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基于非线性薛定谔方程的一种孤子特性的分析

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摘 要:光孤子通信是解决光信息在光纤中长距离传输时衰减和色散问题的一种较为有效的方法.本文在现有带有群速度色散、非线性项、三阶非线性系数以及增益/损耗项的非线性薛定谔方程孤子解的基础上,给出了灵活性的孤子解.采用具有复振幅的行波解作为试探解,将试探解代入原方程,在实部和虚部分离的基础上,引入三个变量函数,最后表征出孤子解波函数的平方,并应用 Matlab 选择不同的变量函数进行数值模拟,得到图示结果.结果表明孤子解对于参变量变化是敏感的.选择适当的参量,得到合适的孤子,这一结论对光纤中孤子通信具有重要意义.

关键词:信息光学;色散;光孤子;非线性薛定谔方程
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Characteristics for the Soliton Based on Nonlinear Schrödinger Equation

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Abstract: For the problems of the attenuation and the dispersion in long distance fiber communication of optical information, the optical soliton communication is an effective method. The flexible soliton solution has been obtained from the solution of the nonlinear Schrödinger equation with the group-velocity dispersion, linear potential, three-order nonlinearity term, and the gain/loss term. A trial solution that was a travelling wave solution with complex amplitude was used and it was replaced into the nonlinear Schrödinger equation. The three new variables were introduced after the nonlinear Schrödinger equation was separated into imaginary part and real part. Finally the square of wave-function of soliton solution and the consequence figures was got with Matlab. The results show that the soliton solution is sensible for various parameters. The appropriate soliton was obtained by reasonable parameters choice and it made the foundation for the soliton communication in the optical fiber.

Key words: Information optics; Dispersion; Optical soliton; Nonlinear Schrödinger equation

0 Introduction

The soliton was put forward for the first time in 1844[1], which can be found in many fields, such as ocean science[2], optical fiber communication[3-4], condensed matter physics and so on. Because the nonlinearity of solution, many questions were resolved in the process of transmission such as signal attenuation and distortion. Optical soliton had almost no energy loss in the process of transmission and have a strong anti-jamming property. A series of systematic method for researching the nonlinear equation was found except the inverse scattering method[4] and inverse method[5-6], such as Adomian decomposition method[7] and the fractional Fourier integral method[8]. It also has been found that many nonlinear partial differential equations had the soliton solution, through computational experiment was combined with ordinary solving process. Through above the method the solution was got and these solutions

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with a fixed form of the soliton solution have some limitations for studying of the characteristics of soliton. In the work, the property of the flexible soliton solution was focused and the foundation was made for soliton communication in the future.

1 The solution of nonlinear Schrödinger equation

The nonlinear Schrödinger equation (NLS) was a typical dispersive nonlinear partial differential equation that was distinguished from Schrödinger equation for being inserted nonlinear term that was in proportion to wave function, so it was commonly known as nonlinear wave envelope function.

It described the spatio-temporal evolution of the complex field $\psi=\psi(x,t)\in C$ and that it has the dimensionless form[7]

$$i\partial_t\psi+\partial_x^2\psi+m|\psi|^2\psi=0 \quad (x\in R, t>0) \tag{1}$$

There was the parameter $m\in R$ that corresponded to focusing ($m>0$) or defocusing ($m<0$) effect of nonlinearity. In this paper, the nonlinear Schrödinger equation was expanded with group-velocity dispersion $\beta(t)$, linear potential $v(x,t)$, nonlinearity $g_{p-1}(t)$ and the gain/loss term $\gamma(t)$, moreover $m=1$, in the form[2, 9-10]

$$i\partial_t\psi+\frac{\beta(t)}{2}\partial_x^2\psi+v(x,t)\psi+g_{p-1}(t)|\psi|^{p-1}\psi=i\gamma(t)\psi \tag{2}$$

Nowadays many physical phenomena were found for the different linear potential $v(x,t)$ and nonlinearity coefficient $g_{p-1}(t)$, which illuminated my idea that was analyzed the soliton solution of the extended nonlinear Schrödinger equation and obtained the influence by the different parameters to research the features of the soliton.

For the sake of simplicity, the situation of $p=3$ was discussed and got

$$i\partial_t\psi+\frac{\beta(t)}{2}\partial_x^2\psi+v(x,t)\psi+g_2(t)|\psi|^2\psi=i\gamma(t)\psi \tag{3}$$

We employ the envelope field in the gauge form[11]

$$\psi(x,t)=[R(x,t)+iI(x,t)]e^{i\theta(x,t)} \tag{4}$$

First, in order to get the specific form, the envelope field (4) was integrated to spatial and temporal variables and then the consequence into Eq. (3) yields a plural function. If it was zero then the imaginary part and real part were zero.

Second, $R(x,t)$, $I(x,t)$, $\theta(x,t)$ were described by using the new variables that were $\mu(x,t)$, $\varphi(t)$. Now we must define $R_A(t)$, $R_B(t)$, $I_C(t)$, $\varphi(t)$ that only were the function with t and the $R_M(\mu(x,t),\varphi(t))$, $\theta_\chi(x,t)$, $I_N(\mu(x,t),\varphi(t))$ that were the function with x,t to describe it, then we will get (the variables were left out below the calculating).

$$R=R_A+R_BR_M, I=I_CI_N, \theta=\theta_\chi+C\varphi \tag{5}$$

There was C that was a constant.

The Eq. (5) was substituted into the plural function and the equivalent solutions were obtained.

$$\frac{\partial^2\mu}{\partial x^2}=0 \tag{6a}$$

$$\frac{\partial\mu}{\partial t}+\beta(t)\frac{\partial\theta_\chi}{\partial x}\cdot\frac{\partial\mu}{\partial x}=0 \tag{6b}$$

$$2\frac{\partial\theta_\chi}{\partial t}+\beta(t)(\frac{\partial\theta_\chi}{\partial x})^2-2\nu(x,t)=0 \tag{6c}$$

$$2\frac{\partial R_\sigma}{\partial t}+[\beta(t)\frac{\partial^2\theta_\chi}{\partial x^2}-2\gamma(t)]R_\sigma=0 \quad (\sigma=A,B,C) \tag{6d}$$

$$\frac{\partial\varphi}{\partial t}R_B\frac{\partial R_M}{\partial\varphi}+\frac{\beta(t)}{2}(\frac{\partial\mu}{\partial x})^2I_C\frac{\partial I_N}{\partial\mu^2}-C\frac{\partial\varphi}{\partial t}I_CI_N+g_2(t)I_CI_N[I_C^2I_N^2+(R_A+R_BR_M)^2]=0 \tag{6e}$$

$$-\frac{\partial\varphi}{\partial t}I_C\frac{\partial I_N}{\partial\varphi}+\frac{\beta(t)}{2}(\frac{\partial\mu}{\partial x})^2R_B\frac{\partial R_M}{\partial\mu^2}-C\frac{\partial\varphi}{\partial t}(R_A+R_BR_M)+g_2(t)(R_A+R_BR_M)\cdot[I_C^2I_N^2+(R_A+R_BR_M)^2]=0 \tag{6f}$$

According to the eight equations, the new variables were defined to obtain functions that $R(x,t)$, $I(x,t)$, $\theta(x,t)$ and the square of wave-function with variables. Then the new variables were reasonably transformed so that numerical solution was obtained with Matlab. In order to realize it, the functions $n(t)$, $q(t)$ were defined.

1) Through ordering the $\mu(x,t)=n(t)x+q(t)$ and associating with Eq. (6b), then we got

$$\theta_\chi=\frac{1}{\beta n}[-\frac{\partial q}{\partial t}x-\frac{1}{2}\frac{\partial n}{\partial t}x^2]+c_1 \tag{7}$$

where the symbol c_1 was a constant what the integration got. For the convenient, we ordered that $c_1=0$.

2) Through Eq. (6c), we obtained

$$\nu(x,t)=\frac{\partial\theta_\chi}{\partial t}+\frac{\beta(t)}{2}(\frac{\partial\theta_\chi}{\partial x})^2 \tag{8}$$

3) Associating Eq. (7) and Eq. (6d), the consequence was got by the method of separation of variables.

$$2\frac{dR_A}{R_A}=\frac{dn}{n}+2\gamma(t)dt, \; R_A=A_0\sqrt{|n|}\cdot\exp(\int_0^t\gamma(p)dp) \tag{9}$$

4) The Eq. (6e) and the Eq. (6f) were partial differential equation with constant coefficients. If we want to get it, the two equations must be defined.

$$\frac{\partial\varphi}{\partial t}=\frac{\beta(t)}{2}\cdot(\frac{\partial\mu}{\partial x})^2, \; g_2(t)=\frac{\beta(t)}{2}\cdot\frac{G}{R_A^2}(\frac{\partial\mu}{\partial x}) \tag{10}$$

where G is a constant and for convenient we ordered that $G=1$, $C=1$. It also had $R_B=bR_A$, $I_C=cR_A$. Then through Eq. (6e) and Eq. (6f) got

$$R_M=\frac{-4}{b(1+2\mu^2+4\varphi^2)},I_N=\frac{-8\varphi}{c(1+2\mu^2+4\varphi^2)} \quad (11)$$

Through integrating Eq. (10), it was got

$$\varphi(t)=\frac{1}{2}\int_0^t n^2(p)\beta(p)\mathrm{d}p,g_2(t)=\frac{n(t)\beta(t)}{2R_M^2|n|} \quad (12)$$

5) The function $\gamma(t)$ was extended item of nonlinear Schrödinger equation and defined it (c_2 is a constant) as

$$\gamma(t)=c_2\tan(t)\mathrm{sech}(t) \quad (13)$$

Finally the consequence was got

$$|\psi(x,t)|^2=\frac{A_0^2|n(t)|\exp(2\int_0^t\gamma(p)\mathrm{d}p)}{[1+2(n(t)x+q(t))^2+4\varphi^2(t)]^2}\cdot$$

$$\{[2(n(t)x+q(t))^2+4\varphi^2(t)-3]^2+64\varphi^2(t)\} \quad (14)$$

where A_0 was the totality of constant in exponential function and we ordered it equaled to one.

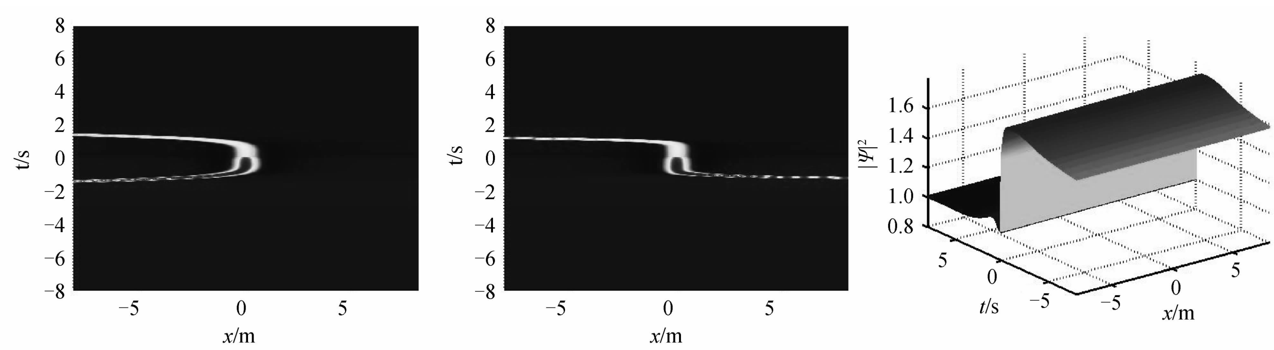
2 Discussion

Changed $n(t)$, $q(t)$, $\varphi(t)$ of the square of wave-function and numerical solution was obtained by Matlab[12]. Through the influence the square of wave-function was observed, the influence of the soliton solution can be boldly guessed.

1) First, ordering function $n(t)$ was a constant, function $\beta(t)$ was polynomial.

(i) The function is $q(t)=t^m$. As we can see from Fig. 1 (a) and (b). it only showed that the $x-t$ plane of the square of wave-function, when the parameter $m=2k+1(k=0,1,2\cdots)$, the picture were similar to the centrosymmetric graph at point (0,0), while when $m=2k(k=1,2\cdots)$, it was similar to axisymmetric graph, in addition, the graph was discrete following the increase of value.

(ii) The function is $q(t)=c$ (The symbol c was the constant). It showed that the graph was on the center of the $x-t$ plane, when the constant was become smaller, until $q(t)=0.1$, the graph was shifted along the x axis, but the shape did not change. When the $q(t)=9$, it was shifted out. Following the increase of the constant, the stabilized graph was got, like Fig. 1(c).

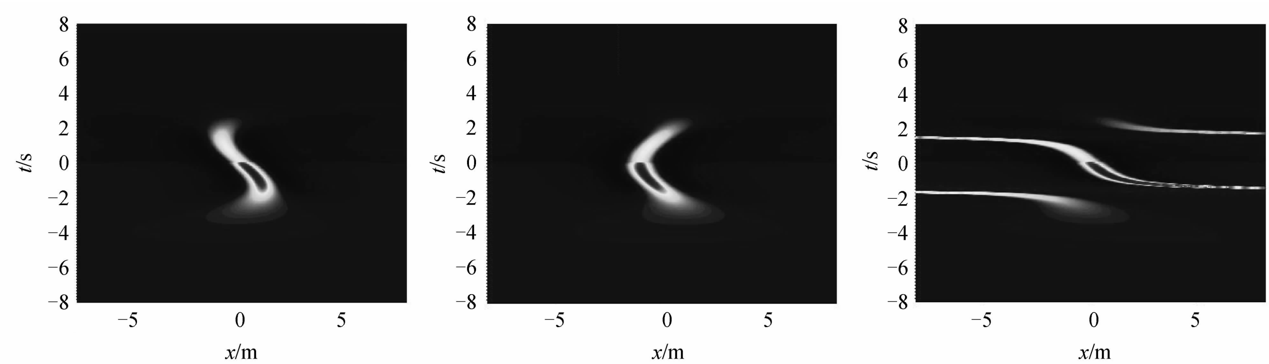


(a) $q(t)=t^{10}$ (b) $q(t)=t^{11}$ (c) $q(t)=10^{10}$

Fig. 1 Those pictures were under the condition that was $n(t)=0.1$, $\beta(t)=0.5t^2$ with different $q(t)$

(iii) The function is $q(t)=\exp(c+t+t^2+\cdots)$. We also got the graphs that were stabilized along with the increase of constant and the shape of graph was the same as Fig. 1 (c). Furthermore, the square of wave-function was nearly unvarying.

(iv) The function $q(t)$ was trigonometric function When $q(t)$ was trigonometric function, it will find that the graph of $x-t$ plane was similar to the graph of $q(t)$ function, like Fig. 2; furthermore, the influence to the square of wave-function was so small.



(a) $q(t)=\sin t$ (b) $q(t)=\cos t$ (c) $q(t)=\tan t$

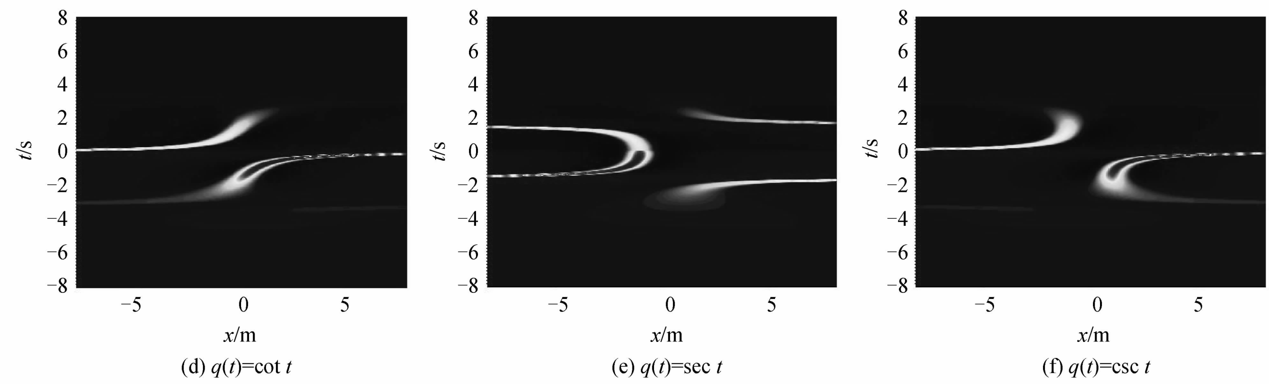


Fig. 2　Those pictures were under the condition that was $n(t)=0.1$, $\beta(t)=0.5t^2$ with different $q(t)$

As we all known the consequence was caused by the function $q(t)$ and what was induced by $n(t)$ and $\beta(t)$, so we will change $n(t)$ and $\beta(t)$ to obtain the conclusion.

2) Second, the $q(t)=t$, the $\beta(t)=0.5t^2$ and the $n(t)$ was changed.

(i) The function $n(t)$ was polynomial. It was found that the graph of $t-z$ plane was related to the graph of $|n(t)|$ function from the Fig. 3 when the graph verged to stabilization, and it can get stable graph that identical to Fig. 1 (c) with increasing of constant, moreover, when the constant increased the square of wave-function was accordingly increased, which proved the stability of soliton. Meanwhile, it also proved that the graph of $x-t$ plane was similar to the graph of $q(t)$, like Fig. 1 (a) and (b).

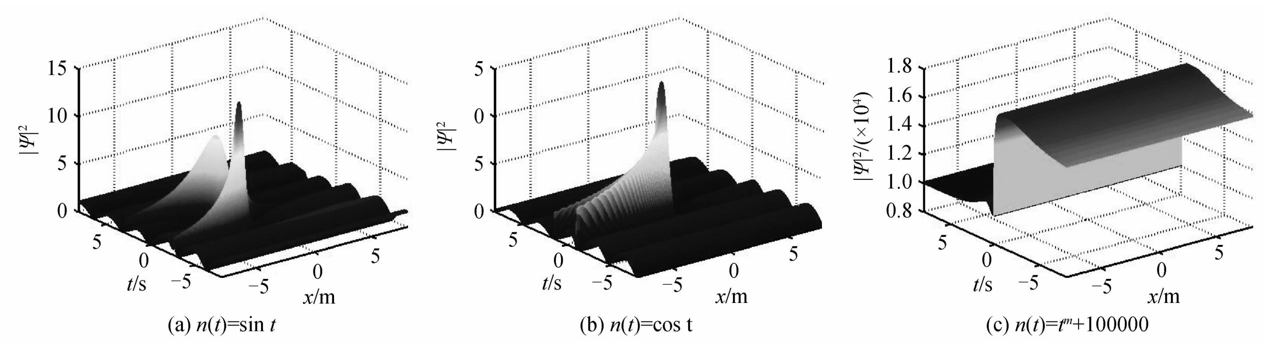


Fig. 3　Those pictures were under the condition that was $q(t)=t$, $\beta(t)=0.5t^2$ with different $n(t)$

(ii) The symbol $n(t)$ was exponential function and trigonometric

3) When functions $n(t)$, $q(t)$ were invariant, the dark soliton was discussed, so ordered the $\beta(t)>0$ and it found that the influence was small with change function $\beta(t)$, just as the literatures[3, 6-7], the value of function $\beta(t)$ was 1.

3　Conclusion

In the new-style nonlinear Schrödinger equations, the method that was appeared in this work was used to get the flexible soliton solution. This method is superior to the Adomian decomposition method [7] and the fractional Fourier integral method. There are three variable to describe the soliton solution. It has been found that the function $q(t)$ was transformed for square of wave-function hardly any influence, but change of the $q(t)$ play an important role in $x-t$ plane graph. The square of wave-function was changed when the function $n(t)$ was altered and the $t\rightarrow z$ plane graph was similar to the graph of $|n(t)|$ function. Meanwhile, as far as some condition of transformed the patterning close to stabilization, the soliton was put to good use in far distance communication. It described the phenomenon of the soliton can provide more precious information for researching nonlinear system in the future.

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·下期预告·

基于运动剧烈程度的无参考视频质量评价模型

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摘　要:网络视频质量评估具有无参考性、实时性、网络传输状态依赖性和主观视觉性等需求.本文以网络丢包引发的视频失真为研究重点,针对不同解码类型帧上的网络丢包引发的视频失真持续效应不同和网络丢包引发的人眼视觉感受与丢包所在帧视频内容的运动剧烈程度有着显著的关联等现象,提出了一种基于运动剧烈程度的无参考视频质量评价模型.该模型无需原始参考视频序列亦无需视频解码,对客户端接收到的码流分析其由丢包引起的视频损伤,标记受损宏块,建立受损宏块的失真持续效应和运动剧烈程度与视频质量间的关联,并完成视频质量评估.实验结果表明,该方法计算量小,实时性高,与主观评价结果一致性程度较高.

关键词:视频质量评价;无参考质量评价;丢包;宏块;运动剧烈程度