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基于两光子宇称门接近确定性的制备偏振 Cluster 态

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摘 要:基于弱的交叉克尔效应和线性光学元件,利用光子偏振宇称门呈现了一个关于制备 Cluster 态的方案.本方案引入了交叉克尔非线性介质,采用了对相干探测模的非破坏测量,有效实现了光子与光子间的相互作用,进而获得所期望的光子纠缠态,同时可以提高制备的成功概率和制备态的保真度.通过本方案可以制备接近完美保真度的四光子偏振 Cluster 态,并具有接近 1 的成功概率.在本方案的基础上通过利用线性光学元件还可以获得 $3N+1$ 个光子的偏振 Cluster 态.

关键词:交叉克尔非线性;量子非破坏测量;Cluster 态
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Near-deterministic Preparation for Four-qubit Polarization Cluster State Based on Two-photon Parity Gate

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Abstract: A scheme for generating Cluster state with the quantum non-demolition polarization parity gate was presented based on weak cross-Kerr nonlinearity and linear-optics elements. The nonlinearity of cross-Kerr medium was introduced in the present scheme in order to achieve effectively the interaction between photons and achieve the desired photon entangled state. The success probability and the perfect fidelity for generating a cluster state could be enhanced due to introducing the nonlinearity of cross-Kerr and quantum non-demolition measurement on the coherence probe modes. The distinct feature of the present scheme is its preparation for a polarization four-photon cluster state with near unity success probability and near perfect fidelity. Furthermore, the cluster state of $3N+1$ photons could be obtained with the help of linear optical elements based on the present scheme.

Key words: Cross-Kerr nonlinearity; Quantum non-demolition measurement; Cluster state

0 Introduction

Entanglement in bipartite quantum systems is well understood and can be easily quantified. In contrast, multipartite quantum systems offer a much richer structure and various types of entanglement, and may play an important role in quantum computation and communication network[1-3]. There is only one class of entanglement for two qubits, there are two classes of genuine entangled states for three qubits, and there are at least nine different classes of entanglement for four qubits[4]. Multipartite entanglement is at the core of studies probing the foundations of quantum physics and represents a key component in a wide range of quantum information processing (QIP) tasks[5]. Much

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attention has been devoted to a class of multipartite entangled states, so-called cluster states, due to the application of cluster states as a fundamental resource aimed at the one-way quantum computation by Briegel and Raussendorf[6] and at the realization of important quantum information tasks, such as quantum error correction and quantum communication protocols[7]. As a universal resource for quantum computation and quantum information, they are proven to be more immune to decoherence than GHZ states[8]. The proof of Bell's theorem without the inequalities has already been given for cluster states[9]. It has been shown[6] that one key advantage of using cluster states is that the quantum gates are implemented with unit probability, rather than the "asymptotically unit" probability of the original Knill, Laflamme and Milburn's[10] scheme. Because of these advantages, cluster states have attracted much attention recently and various schemes for generating them via different physical systems have been proposed.

In the optical system, we propose a simple and efficient scheme for the preparation of a four-qubit polarization cluster state with the two-photon parity gate, which is the analogue of the one in Ref. [11], based on cross-Kerr medium (CKM). The distinct feature of the present scheme is its preparation with near unity success probability on the condition that four single-photons are generated in the four corresponding input modes and with near perfect fidelity due to quantum non-demolition (QND) measurement on the coherence probe modes.

1 Preparation of the entangled states

Before we begin our detailed discussion on the preparation scheme, let briefly review the useful weak cross-Kerr nonlinearity which has been used in Refs. [11- 12]. Suppose a nonlinear weak cross-Kerr interaction between a signal mode initially in a superposition of photon-number states $|\varphi\rangle_s = c_0|0\rangle_s + c_1|1\rangle_s$ and a probe mode initially in a coherent state $|\alpha\rangle_p$. The cross-Kerr interaction causes the combined signal-probe system to evolve as

$$|\varphi\rangle_s \otimes |\alpha\rangle_p \rightarrow c_0|0\rangle_s|\alpha\rangle_p + c_1|1\rangle_s|\alpha e^{i\theta}\rangle_p \quad (1)$$

where θ is induced by the nonlinearity. Conditioned on the results of QND measurement[11], the signal state will be projected into a definite number state or superposition of number states with high fidelity.

Now, let us move our attention to preparing four-qubit polarization Cluster state. Schematic diagram of the preparation is illustrated in Fig. 1, with the help of three two-qubit polarization parity gates based on weak cross-Kerr nonlinearity. Four

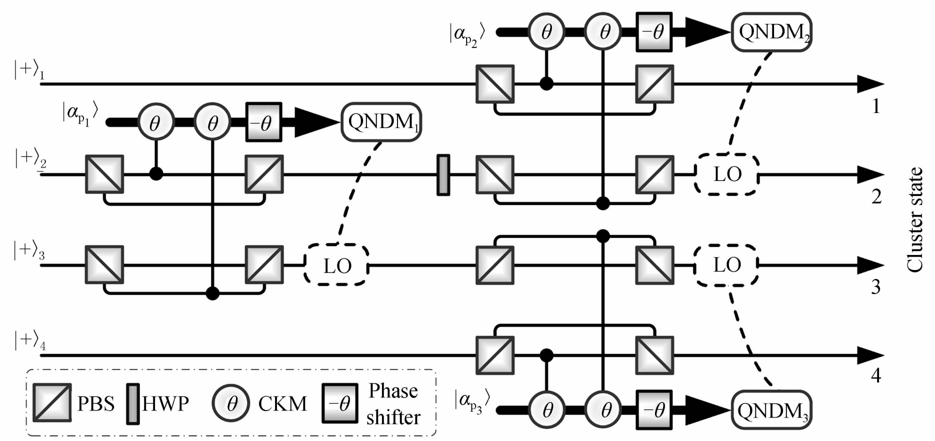

Fig. 1 Schematic setup for the state preparation

input modes are prepared in the single polarization state of the form as $|+\rangle_i \equiv (|H\rangle_i + |V\rangle_i)/\sqrt{2}$ with $i=1, 2, 3, 4$. Here H and V denote, respectively, horizontal and vertical linear polarizations. The parity gate, which is the analogue of Nemoto *et al.*'s[11] and Munro *et al.*'s[13] ones, is the crucial ingredient for the present scheme. Thus, it is necessary to illustrate the principle of work about the parity gates. Take the left-hand side parity gate as depicted in Fig. 1 for an example; expound the implementation of entanglement between two signal modes. The photons of input modes 2 and 3 are split individually on the PBSs into two spatial modes which transmit $|H\rangle$ and reflect $|V\rangle$. Corresponding spatial modes then interact with the probe beam in the Kerr nonlinear medium. The action of the first parity gate on the left-hand side of Fig. 1 evolves the initially combined system $|+\rangle_2 \otimes |+\rangle_3 \otimes |\alpha_{p_1}\rangle$ to

$$[(|H\rangle_2|H\rangle_3 + |V\rangle_2|V\rangle_3)|\alpha_{p_1}\rangle + |H\rangle_2|V\rangle_3|\alpha_{p_1}e^{i\theta}\rangle +$$

$$|V\rangle_2|H\rangle_3|\alpha_{p_1}e^{-i\theta}\rangle]/2 \tag{2}$$

The probe measurement will produce an unconditional displacement $D(-\alpha_{p_1})$ [14] on the probe beam p_1 followed by a photon number QND measurement. After the displacement the combined state as described by Eq. (2) is

$$[(|H\rangle_2|H\rangle_3+|V\rangle_2|V\rangle_3)|0\rangle+e^{-i\alpha_{p_1}^2\sin\theta}|H\rangle_2\cdot |V\rangle_3|\alpha_{p_1}(e^{i\theta}-1)\rangle+e^{i\alpha_{p_1}^2\sin\theta}|V\rangle_2|H\rangle_3|\alpha_{p_1}\cdot (e^{-i\theta}-1)\rangle]/2 \tag{3}$$

Now a photon number QND measurement will either pick out the vacuum state or project to two amplitudes $|\alpha_{p_1}(e^{\pm i\theta}-1)\rangle$ without distinguishing them. These unexpected phase shifts, *i.e.* $e^{-i\alpha_{p_1}^2\sin\theta}$ and $e^{i\alpha_{p_1}^2\sin\theta}$, can be eliminated through simple local rotations using phase-shifter while no feed-forward is required. The overlap between the probe components of the even parity ($|0\rangle$) and odd parity ($|\alpha_{p_1}(e^{\pm i\theta}-1)\rangle$) amplitudes is very small if $\alpha_{p_1}\theta\sim\pi$. In this case the mean photon number of the odd parity components is not large, being approximately $\bar{n}_{p_1}^{odd}\sim 10$. Hence a measurement of n_{p_1} with a QND photon number resolving detector cannot discriminate the $|\alpha_{p_1}(e^{\pm i\theta}-1)\rangle$ components from each other, but can distinguish these from the $|0\rangle$ components. For a projection on the number basis $|n\rangle$, the state of modes 2 and 3 is

$$|\psi\rangle_{n_{p_1}=0}=(|H\rangle_2|H\rangle_3+|V\rangle_2|V\rangle_3)/\sqrt{2} \tag{4}$$

$$|\psi\rangle_{n_{p_1}>0}=(e^{i\phi(n_{p_1})}|H\rangle_2|V\rangle_3+e^{-i\phi(n_{p_1})}\cdot |V\rangle_2|H\rangle_3)/\sqrt{2} \tag{5}$$

with an equal probability of 1/2 as long as the coherent amplitudes $|\alpha_{p_1}(e^{\pm i\theta}-1)\rangle$ do not contribute significantly to the vacuum, where $\phi(n_{p_1})=n_{p_1}\arctan[\cot(\theta/2)]$. The parity gate can allow the states $|H\rangle|H\rangle$ and $|V\rangle|V\rangle$ to be distinguished effectively from $|H\rangle|V\rangle$ and $|V\rangle\cdot|H\rangle$. Above we have chosen to call the even parity state $\{|H\rangle|H\rangle, |V\rangle|V\rangle\}$ and the odd parity states $\{|H\rangle|V\rangle, |V\rangle|H\rangle\}$. Due to the overlap of these coherent states $|\alpha_{p_1}(e^{\pm i\theta}-1)\rangle$ with the vacuum $|0\rangle$, an error probability of $P_{err}=\exp(-|\alpha_{p_1}\theta|^2)$ which can be made quite small with a suitable choice of α_{p_1} and θ[15]. For example with small θ(*i.e.* $\theta\sim 10^{-5}$) we can choose $\alpha_{p_1}\theta=\pi$ which leads to an error probability as low as $P_{err}=0.53\times 10^{-4}$. This phase shift $\phi(n_{p_1})$, which dependents on the result of the measurement, in Eq. (5) can then simply be eliminated and a simple bit flip on the qubit of mode 3 is needed via a classical information feed-forward operation to transform this state to one in Eq. (4). Thus the entangled state, as described in Eq. (4), can be obtained near deterministically through the two-qubit polarization parity gate on the left-hand side of Fig. 1. It is still possible to operate in the regime of weak cross-Kerr nonlinearities. For example, the nonlinearity[16] might be largely improved, say $\theta\sim 10^{-2}$, with electromagnetically induced transparency (EIT). And the amplitude of the probe coherent beam is physically reasonable with current technology.

The photons of modes 2 and 3 through the left-hand side parity gate can be evolved near-deterministically into the state in Eq. (4) as mentioned above. Then the HWP takes a $-45°$ rotation of polarization for the photon of mode 2, *i.e.*, it changes $|H\rangle$ into $(|H\rangle+|V\rangle)/\sqrt{2}$ and $|V\rangle$ into $(|H\rangle-|V\rangle)/\sqrt{2}$[17]. So the state in Eq. (4) will be evolved into

$$(|H\rangle_2|H\rangle_3+|H\rangle_2|V\rangle_3+|V\rangle_2|H\rangle_3-|V\rangle_2|V\rangle_3)/2 \tag{6}$$

The action of either of the right-hand side parity gates is similar to one on the left-hand side of Fig. 1 as analyzed above.

Subsequently, the action of the top right corner parity gate evolves near deterministically the system of modes 1, 2, and 3 to

$$(|H\rangle_1|H\rangle_2|H\rangle_3+|H\rangle_1|H\rangle_2|V\rangle_3+|V\rangle_1|V\rangle_2|H\rangle_3-|V\rangle_1|V\rangle_2|V\rangle_3)/2 \tag{7}$$

In particular, for the QNDM result of $n_{p_2}>0$, in order to obtain the state as described in Eq. (7), we need the operations of eliminating the unwanted phase factors that have arisen and a simple bit flip on the photon of mode 1 via classical feed-forward process.

The action of the lower right corner parity gate evolves near deterministically the system of modes 1, 2, 3, and 4 to

$$(|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4+|H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4+|V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4-|V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4)/2 \tag{8}$$

Similarly, for the QNDM result of $n_{p_3}>0$, in order to obtain the state as described in Eq. (8), we need the operations of eliminating the unwanted phase factors and a simple bit flip on the photon of mode 4 via classical feed-forward process.

Apparently, Eq. (8) describes a four-photon polarization cluster state. The polarization cluster state can be obtained with near perfect fidelity due to quantum non-demolition measurement on the coherence probe modes and with high success probability close to unit on the condition that four single-photons are generated in the four corresponding input mode. If the result of QNDM is $n_{p_i}>0$ ($i=1$, 2, or 3) in the preparation scheme, classical information feed-forward process would be needed to eliminate this unwanted phase factor and to implement a single qubit flip. Of course, the single-qubit operation on the photon mode can be implemented rather easy with high precision. However, in order to remove the unwanted phase factor, we are sure that the photon number QND measurement of the parity gate must be accurate enough, *i. e.*, n_{p_i} and θ is known through QNDM. Otherwise this unwanted phase factor cannot be eliminated.

Finally, it is noteworthy that, if N four-photon polarization cluster states in Eq. (8) have been prepared, we can generate the cluster state of $3N+1$ photons by using linear optical elements. The procedure is given in Ref. [18].

2 Discussion

We have presented an efficient scheme for the preparation of the four-photon polarization cluster state with high success probability close to unit on the condition that four single-photons are generated in the four corresponding input mode. The core of the present scheme are two-qubit polarization parity gates, which are first developed by Nemoto *et al.*, based on weak cross-Kerr nonlinearity that allow us to prepare near deterministically the entangled states which are desired. Such nonlinearities are potentially available today using doped optical fibers, cavity quantum electrodynamics systems, with electromagnetically induced transparencies which are capable of producing much larger nonlinearities.

3 Conclusion

This scheme has the following distinct advantages. 1) It is possible to preparing the cluster state with the near unity success probability but as a cost of more feed-forward operations. The two-qubit parity gate is acted as a fermonic PBS; that is, it is robust against the photon bunching effects, so that the success probability can be enhanced to near unit by applying a feed-forward technique. Considering that it is difficult to obtaining deterministic single-photon sources for current experimental technology, the realistic success probability of the present preparation scheme is rather low but is preponderant in contrast to the existing schemes with the linear optical system and with the cross-Kerr nonlinearity. 2) The near perfect fidelity of the preparation scheme profits from the QND measurement, which can be made much more efficient than the single-photon detection, on the coherence probe modes. Photon loss for the signal modes can be treated efficiently through "indirect measurements". 3) It is not necessary in the present scheme that the cross-Kerr nonlinearity is very large because the weak nonlinearity can be compensated by using a probe coherent state field with very large amplitude. Therefore, this scheme uses only the basic tools in quantum optical laboratories and can be implemented in the regime of weak cross-Kerr nonlinearity. These make us more confident in the feasibility of the proposed scheme.

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