第 42 卷第 12 期
2013 年 12 月

光　子　学　报
ACTA PHOTONICA SINICA

Vol. 42 No. 12
December 2013

doi:10.3788/gzxb20134212.1387

三阶非线性色散异向介质中的调制不稳定性

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摘　要:基于异向介质中包一到三阶非线性色散的非归一化扩展非线性传输方程,利用德鲁德电磁模型和线性稳定性分析法,导出了调制不稳定的色散关系和增益谱.以实际单位形式计算和讨论了传输方程系数、正负折射区及自聚焦和自散焦情况下调制不稳定增益谱随归一化角频率的变化关系.结果表明:随归一化频率增加,负折射区在自聚焦时的增益谱谱宽和谱峰都先增加再减为零;负折射区在自散焦时调制不稳定只出现在禁带附近,且谱宽和谱峰都单调减小.正折射区调制不稳定只出现在自聚焦情形,且谱宽一直增加,而谱峰先减小后增加.总体而言,三阶非线性色散的存在不利于调制不稳定的产生.

关键词:调制不稳定性;三阶非线性色散;异向介质;负折射
中图分类号:O437　　**文献标识码**:A　　**文章编号**:1004-4213(2013)12-1387-5

Modulation Instability in Metamaterials with Third-order Nonlinear Dispersion

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Abstract: On the basis of the extended nonlinear propagation equation without normalization in metamaterials with the first-order to third-order nonlinear dispersion effects, the dispersion relation and gain spectra of modulation instability were deduced by adopting the linear stability analysis and the Drude electromagnetic model. Variations of the equation coefficients and gain spectra with the normalized angular frequencies were calculated and discussed in real units in both the positive and negative refractive-index regions in cases of self-focusing and self-defocusing. The results show that, the third-order nonlinear dispersion coefficients are all along positive in both the positive and negative refractive-index regions. In the negative refractive-index region, with increase of normalized angular frequencies, the spectral widths of the gain spectra increase before become zero and the peak gains increase before decrease and then become zero in the self-focusing case. While in the case of self-defocusing, modulation instability can only occur near the forbidden band and both the spectral widths and peak gains decrease monotonically. In the positive refractive-index region, modulation instability can only occur in the self-focusing case. And the spectral widths increase monotonically and the peak gains decrease before increase with the normalized angular frequencies. Generally, the third-order nonlinear dispersion is detrimental to modulation instability generation.

Key words: Modulation instability; Third-order nonlinear dispersion; Metamaterials; Negative-refractive

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Foundation item: The Key Project of Chinese Ministry of Education (No. 210186), the Major Project of Natural Science Supported by the Educational Department of Sichuan Province (Nos. 13ZA0081, 12ZB019), and the Scientific Research Foundation of CUIT (No. 2010d1)

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Received: Apr. 1, 2013; **Accepted**: Jun. 19, 2013

0 Introduction

Nonlinear effects such as modulation instability (MI)[1-3], optical wave breaking[4] in optical fibers have been investigated extensively and now people have paid much more attention to the nonlinear propagation of electromagnetic waves in novel composite materials. These typical materials are photonic crystal fibers[5-7] and metamaterials (MMs)[8]. As we know that study on nonlinear metamaterials is an international hot spot at present. Up to now, people have investigated metamaterials in terms of their propagating models of electromagnetic waves[9-10], propagation of electromagnetic waves and soliton properties[11-12], generation of second-order or third-order harmonic[13], and modulation instability[14-23]. Many singular properties have already been discovered, which will inevitably enrich the contents of nonlinear optics and provide more opportunity for designing much more new-typed optoelectronic devices as well. Among them, MI generation under different circumstances means that there are different new thoughts and methods to generate and control the solitons. Thus, many researchers have already investigated effects of the first-order and second-order nonlinear dispersions, saturable nonlinearity, and fake quintic nonlinearity on MI in MMs[14-23]. And the previous studies have shown that, in comparison with the ordinary materials, MMs are essentially magnetic and dispersive, and that the nonlinear dispersion effects originating from the dispersive permeability can influence MI and propagation properties of ultra-short electromagnetic pulses considerably. In particular, when the pulse width gets shorter and shorter, more and more higher-order nonlinear dispersions will take effects and influence the pulse propagation and MI characteristics. When the femtosecond pulse propagating in MMs, for instance, the third-order nonlinear dispersion effect should be taken into account. Up to now, however, there are rare reports on effect of the third-order nonlinear dispersion on MI[22].

In this paper, after taking into account the first-order to third-order nonlinear dispersion effects, we will calculate the equation coefficients and MI characteristics in different refractive regions for both the self-focusing and self-defocusing MMs. Moreover, in convenient for the future experimental verification, we will directly start from the nonlinear propagation equation instead of the normalized one, and we will carry out the calculation in real units instead of dimensionless one. Although the experimentally investigation on MI in MMs is still a hard task at present for their technically difficulties in manufacture.

1 Nonlinear propagation model and its coefficient analysis

After taking into account the first-order to third-order nonlinear dispersion effects, the extended nonlinear propagation equation can be written as follows

$$\frac{\partial A}{\partial z}+\frac{\mathrm{i}}{2}\beta_2\frac{\partial^2 A}{\partial T^2}=\mathrm{i}\gamma_0\left\{|A|^2A+\mathrm{i}S_1\frac{\partial}{\partial T}[|A|^2A]-S_2\frac{\partial^2}{\partial T^2}[|A|^2A]+\mathrm{i}S_3\frac{\partial^3}{\partial T^3}[|A|^2A]\right\}\tag{1}$$

where β_2、γ_0、S_1、S_2、S_3、T、z are respectively the second-order group velocity dispersion coefficients, the third-order nonlinear coefficient, the first-order nonlinear dispersion coefficient, the second-order nonlinear dispersion coefficient, the third-order nonlinear dispersion coefficient, time in the pulse coordination system, propagation distance. Adopting definitions of the equation coefficients [16] and Drude electromagnetic model, one can derive the refractive index $n(\omega_0)$ at the carrier frequency ω_0 and other equation coefficients as

$$n(\omega_0)=\sqrt{1-\omega_{\mathrm{pe}}^2/\omega_0^2}\sqrt{1-\omega_{\mathrm{pm}}^2/\omega_0^2}\tag{2}$$

$$\beta_2=\frac{1}{cn\omega_0}\left[(1+3\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2/\omega_0^4)-\frac{1}{n^2}(1-\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2/\omega_0^4)^2\right]\tag{3}$$

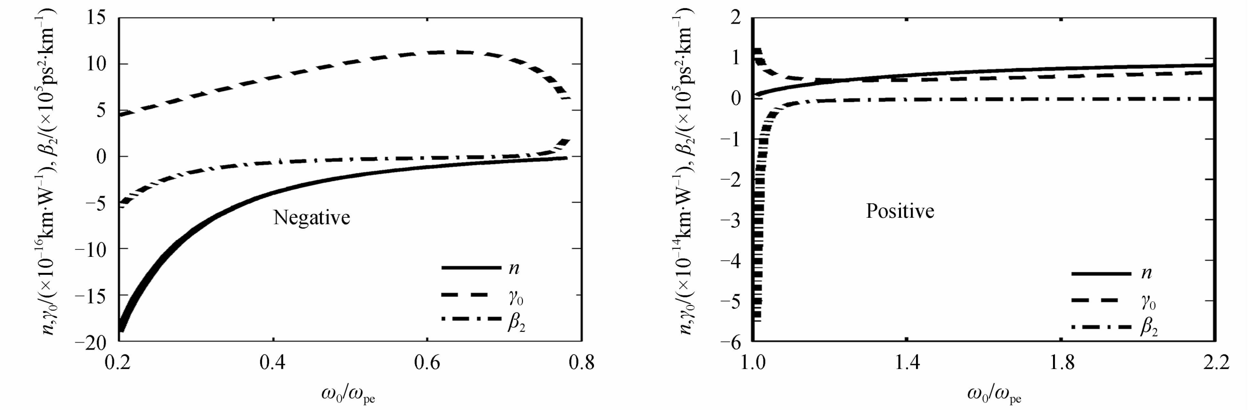
$$\gamma_0=\frac{\chi^{(3)}\omega_0}{2nc}(1-\omega_{\mathrm{pm}}^2/\omega_0^2)\tag{4}$$

$$S_1=\frac{1}{\omega_0}\left(\frac{\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2-\omega_0^4}{n^2\omega_0^4}-\frac{2\omega_0^2}{\omega_{\mathrm{pm}}^2-\omega_0^2}\right)\tag{5}$$

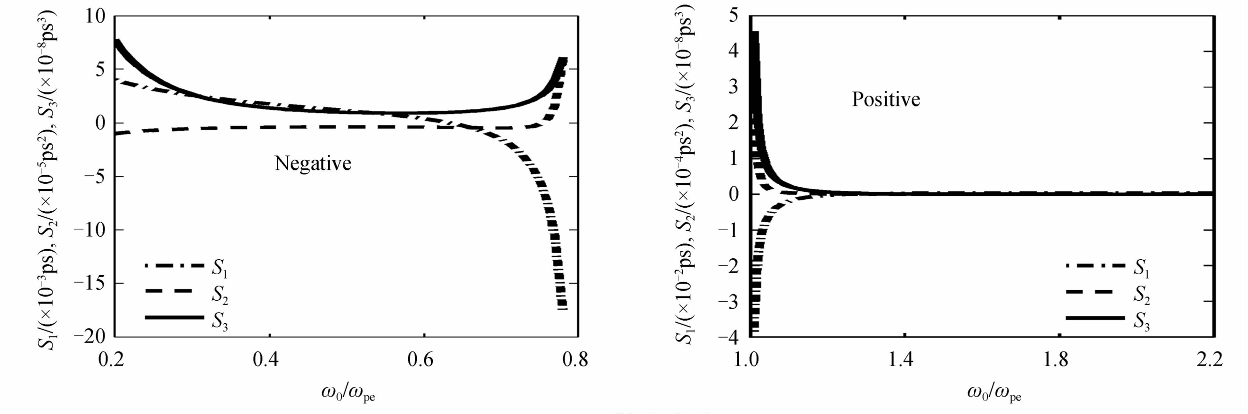
$$S_2=\frac{1}{\omega_0^2}\left[\frac{\omega_0^2}{\omega_0^2-\omega_{\mathrm{pm}}^2}-\frac{1}{4n^2}\left(1+\frac{3\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2}{\omega_0^4}\right)+\frac{1}{4n^4}\left(1-\frac{\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2}{\omega_0^4}\right)^2\right]\tag{6}$$

$$S_3=\frac{1}{n^2\omega_0^3}\frac{\omega_{\mathrm{pm}}^2\omega_{\mathrm{pe}}^2}{\omega_0^4}\tag{7}$$

where c is the light velocity in the vacuum, ω_{pe} and ω_{pm} are electronic and magnetic plamas frequencies, respectively. And $\chi^{(3)}$ is the third-order electric susceptibility. $\chi^{(3)}>0$ and $\chi^{(3)}<0$ respectively stand for the self-focusing and self-defocusing cases. Variations of the equation coefficients with the normalized angular frequency have been shown in real units in Fig. 1. The common parameters are set as $\omega_{\mathrm{pm}}/\omega_{\mathrm{pe}}=0.8$, $\chi^{(3)}=10^{-10}$ esu. during calculation. It can be seen from the figure that, the frequency regions $0<\omega_0/\omega_{\mathrm{pe}}<0.8$, $0.8<\omega_0/\omega_{\mathrm{pe}}<1$, and $\omega_0/\omega_{\mathrm{pe}}>1$ are the negative refractive region, the forbidden band region, and the positive refractive region, respectively. It also can be seen from Fig. 1 and Eq. (7) that S_3 is positive in both the negative and positive regions. With increase of $\omega_{\mathrm{pm}}/\omega_{\mathrm{pe}}$, S_3 decreases before increase in the negative refractive region while decreases monotonically to zero in the positive refractive region. Moreover, as shown in Eq. (4), the sign of parameter γ_0 will change with $\chi^{(3)}$. In addition, the equation coefficients are obviously associated with each other though $\omega_0/\omega_{\mathrm{pe}}$ and can not vary independently.



(a) n, γ_0, β_2



(b) S_1, S_2, S_3

Fig. 1　Variations of the refractive index and equation coefficients with the normalized angular frequency $\omega_0/\omega_{\mathrm{p}}$

2　Theoretical analysis and calculations

Perturbing the steady-state solution of Eq. (1) by adding a perturbation $a\ (z,\ T)$ with angular frequency Ω and wave number K and utilizing the standard linear stability analysis, the linearized nonlinear propagation equation can be obtained as follows

$$\frac{\partial a}{\partial z}=\mathrm{i}c_1(a+a^*)-c_2\frac{\partial a}{\partial T}-c_3\frac{\partial a^*}{\partial T}-\mathrm{i}c_4\frac{\partial^2 a}{\partial T^2}-\mathrm{i}c_5\frac{\partial^2 a^*}{\partial T^2}+c_6\frac{\partial^3 a}{\partial T^3}-c_7\frac{\partial^3 a^*}{\partial T^3}\tag{8}$$

where the parameters $c_j\,(j=1,2,3,4,5,6,7)$ are defined as $c_1=\gamma_0P_0$, $c_2=2\gamma_0S_1P_0$, $c_3=\gamma_0S_1P_0$, $c_4=0.5\beta_2+2\gamma_0S_2P_0$, $c_5=\gamma_0S_2P_0$, $c_6=-2\gamma_0S_3P_0$, $c_7=\gamma_0S_3P_0$, and P_0 is the optical power density. According to the procedure and step of the linear stability analysis, one can derive the following dispersion relation of MI

$$K=c_2\Omega+c_6\Omega^3\pm\Omega[c_7^2\Omega^4+(c_4^2-c_5^2-2c_3c_7)\Omega^2+c_3^2+2c_1c_4-2c_1c_5]^{1/2}\tag{9}$$

when the inner part of the radical is negative, K becomes complex and MI occurs. The corresponding MI condition and power gain can be respectively obtained as

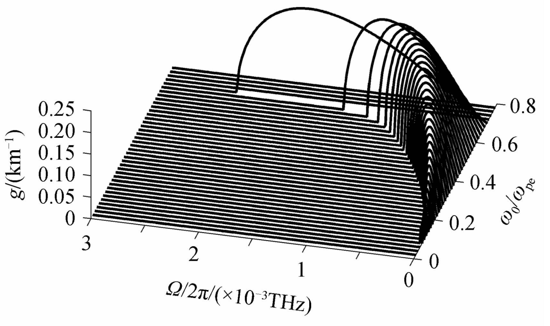
$$c_7^2\Omega^4+(c_4^2-c_5^2-2c_3c_7)\Omega^2+c_3^2+2c_1c_4-2c_1c_5<0\tag{10}$$

$$g(\Omega)=2\mathrm{Im}(K)=2|\Omega|[-c_7^2\Omega^4-(c_4^2-c_5^2-2c_3c_7)\Omega^2-c_3^2-2c_1c_4+2c_1c_5]^{1/2}\tag{11}$$

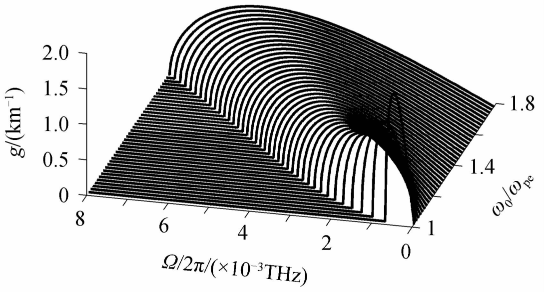
the signal Im stands for the imaginary part.

Variations of the MI gain spectra with the normalized angular frequency $\omega_0/\omega_{\mathrm{pe}}$ in the negative- ((a), (c)) and positive-refractive (b) regions for the self-focusing ((a), (b)) and self-defocusing (c) cases are shown in Fig. 2. It can be seen that, in the negative-refractive region, the spectral widths of the gain spectra increase with

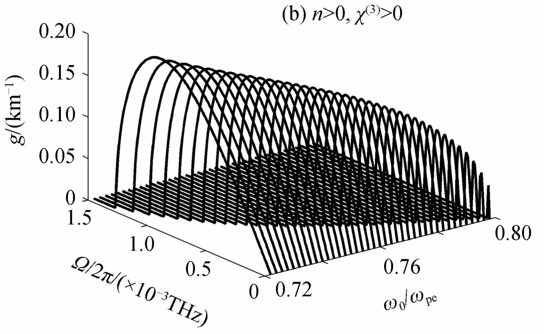
ω_0/ω_{pe} for the self-focusing case. And MI disappears near the forbidden band region. For the self-defocusing case, however, MI only occurs near the forbidden band region and the spectral width decreases with increasing of ω_0/ω_{pe}. In the positive-refractive region, the spectral widths of the gain spectra increase with ω_0/ω_{pe} for the self-focusing case while no MIs occur for the self-defocusing case.



(a) $n<0, \chi^{(3)}>0$



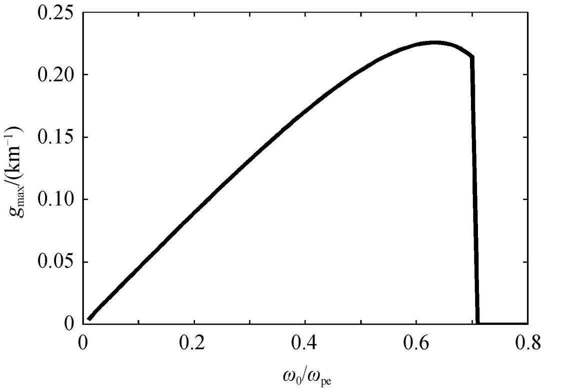
(b) $n>0, \chi^{(3)}>0$



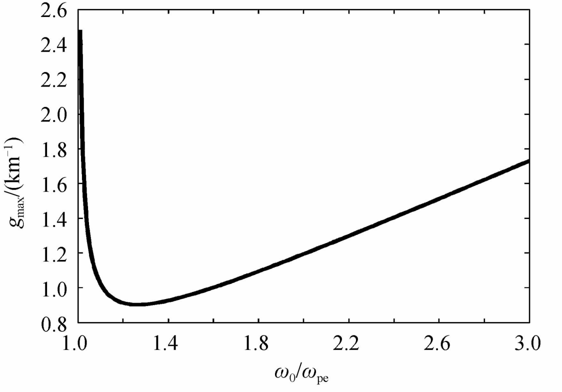
(c) $n<0, \chi^{(3)}<0$

Fig. 2 Variations of gain spectra with the normalized angular frequency ω_0/ω_{pe} in the negative (a)(c) and positive (b) refractive regions in cases of self-focusing (a)(b) and self-defocusing (c) nonlinearity

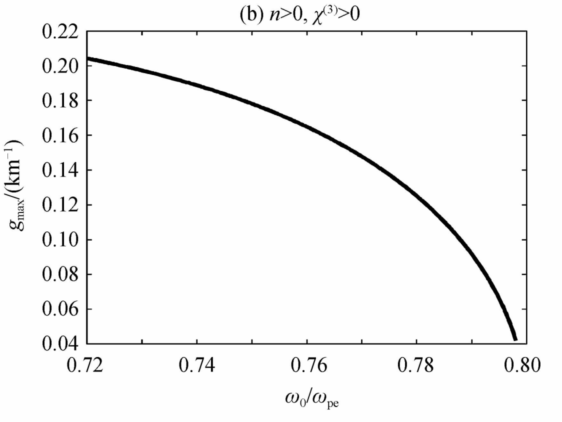
In correspondence with Fig. 2, variations of the peak gains $g_{\max}$ with ω_0/ω_{pe} are shown in Fig. 3. It can be seen clearly that, in the negative-refractive region, with increasing of ω_0/ω_{pe}, $g_{\max}$ increases before decrease for the self-focusing case and decreases for the self-defocusing case, respectively. In the positive-refractive region, however, $g_{\max}$ decreases before increase. The common parameters has been set as $\omega_{pm}/\omega_{pe}=0.8$ and during calculation in Fig. 2 and Fig. 3. In addition, for the reason that the spectral widths increase (decrease) monotonically with ω_0/ω_{pe} for the self-focusing (self-defocusing) case, one does not have to show them.



(a) $n<0, \chi^{(3)}>0$



(b) $n>0, \chi^{(3)}>0$



(c) $n<0, \chi^{(3)}<0$

Fig. 3 Variations of the peak gains of gain spectra with the normalized angular frequency ω_0/ω_{pe} in the negative (a)(c) and positive (b) refractive regions in cases of self-focusing (a)(b) and self-defocusing (c) nonlinearity

Further investigation shows that it is difficult for MI to occur when the first-order and second-order nonlinear dispersion effects are omitted, which means that the third-order nonlinear dispersion is detrimental to MI generation.

3 Conclusions

MI is analytically studied by adopting the linear stability analysis and the Drude

electromagnetic model by utilizing the extended nonlinear propagation equation without normalization in metamaterials with the first-order to third-order nonlinear dispersion effects. Variations of the equation coefficients and gain spectra with the normalized angular frequencies are calculated and discussed in real units in both the positive and negative refractive-index regions for both the self-focusing and self-defocusing cases. The results show that, the third-order nonlinear dispersion coefficients are all along positive in both the positive and negative refractive-index regions. In the negative refractive-index region, with increasing of normalized angular frequencies, the spectral widths of the gain spectra increase before decrease to zero and the peak gains increase before decrease and then become zero in the self-focusing case. While in the case of self-defocusing, MI can only occur near the forbidden band and both the spectral widths and peak gains decrease monotonically. In the positive refractive-index region, MI can only occur in the self-focusing case. And the spectral widths increase monotonically and the peak gains decrease before increase with the normalized angular frequencies. When the first-order and second-order nonlinear dispersion effects are omitted, no MIs occur, which means the third-order nonlinear dispersion is detrimental to MI generation.

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