]指出,Compton 散射使等离子体 MI 最大时间增长率显著减小,界面处时间增长率显著增大,加速激光场坍塌[14]. 郝东山[15]指出,Compton 散射在等离子体中形成的强朗缪尔湍动使激光场出现明显成丝现象. 但以上对等离子体零色散附近 MI 的研究并未涉及 Compton 散射因素. 实验表明,等离子体中光强在 $10^{16}\,\mathrm{W/cm^2}$ 量级以上时,非线性 Compton 效应开始显现[16],Compton 散射对零色散附近 MI 的影响不可忽略. 本文应用 Compton 散射模型,给出了零色散附近 MI 修正色散关系,得出了一些重要结论.

1 Compton 散射下的色散关系

若等离子体中发生多光子非线性 Compton 散射(简称散射),则散射光频为[15]

$$\omega_s=\frac{N\omega(1+\beta\cos\theta)(1-\beta_f\cos\theta_1')}{\eta^2+\frac{\eta N\hbar\omega(1+\beta\cos\theta)}{mc^2(1-\cos\theta')^{-1}}}\tag{1}$$

式中,$\eta=(\gamma-\gamma_f)/|\gamma-1|$ 为散射非弹性参量;$\gamma=[-(\gamma/c)^2]^{-1/2}=(1-\beta^2)^{-1/2}$,$\upsilon_f=[1-(\upsilon_f/c)^2]^{-1/2}=(1-\beta_f^2)^{-1/2}$、$\upsilon$ 和 υ_f 分别为电子散射前后的 Lorentz 因子、速度;N、θ、θ_1' 和 θ'、c、$h=2\pi\hbar$、ω、m 分别为与电子同时作用光子数、散射前电子和光子运动方向夹角、电子静止系中电子和散射光子运动方向夹角和散射角、真空中光速、普朗克常量、入射光频、电子静质量. 取入射和散射光耦合频率 $\omega_c=\omega_s+\omega$,有

$$\omega_c=\omega\left[\frac{N(1+\beta\cos\theta)(1-\beta_f\cos\theta_1')}{\eta^2+\frac{\eta N\hbar\omega(1+\beta\cos\theta)}{mc^2(1-\cos\theta')^{-1}}}+1\right]\tag{2}$$

可见,耦合光频明显大于入射光频,散射光对等离子体零色散附近 MI 影响不能忽略. 设散射前入射光在等离子体中传播距离、时间、群速度倒数、2~4 阶色散系数、非线性系数、缓变包络振幅、损耗系数分别为 z、t、α_1、$\alpha_2\sim\alpha_4$、μ、A、ζ,散射光作用下,若忽略 z 和 t 的变化,其余参量变化量分别为 $\Delta\alpha_1$、$\Delta\alpha_2\sim\Delta\alpha_4$、$\Delta\mu$、$\Delta A$、$\Delta\zeta$,耦合光脉冲在 4 阶色散和等离子体损耗下传输的广义非线性 Schrödinger 方程为

$$\left(\frac{\partial A}{\partial z}+\alpha_1\frac{\partial A}{\partial t}+\frac{\mathrm{i}}{2}\alpha_2\frac{\partial^2 A}{\partial t^2}+\frac{\zeta}{2}A\right)+\left(\frac{\partial\Delta A}{\partial z}+\Delta\alpha_1\frac{\partial\Delta A}{\partial t}+\frac{\mathrm{i}}{2}\Delta\alpha_2\frac{\partial^2\Delta A}{\partial t^2}\right)\approx\left(\mathrm{i}\frac{\mu}{A^{-1}}|A|^2+\frac{1}{6}\alpha_3\frac{\partial^2 A}{\partial t^2}+\frac{\mathrm{i}}{24}\alpha_4\frac{\partial^4 A}{\partial t^4}\right)+\left[\mathrm{i}\left(\frac{\Delta\mu}{A^{-1}}|A|^2+2\mu|A\Delta A|A+\mu|A|^2\Delta A+\frac{1}{6}\left(\Delta\alpha_3\frac{\partial^2 A}{\partial t^2}+\alpha_3\frac{\partial^2\Delta A}{\partial t^2}\right)+\frac{\mathrm{i}}{24}\left(\Delta\alpha_4\frac{\partial^4 A}{\partial t^4}+\alpha_4\frac{\partial^4\Delta A}{\partial t^4}\right)\right]\tag{3}$$

式(3)变换及得到的 Schrödinger 方程分别为

$$\begin{cases}T\approx t-\alpha_1 z,\Delta T\approx-\Delta\alpha_1 z\\ \sigma\approx\dfrac{1-\zeta z}{\alpha},\Delta\sigma\approx-\dfrac{\Delta\zeta z}{\alpha}\\ q\approx\left(1+\dfrac{\zeta}{2}z\right)A,\Delta q\approx\dfrac{\Delta\zeta zA}{2}+\Delta A\end{cases}\tag{4}$$

$$\left(\frac{\partial q}{\partial\sigma}+\frac{\mathrm{i}\alpha_2\mathrm{e}^{\zeta z}}{2}\frac{\partial^2 q}{\partial T^2}\right)+\left[\frac{\partial\Delta q}{\partial\sigma}+\frac{\mathrm{i}\mathrm{e}^{\zeta z}}{2}\left(\Delta\alpha_2\frac{\partial^2 q}{\partial T^2}\alpha_2\frac{\partial^2\Delta q}{\partial T^2}\right)\right]\approx\left[\frac{\mathrm{i}\mu q}{|q|^{-2}}+\mathrm{e}^{\zeta z}\left(\frac{\alpha_3}{6}\frac{\partial^3 q}{\partial T^3}+\frac{\mathrm{i}\alpha_4}{24}\frac{\partial^4 q}{\partial T^4}\right)\right]+\left[3\mu|q|^2\Delta q+\frac{\mathrm{e}^{\zeta z}}{6}\left(\Delta\alpha_3\frac{\partial^3 q}{\partial T^3}+\alpha_3\frac{\partial^3\Delta q}{\partial T^3}\right)+\frac{\mathrm{i}\mathrm{e}^{\zeta z}}{24}\left(\Delta\alpha_4\frac{\partial^4 q}{\partial T^4}+\alpha_4\frac{\partial^4\Delta q}{\partial T^4}\right)\right]\tag{5}$$

式中,$\mathrm{e}^{\mathrm{i}(\Delta\mu p_0\zeta+\mu\Delta p_0\zeta+\mu p_0\Delta\zeta)}\to 1$. 式(5)解为

$$\tilde{q}+\Delta\tilde{q}\approx\frac{\sqrt{p_0}+a}{\mathrm{e}^{-\mu p_0\zeta}}+\frac{\sqrt{\Delta p_0}+\Delta a}{\mathrm{e}^{-\mathrm{i}\mu p_0\zeta}}\tag{6}$$

式中 a 和 Δa 为连续光微扰及其增量,式(5)和(6)两端第二项为散射修正项. 可见,散射使扰动加剧. a 和 Δa 线性化,由式(4)和(6),得

$$\left(\frac{\partial a}{\partial\sigma}+\frac{\mathrm{i}\alpha_2\mathrm{e}^{\zeta z}}{2}\frac{\partial^2 a}{\partial T^2}\right)+\left[\frac{\partial\Delta a}{\partial\sigma}+\frac{\mathrm{i}\mathrm{e}^{\zeta z}}{2}\left(\Delta\alpha_2\frac{\partial^2 a}{\partial T^2}+\alpha_2\frac{\partial^2\Delta a}{\partial T^2}\right)\right]\approx\frac{\mathrm{i}\mu}{p_0^{-1}}(a+a^*)+\left\{\mathrm{i}\left[\frac{\Delta\mu}{p_0^{-1}}(a+a^*)+\frac{\mu\Delta p_0}{(a+a^*)^{-1}}+\frac{\mu p_0}{(\Delta a+\Delta a^*)^{-1}}\right]+\frac{\mathrm{e}^{\zeta z}}{6}\left(\Delta\alpha_3\frac{\partial^3 a}{\partial T^3}+\alpha_3\frac{\partial^3\Delta a}{\partial T^3}\right)+\frac{\mathrm{i}\mathrm{e}^{\zeta z}}{24}\left(\Delta\alpha_4\frac{\partial^4 a}{\partial T^4}+\alpha_4\frac{\partial^4\Delta a}{\partial T^4}\right)\right]\tag{7}$$

a^* 和 Δa^* 为 a 和 Δa 共轭复数. 设 a 和 Δa 解为

$$a(\sigma,T)=U+\mathrm{i}V,\Delta a=\Delta U+\mathrm{i}\Delta V \tag{8}$$

由式(7)和(8)，得

$$\left(\frac{\partial U}{\partial\sigma}-\frac{\alpha_2\mathrm{e}^{\zeta z}}{2}\frac{\partial^2 V}{\partial T^2}\right)+\left[\frac{\partial\Delta U}{\partial\sigma}-\frac{\mathrm{i}\mathrm{e}^{\zeta z}}{2}\left(\Delta\alpha_2\frac{\partial^2 V}{\partial T^2}+\alpha_2\frac{\partial^2\Delta V}{\partial T^2}\right)\right]\approx\frac{\mathrm{e}^{\zeta z}}{6}\left(\Delta\alpha_3\frac{\partial^3 U}{\partial T^3}+\alpha_3\frac{\partial^3\Delta U}{\partial T^3}\right)-\frac{\mathrm{e}^{\zeta z}}{24}\left(\Delta\alpha_4\frac{\partial^4 V}{\partial T^4}+\alpha_4\frac{\partial^4\Delta V}{\partial T^4}\right)\Big] \tag{9}$$

$$\left(\frac{\partial V}{\partial\sigma}+\frac{\alpha_2\mathrm{e}^{\zeta z}}{2}\frac{\partial^2 U}{\partial T^2}\right)+\left[\frac{\partial\Delta V}{\partial\sigma}+\frac{\mathrm{e}^{\zeta z}}{2}\left(\Delta\alpha_2\frac{\partial^2 U}{\partial T^2}+\alpha_2\frac{\partial^2\Delta U}{\partial T^2}\right)\right]\approx\left[\frac{2\mu U}{p_0^{-1}}+\frac{\mathrm{e}^{\zeta z}}{6}(\alpha_3\frac{\partial^2 V}{\partial T^2}+\frac{\alpha_4}{4}\times\frac{\partial^4 U}{\partial T^4})\right]+\left(\left[2\frac{\Delta\mu U}{p_0^{-1}}+\frac{\mu U}{\Delta p_0^{-1}}+\frac{\mu\Delta U}{p_0^{-1}}\right)+\frac{\mathrm{e}^{\zeta z}}{6}\times\left(\Delta\alpha_3\frac{\partial^3 V}{\partial T^3}+\alpha_3\frac{\partial^3\Delta V}{\partial T^3}\right)+\frac{\mathrm{e}^{\zeta z}}{24}\left(\Delta\alpha_4\frac{\partial^4 U}{\partial T^4}+\alpha_4\frac{\partial^4\Delta U}{\partial T^4}\right)\right] \tag{10}$$

设解的形式为

$$\begin{bmatrix}U+\Delta U\\V+\Delta V\end{bmatrix}=\begin{bmatrix}U_0+\Delta U_0\\V_0+\Delta V_0\end{bmatrix}\mathrm{e}^{\mathrm{i}(K\sigma-\Omega T)} \tag{11}$$

式中，K 和 Ω 分别为微扰波数和频率. 由式(9)～(11)，得 U_0 和 ΔU_0 与 V_0 和 ΔV_0 的耦合方程为

$$\left[\left(\mathrm{i}K-\frac{\mathrm{i}\mathrm{e}^{\zeta z}}{6\alpha_3^{-1}}\Omega^2\right)U_0+\frac{\mathrm{e}^{\zeta z}\Omega^2}{2}\left(\alpha_2+\frac{\alpha_4\Omega^2}{12}\right)V_0\right]+\left\{\mathrm{i}\Delta K-\frac{\mathrm{i}\mathrm{e}^{\zeta z}\Omega^2}{6}\left(\Delta\alpha_3+\frac{2\alpha_3\Delta\Omega}{\Omega}\right)U_0+\left(\mathrm{i}K-\frac{\mathrm{i}\alpha_3\mathrm{e}^{\zeta z}}{6\Omega^{-2}}\right)\Delta U_0+\frac{\Omega^2}{\mathrm{e}^{-\zeta z}}\left(\frac{\Delta\alpha_2}{2}+\frac{\alpha_2\Delta\Omega}{\Omega}+\frac{\Delta\alpha_4\Omega^2}{24}+\frac{\alpha_4\Omega\Delta\Omega}{6}\right)V_0+\frac{\mathrm{e}^{\zeta z}\Omega^2}{2}\left(\frac{1}{\alpha_2^{-1}}+\frac{\alpha_4\Omega^2}{12}\right)\Delta V_0\right\}=0 \tag{12}$$

$$\left[\left(\frac{\alpha_2\Omega^2}{2}+\frac{2\mu}{p_0^{-1}}+\frac{\alpha_4\Omega^4}{24}\right)\mathrm{e}^{\zeta z}U_0+\frac{\mathrm{i}}{6}\left(\frac{\alpha_3\Omega^3}{\mathrm{e}^{-\zeta z}}-K\right)V_0\right]+\left\{\mathrm{e}^{\zeta z}\left[\frac{\Delta\alpha_2\Omega^2}{2}+\alpha_2\Delta\Omega+\frac{\Delta\alpha_4\Omega^3\Delta\Omega}{6}+2(\Delta\mu p_0+\Delta p_0\mu)U_0+\frac{\mathrm{e}^{\zeta z}}{2}\left(\alpha_2\Omega^2+4\mu p_0+\frac{\alpha_4\Omega^4}{12}\right)\Delta U_0+\frac{\mathrm{i}\mathrm{e}^{\zeta z}}{\Omega^{-2}}\left(\frac{\Delta\alpha_3\Omega}{6}+\frac{\alpha_3\Delta\Omega}{3}-K\right)V_0=0\right.\right. \tag{13}$$

式(12)和(13)有非奇异解时，K 和 ΔK 及 Ω 和 $\Delta\Omega$ 满足色散关系，即

$$K+\Delta K\approx\left[\frac{\alpha_3\mathrm{e}^{\zeta z}\Omega^3}{6}\pm\mathrm{e}^{\zeta z}\Omega^2\left(\frac{\alpha_2^2}{4}+\frac{\alpha_4^2\Omega}{24^2}+\frac{\alpha_2\alpha_4\Omega^2}{24\mathrm{e}^{\zeta z}}+\frac{\mu p_0\alpha_2}{\mathrm{e}^{\zeta z}\Omega^2}+\frac{\mu p_0\alpha_4}{12\mathrm{e}^{\zeta z}}\right)^{1/2}\right]+\left\{\frac{\mathrm{e}^{\zeta z}\Omega^2}{6}(\Delta\alpha_{3\Omega}+3\alpha_3\Delta\Omega)\pm\left[\alpha_2\mathrm{e}^{\zeta z}\Omega^2\left(\frac{\Delta\alpha_2}{2}+\frac{\alpha_2\Delta\Omega}{\Omega^2}\right)+\frac{\mathrm{e}^{\zeta z}\Omega^2}{24}\left(\frac{2\alpha_4\Delta\alpha_4}{\Omega^{-1}}+5\alpha_4^2\Delta\Omega+24\Delta\alpha_2\alpha_4\Omega^2+24\alpha_2\Delta\alpha_4\Omega^2+216\alpha_2\alpha_4\Omega^3\Delta\Omega+\frac{\Delta\mu p_0\alpha_2+\mu\Delta p_0\alpha_2+\mu p_0\Delta\alpha_2}{\mathrm{e}^{\zeta z}}+\frac{2\mu p_0\alpha_2\Delta\Omega}{\Omega\mathrm{e}^{\zeta z}}\right)+\frac{\Omega^2}{12}\left(\frac{\Delta\mu\alpha_4}{p_0^{-1}}+\mu\Delta p_0\alpha_4+\mu p_0\Delta\alpha_4+\frac{3\mu p_0\alpha_4\Delta\Omega}{\Omega}\right)\right]^{1/2}\right\} \tag{14}$$

可见，仅在负色散区，即$(\alpha_2+\Delta\alpha_2)\leqslant 0$ 及$(\alpha_4+\Delta\alpha_4)\leqslant 0$ 时发生 MI，三阶色散对 MI 不起作用.

单模等离子体 MI 中引入耦合增益谱 $g_c=g+\Delta g$，g 和 Δg 为散射前增益谱及增量. 定义 $g\equiv 2\mathrm{Im}(K)$，$\Delta g\equiv 2\mathrm{Im}(\Delta K)$. 由式(14)，得

$$g+\Delta g\approx 2\mathrm{e}^{2\zeta z}\left[\frac{\mu p_0|\alpha_2|\Omega^2}{\mathrm{e}^{\zeta z}}+\frac{\mu p_0|\alpha_4|\Omega^4}{12\mathrm{e}^{\zeta z}}-\frac{\alpha_2^2\Omega^4}{4}-\frac{\alpha_4^2\Omega^3}{24^2}-\frac{\alpha_2\Omega^6}{24\alpha_4^{-1}}\right]^{1/2}+2\left[\frac{\Omega^2}{\mathrm{e}^{-\zeta z}}\times(\Delta\mu p_0|\alpha_2|+\mu\Delta p_0|\alpha_2|+\mu p_0|\Delta\alpha_2|)+\mu p_0\Omega\mathrm{e}^{\zeta z}(|\Delta\alpha_2|\Omega+2|\alpha_2|\Delta\Omega)+\frac{\mathrm{e}^{\zeta z}\Omega^4}{12}\times|\alpha_4|(\Delta\mu p_0+\mu\Delta p_0+\mu p_0+4\mu p_0)\Omega^3\Delta\Omega-\frac{\mathrm{e}^{2\zeta z}\alpha_2\Omega^3}{2}(\alpha_2\Omega+2\alpha_2\Delta\Omega)+\frac{\mathrm{e}^{2\zeta z}\Omega^2}{24^2}\times(2\alpha_4\Delta\alpha_4\Omega+3\alpha_4^2\Delta\Omega)-\frac{\mathrm{e}^{2\zeta z}\Omega^5}{24}\left(\frac{\Delta\alpha_2\Omega}{\alpha_4^{-1}}+\alpha_2\Delta\alpha_4\Omega+\frac{6\alpha_2\Delta\Omega}{\alpha_4^{-1}}\right)\right]^{1/2} \tag{15}$$

若 $|\alpha_4|=|\Delta\alpha_4|=0$，式(14)和(15)为

$$K_1+\Delta K_1\approx\left\{\frac{\alpha_3\mathrm{e}^{\zeta z}\Omega^3}{6}\pm\frac{|\alpha_2|\mathrm{e}^{2\zeta z}\Omega}{2}\left[\frac{1}{\Omega^{-2}}+\mathrm{sgn}(\alpha_2)\frac{1}{\Omega_{\mathrm{e}1}^{-2}}\right]^{1/2}\right\}+\left\{\frac{\Omega^2(\Delta\alpha_3\Omega+3\alpha_3\Delta\Omega)}{6\mathrm{e}^{-\zeta z}}\pm\frac{|\Delta\alpha_2|\Omega+|\alpha_2|\Delta\Omega}{2\mathrm{e}^{-\zeta z}}[\Omega^2+\mathrm{sgn}(\alpha_2)\Omega_{\mathrm{e}1}^2]^{1/2}+\frac{|\alpha_2|\Omega}{2\mathrm{e}^{-\zeta z}}\left[\frac{2\Omega}{\Delta\Omega^{-1}}+\frac{\mathrm{sgn}(\Delta\alpha_2)}{\Omega_{\mathrm{e}1}^{-2}}+\frac{2\mathrm{sgn}(\alpha_2)}{\Omega_{\mathrm{e}}^{-1}\Delta\Omega_{\mathrm{e}}^{-1}}\right]^{1/2}\right\} \tag{16}$$

$$g_c\approx\left|\frac{\alpha_2\Omega}{\mathrm{e}^{-\zeta z}}\right|(\Omega_{\mathrm{e}}^2-\Omega)^{21/2}+\left[\left|\frac{\Delta\alpha_2\Omega+\alpha_2\Delta\Omega}{\mathrm{e}^{-\zeta z}}\right|(\Omega_{\mathrm{e}}^2-\Omega^2)^{1/2}+2\left|\frac{\alpha_2\Omega}{\mathrm{e}^{-\zeta z}}\right|\left(\frac{\Delta\Omega_{\mathrm{e}}}{\Omega_{\mathrm{e}}^{-1}}-\frac{\Delta\Omega}{\Omega^{-1}}\right)^{1/2}\right] \tag{17}$$

$\Omega_{\mathrm{e}1}=(4\mu p_0/|\alpha_2|\mathrm{e}^{\zeta z})^{1/2}$ 和 $\Delta\Omega_{\mathrm{e}1}=[4(\Delta\mu p_0+\mu\Delta p_0)/|\alpha_2|\mathrm{e}^{\zeta z}]^{1/2}$ 为临界扰频及其增量. 可见，$\Omega_{\mathrm{e}1}>\Omega$ 及 $\Delta\Omega_{\mathrm{e}1}>\Delta\Omega$ 时 MI 才发生. 由式(17)知，$(\Omega+\Delta\Omega)_{\max1}$ 和 $(g+\Delta g)_{\max1}$ 分别为

$$(\Omega+\Delta\Omega)_{\max1}\approx(\Omega_{\mathrm{e}1}+\Delta\Omega_{\mathrm{e}1})/\sqrt{2} \tag{18}$$

$$(g+\Delta g)_{\max1}\approx 2\mu p_0+2(\Delta\mu p_0+\mu\Delta p_0) \tag{19}$$

$|\alpha_2|=|\Delta\alpha_2|=0$,式(14)和(15)分别为

$$K_2+\Delta K_2\approx\left\{\frac{\alpha_3 e^{\zeta z}\Omega^3}{6}\pm\frac{\Omega^2|\alpha_4|e_{\zeta z}}{24}\left[\frac{1}{\Omega^{-2}}+\mathrm{sgn}(\alpha_4)\frac{1}{\Omega_{e2}^{-4}}\right]^{1/2}\right\}+\left\{\frac{\Delta\alpha_3\Omega+3\alpha_3\Delta\Omega}{6e^{-\zeta z}\Omega^{-2}}\pm\frac{\Omega\Delta\Omega|\alpha_4|+\Omega^2|\Delta\alpha_4|}{24e^{-\zeta z}}\cdot\right.$$
$$\left.[\Omega^4+\mathrm{sgn}(\alpha_4)\Omega_{e2}^4]^{3/2}+\frac{\Omega^2|\alpha_4|e^{\zeta z}}{24}[4\Omega^3\Delta\Omega+\mathrm{sgn}(\Delta\alpha_4)\Omega_{e2}^4]^{3/2}+\frac{\Omega^2|\alpha_4|e^{\zeta z}}{6}\left[\Omega^4+\frac{\mathrm{sgn}(\alpha_4)\Delta\Omega_{e2}}{\Omega_{e2}^{-3}}\right]^{3/2}\right\} \tag{20}$$

$$g_2+\Delta g_2\approx\left[\frac{\Omega^2|\alpha_4|e^{\sigma z}}{12}(\Omega_{e2}^4-\Omega)^{4\,3/2}\right]+\left[\frac{(2\Omega\Delta\Omega|\alpha_4|+\Omega^2|\Delta\alpha_4|)e^{\zeta z}}{12}(\Omega_{e2}^4-\Omega^4)^{3/2}+\right.$$
$$\left.\frac{\Omega^2|\alpha_4|e^{\zeta z}}{2}(\Omega_{e2}^3\Delta\Omega_{e2}-\Omega^3\Delta\Omega)^{3/2}\right] \tag{21}$$

$\Omega_{e1}=(4\mu p_0/|\alpha_2|e^{\zeta z})^{1/4}$ 和 $\Delta\Omega_{e1}=[4(\Delta\mu p_0+\mu\Delta p_0)/|\alpha_2|e^{\zeta z}]^{1/4}$ 为临界扰频及增量.可见,$\Omega_{e2}+\Delta\Omega_{e2}>\Omega+\Delta\Omega$ 时 MI 才发生.由式(21)可知

$$(\Omega+\Delta\Omega)_{\max}\approx(\Omega_{e2}+\Delta\Omega_{e2})/2^{3/4} \tag{22}$$

时,增益谱最大值为

$$(g+\Delta g)_{\max 2}\approx 2\mu p_0+2(\Delta\mu p_0+\mu\Delta p_0) \tag{23}$$

由式(18)~(23)可知,零色散和远离零色散附近增益谱相等,与 Ω_e 相比,$(\Omega+\Delta\Omega)_{\max}$ 由 $(\Omega_{e1}+\Delta\Omega_{e1})/\sqrt{2}$ 变为 $(\Omega_{e2}+\Delta\Omega_{e2})/2^{3/4}$,即零散射附近,四阶色散使扰动频率最大值向临界扰动频率靠近效果由耦合光决定.

为考察零散射附近损耗对 MI 影响,由式(21),得常规等离子体($\zeta\neq0$)与理想等离子体($\zeta=0$)的 MI 增益谱 ΔG_1 和 ΔG_2 之比为

$$\Delta G_1/\Delta G_2\approx\exp(-\mu z/4) \tag{24}$$

因 μ 和 z 恒大于 0 及散射使等离子体辐射阻尼增大[17],故 $\Delta G_2\leqslant\Delta G_1$ 恒成立,且导致等离子体损耗增大,散射有效地减小了产生 MI 频率范围.

2 数值计算

取 ζ、p_0、μ、α_2、α_4 及其增量、传输距离分别为 $\zeta=0.19\ \mathrm{km^{-1}}$ 和 $\Delta\zeta=0.19\ \mathrm{km^{-1}}$、$p_0=4.9$ W 和 $\Delta p_0=0.01$ W、$\mu=3/\mathrm{W\cdot km}$ 和 $\Delta\mu=0.5/\mathrm{W\cdot km}$、$\alpha_2=0.09\ \mathrm{ps^2/km}$和 $\Delta\alpha_2=0.01\ \mathrm{ps^2/km}$、$\alpha_4=6\times10^{-4}\ \mathrm{ps^4/km}$ 和 $\Delta\alpha_4=10^{-4}\ \mathrm{ps^4/km}$、2 km. $|\alpha_4+\Delta\alpha_4|=0$ 时,远离零色散附近MI增益谱如图1.由图1知,当 $|\alpha_2+\Delta\alpha_2|$ 较大时,增益谱由耦合光二阶色散定,随 $|\alpha_2+\Delta\alpha_2|$ 增大,$|\alpha_4+\Delta\alpha_4|$ 作用迅速较小. $|\alpha_2+\Delta\alpha_2|\to 1\ \mathrm{ps^2/km}$时曲线重合,四阶色散无作用,这是因散射有效削弱了四阶效应的缘故.

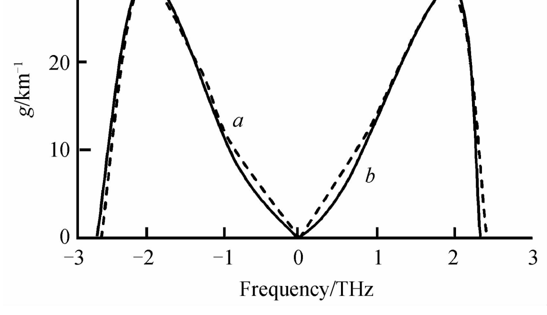

图 1 零色散时调制不稳定性增益谱曲线

Fig. 1 Gain spectra of modulation instability in the region of minimum dispersion

当 $\alpha_2=9\times10^{-4}\ \mathrm{ps^2/km}$ 和 $\Delta\alpha_2=10^{-4}\ \mathrm{ps^2\cdot km^{-1}}$,即脉冲在零色散附近,其它条件不变时,远离零色散附近 MI 增益谱如图 2.由图 2 知,MI 增益谱由耦合光的四阶色散确定,二阶色散被湮灭,$(\Omega+\Delta\Omega)_{\max}$ 较快向 Ω_e 靠近.这是因散射使等离子体非线性迅速增强,导致四阶色散迅速增强了的缘故.

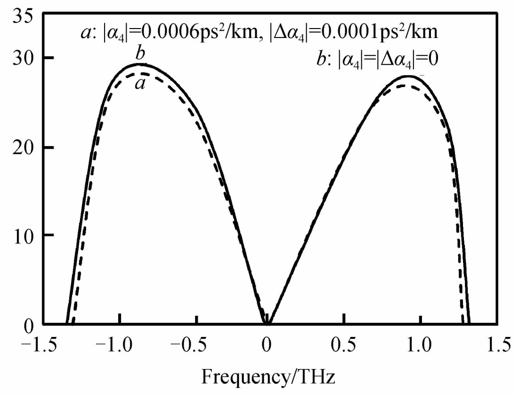

图 2 远离零色散时调制不稳定性增益谱曲线

Fig. 2 Gain spectra of modulation instability in the region far from the point of minimum dispersion

当 $\alpha_2=\Delta\alpha_2=0$,损耗及增量 $\zeta=0.19$/km 和 $\Delta\zeta=0.01$/km 及 $\zeta=0$ 时,Ω_e 随 p_0 变化如图 3.由图 3 知,随

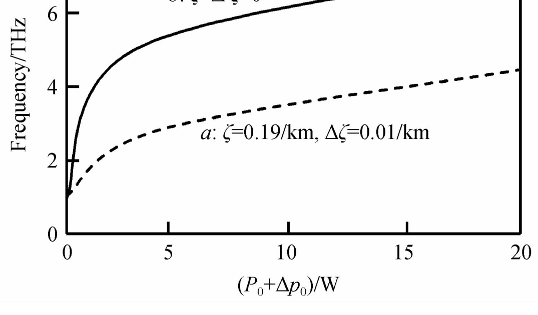

图 3 临界微扰频率随初始峰值功率及其增量变化曲线

Fig. 3 Critical modulation frequency as a function of the incident power and its increased number

p_0 增大，损耗对谱宽影响迅速增大，这是因功率增大使散射增强，导致带电粒子辐射阻尼效应增强的缘故.

3 结论

本文基于多光子非线性 Compton 散射模型，研究了 Compton 散射对等离子体零色散附近调制不稳定性的影响，给出了耦合脉冲作用下色散的理论公式，数值计算发现：远离零色散附近和零色散附近的 MI 增益谱均与散射光密切相关，当二阶色散较大时，远离零色散附近 MI 增益谱由入射光和散射光共同决定. 随二阶色散增大，四阶色散作用迅速减小，这是因散射有效地削弱了四阶色散效应的缘故. 零色散附近 MI 增益谱由耦合光得四阶色散确定，二阶色散被湮灭，微扰频率最大值较快向临界微扰频率靠近. 这是因散射有效地增强了四阶效应的缘故. 随等离子体损耗增大，MI 增益谱宽迅速增大. 这是因功率增大使散射增强，导致带电粒子辐射阻尼效应增强的缘故. 由此，提出了散射是形成等离子体零色散附近 MI 新机制. 这对于人们进一步了解色散产生的微观机制及粒子与场作用的本质应具有一定的参考价值.

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