第 42 卷第 10 期
2013 年 10 月

光 子 学 报
ACTA PHOTONICA SINICA

Vol. 42 No. 10
October 2013

doi:10.3788/gzxb20134210.1248

运动原子和不同初始场相互作用系统中熵和纠缠的研究

廖庆洪[1,2],鄢秋荣[1a],刘晔[1a],Muhammad Ashfaq Ahmad[3]

(1 南昌大学 a.电子信息工程系;b. 材料科学与工程学院,南昌 330031)
(2 北京师范大学 射线束技术与材料改性教育部重点实验室,北京 100875)
(3 COMSATS 信息技术学院物理系,拉合尔 54000,巴基斯坦)

摘　要:通过计算线性熵研究了当场初始处于薛定谔猫态和压缩相干态时 Jaynes-Cummings 模型中原子线性熵随时间的演化特性,讨论了原子运动和场模结构参数对原子线性熵的影响.结果发现原子和场之间的纠缠对压缩参数非常敏感,并且结果表明当腔场初始处于 Yurke-Stoler 态时,原子的运动导致了线性熵的周期性演化,随着场模结构参数的增加,不但线性熵的演化周期缩短,而且线性熵的的幅值减小.当腔场初始处于偶相干态时,场模结构参数的增加导致了线性熵的演化周期缩短,然而对线性熵的幅值没有影响.

关键词:线性熵;原子运动;薛定谔猫态;压缩相干态

中图分类号:O431.2　　**文献标识码**:A　　**文章编号**:1004-4213(2013)10-1248-8

Entropy and Entanglement of a Moving Atom Interacting with Different Initial Fields

LIAO Qing-hong[1,2], YAN Qiu-rong[1a], LIU Ye[1a], Muhammad Ashfaq Ahmad[3]

(1 a. *Department of Electronic Information Engineering*; b. *School of Materials Science and Engineering*, *Nanchang University*, *Nanchang* 330031, *China*)
(2 *Key Laboratory of Beam Technology and Material Modification of Ministry of Education*, *Beijing Normal University*, *Beijing* 100875, *China*)
(3 *Department of Physics*, *COMSATS Institute of Information Technology*, *Lahore* 54000, *Pakistan*)

Abstract: The effects of the atomic motion and the field-mode structure, on the evolution of the linear entropy of the atom were examined in the Jaynes-Cummings model by means of linear entropy when the field was initially prepared in Schrödinger cat state and squeezed coherent state. It was found that the degree of entanglement between the atom and the field is very sensitive to the squeezing parameter. It was shown that the atomic motion leads to the periodic evolution of the linear entropy and an increase in field-mode structure parameter results in shortening of the evolution period of the linear entropy and decreasing in the amplitude of the linear entropy when the field is initially in Yurke-Stoler state. Furthermore, the increase of field-mode structure parameter leads to the shortening of the evolution periodicity of the linear entropy, but has no effect on the amplitude of the linear entropy when the field is initially in even coherent state.

Foundation item: Project supported by the National Natural Science Foundation of China(Nos. 11247213, 10664002, 11264030, 61368002), China Postdoctoral Science Foundation(No. 2013M531558), the Natural Science Foundation of Jiangxi Province(No. 20122BAB201031), the Foundation for Young Scientists of Jiangxi Province(Jinggang Star)(No. 2012BCB23002), the Research Foundation of the Education Department of Jiangxi Province(Nos. GJJ12137, GJJ13051, GJJ13057), and Jiangxi Postdoctoral Research Project(No. 2013KY33)

First author: LIAO Qing-hong (1982 —), male, associate professor, Ph. D. degree, mainly focuses on quantum optics and quantum information. Email: nculqh@163.com

Received: Apr. 11, 2013; **Accepted**: Jun. 14, 2013

Key words: Linear entropy; Atomic motion; Schrödinger cat state; Squeezed coherent state

0 Introduction

Quantum entanglement is one of the most remarkable features of quantum theory[1-2]. It plays an essential role in quantum communication and quantum information processes such as quantum key distribution[3], quantum teleportation[4], superdense coding[5-6], quantum computation[7-8] and entanglement swapping[9-11]. An investigation of the atom-field entanglement for Jaynes-Cummings model has been initiated by Phoenix and Knight[12-13] and Gea-Banacloche[14-15]. The time evolution of the field (atomic) entropy reflects the time evolution of the degree of entanglement between the atom and the field. The higher the entropy, the greater the entanglement. Moreover, many papers have focused on the properties of the entanglement and entropy in different models[16-25]. All these theoretical study results are obtained only under the condition that the atomic motion is neglected and the field-mode structure is not taken into account.

Recently, much attention has been paid to the von Neumann entropy in the presence of the atomic motion. For instance the effects of atomic motion and field-mode structure in the one-photon Jaynes-Cummings model have been investigated in Ref. [26] showing that the atomic motion leads to the periodic evolution of the von Neumann entropy. The influences of atomic motion and field-mode structure on the dynamical properties of the von Neumann entropy in the JC model when the field is initially prepared in SU(1, 1)-related coherent fields and thermal field have been examined in Refs. [27-28] respectively. Furthermore, the effects of atomic motion and field-mode structure in the two-photon Jaynes-Cummings model have been investigated in Ref. [29] showing that the atomic motion and field-mode structure have no effect on the amplitude of the von Neumann entropy. The main aim of this paper is to examine the influences of the atomic motion and field-mode structure on the dynamical properties of the linear entropy when the field is initially prepared in Schrödinger cat state and squeezed coherent state. The organization of the paper is arranged as follows: we introduce the model and the basic equations for the system under consideration in section 2 ; by the numerical computation , we investigate the effects of the atomic motion and the field-mode structure on the time evolution of linear entropy when the field is in Schrödinger cat state and squeezed coherent state in section 3; in section 4, the main results are summarized.

1 The model and basic equations

We consider a moving two-level atom interacting with a single mode of the cavity field via the one-photon transition processes. The effective Hamiltonian of the model with the rotating-wave approximation[30-31] can be written as

$$H=\omega a^{\dagger}a+\frac{1}{2}\omega_0\sigma_z+gf(z)(a\sigma_+ + a^{\dagger}\sigma_-)(\hbar=1) \quad (1)$$

where $a^{\dagger}$ and a are the creation and annihilation operators of the field of frequency ω, σ_z and $\sigma_{\pm}$ are the Pauli spin operators of the atom, ω_0 is the atomic transition frequency and the coupling constant g is proportional to the dipole matrix element of the atomic transition. $f(z)$ is the shape function of the cavity field mode. We restrict the investigations to the atomic motion along z-axis, i. e. the z-dependence of the field-mode function is considered. The atomic motion can be incorporated into the system through [26,32]

$$f(z)\rightarrow f(\upsilon t) \quad (2)$$

where υ is the atomic motion velocity. In this regard the transformation TEM_{mnp} is defined as[16]

$$f(z)=\sin(p\pi\upsilon t)/L \quad (3)$$

where p represents the number of half-wavelengths of the field mode inside a cavity of the length L. For reasons of simplicity we will now consider a resonant system, i. e. $\omega=\omega_0$ and assume that the atom enters the cavity at time $t=0$ in the excited state $|+\rangle$ and leaves the cavity again after passing p half-wavelengths of the electric field.

We consider the atom initially in the excited state $|+\rangle$ and the field mode in a general pure state having the form[33]

$$|\gamma\rangle=\sum_{n=0}^{\infty}p_n^{1/2}\exp(\mathrm{i}\beta_n)|n\rangle \quad (4)$$

Hence, the initial state of the system is $|\psi(0)\rangle=|+\rangle|\gamma\rangle$. Then the solution of the Schrödinger equation in the interaction picture is given by

$$|\psi(t)\rangle=\sum_{n=0}^{\infty}p_n^{1/2}\exp(\mathrm{i}\beta_n)\left[\cos(g\theta(t)\sqrt{n+1})|n,e\rangle-\mathrm{i}\cdot\sin(g\theta(t)\sqrt{n+1})|n+1,g\rangle\right] \quad (5)$$

where

$$\theta(t)=\int_0^t f(\upsilon t')\mathrm{d}t'=\frac{L}{p\pi\upsilon}\left[1-\cos\left(\frac{p\pi\upsilon t}{L}\right)\right] \quad (6)$$

For a particular choice of the atomic motion velocity $v=gL/\pi$, $\theta(t)$ becomes

$$\theta(t)=\frac{1}{pg}[1-\cos(pgt)] \tag{7}$$

In order to describe the evolution of the atom alone it is convenient to introduce the reduced density matrix $\rho_A(t)=\mathrm{Tr}_\gamma[\rho(t)]$, where $\rho(t)=|\psi(t)\rangle\langle\psi(t)|$ is the total density matrix and the trace is over a complete set of radiation field states. One easily finds that

$$\rho_A(t)=\rho_{ee}(t)|e\rangle\langle e|+\rho_{gg}(t)|g\rangle\langle g|+\rho_{eg}(t)|e\rangle\langle g|+\rho_{eg}^*(t)|g\rangle\langle e| \tag{8}$$

From Eq. (5) the matrix elements of reduced density operator $\rho_A(t)$ of the atom can be expressed as

$$\rho_{ee}(t)=\sum_{n=0}^{\infty}p_n\cos^2[g\theta(t)\sqrt{n+1}] \tag{9}$$

$$\rho_{eg}(t)=\mathrm{i}\sum_{n=0}^{\infty}\sqrt{p_{n+1}p_n}\exp[\mathrm{i}(\beta_{n+1}-\beta_n)]\cdot\cos[g\theta(t)\sqrt{n+2}]\sin[g\theta(t)\sqrt{n+1}]=\rho_{ge}^*(t) \tag{10}$$

$$\rho_{gg}(t)=1-\rho_{ee}(t) \tag{11}$$

Analytical conclusions about the system state vector dynamics and atom-field entanglement can be verified through examining the linear entropy. The linear entropy of reduced atomic (or field) density matrix can be used as a measurement of the degree of the entanglement between the atom and the field of the system under consideration. The linear entropy of reduced atomic density matrix for considered systems has the following form[34]

$$S(t)=1-\mathrm{Tr}[\rho_a^2(t)]=1-\rho_{ee}^2(t)-\rho_{gg}^2(t)-2|\rho_{eg}|^2 \tag{12}$$

By making use of Eqs. (8)-(12), we are in a position to discuss the effects of the atomic motion and the field-mode structure on the evolution of the linear entropy of the atom. This will be seen in section 3.

2 Discussion of the results

The main purpose of the present section is to discuss the effects of the atomic motion and the field-mode structure on the time evolution of linear entropy of the atom when the field is in Schrödinger cat state and squeezed coherent state.

2.1 The initial field is in a Schrödinger cat state

The field is initially prepared in a Schrödinger cat state having the form[35]

$$|\gamma\rangle=N(|\alpha\rangle+\exp(\mathrm{i}\varphi)|-\alpha\rangle) \tag{13}$$

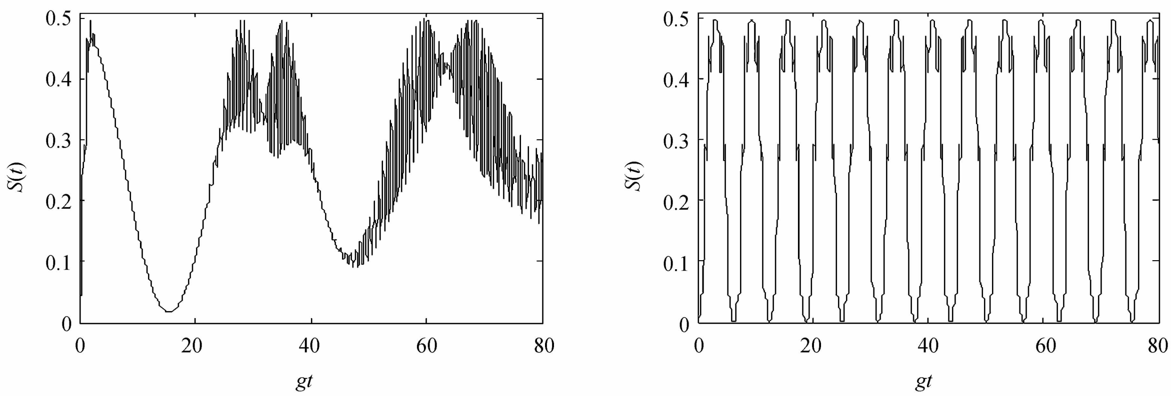
where $|\alpha\rangle$ is a coherent state of amplitude α, and φ is a real local phase factor. Note that the relative phase φ can be approximately controlled by the displacement operation for a given cat state with $\alpha\geqslant 1$. Schrödinger cat states of this type have been realized for a trapped $^9Be^+$ ion[36]. The normalization factor N takes the form

$$N^2=\frac{1}{2+2\cos(\varphi)\exp(-2|\alpha|^2)} \tag{14}$$

So the quantity p_n takes the form

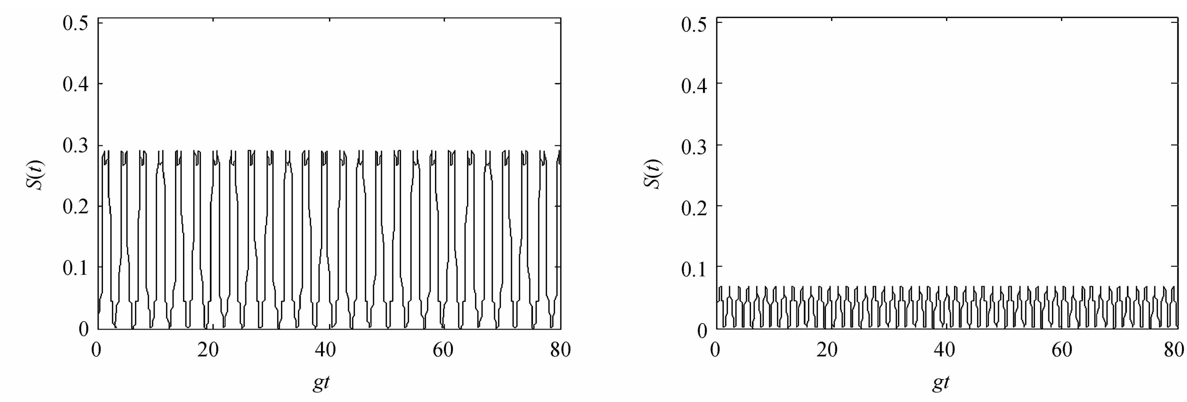
$$p_n=\frac{|\alpha|^{2n}[1+(-1)^n\cos\varphi]}{n!\,[\exp(|\alpha|^2)+\exp(-|\alpha|^2)]} \tag{15}$$

In Fig. 1 we plot the time evolution of the linear entropy of the atom when the field is initially prepared in Yurke-Stoler state ($\varphi=\pi/2$) with the initial mean photon number $\bar{n}=25$ and different values of the field-mode structure p. Fig. 1(a) displays the case when the atomic motion is ignored, i. e. $\theta(t)\to t$. One can observe that the time behavior of the linear entropy is not periodical in the normal Jaynes-Cummings model and the linear entropy evolves to the minimum values and the atom is completely disentangled from the field at the half of the revival time ($t_R=2\pi\sqrt{\bar{n}}/g$). This corresponds to the case of the Jaynes-Cummings



(a) The atomic motion is neglected

(b) The atomic motion is considered where p=1

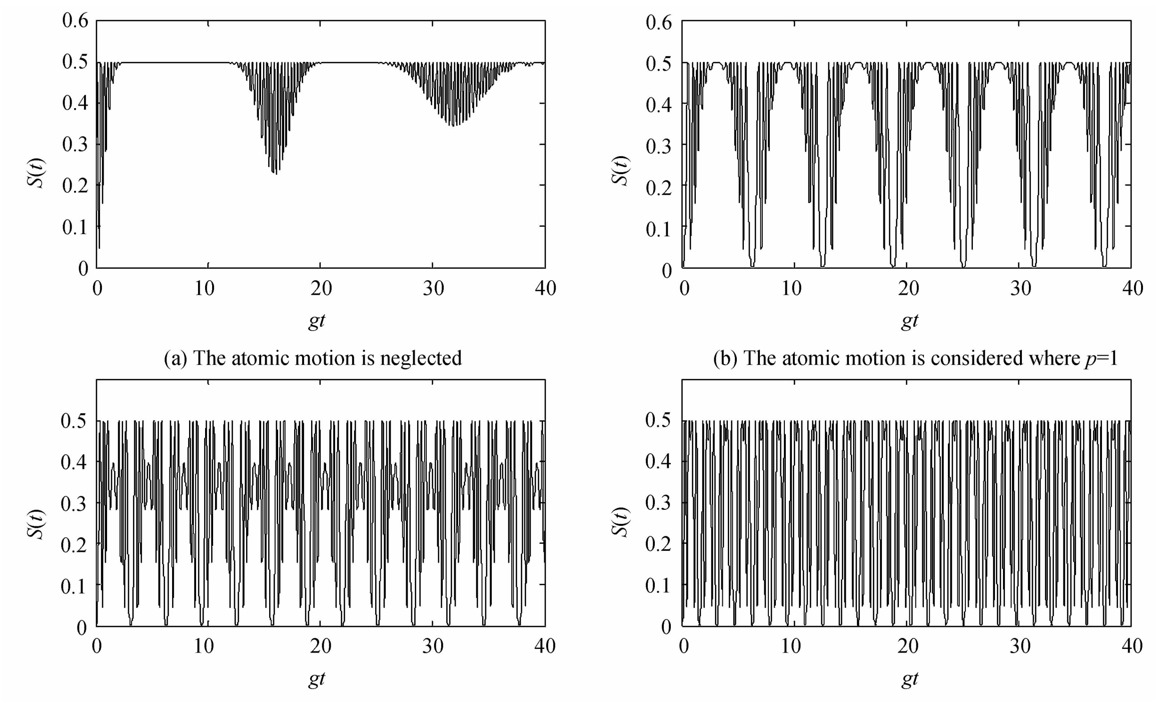


(c) The atomic motion is considered where p=2　　(d) The atomic motion is considered where p=4

Fig. 1　The time evolution of the linear entropy (the field is initially in Yurke-Stoler state with mean photon number $\bar{n}=25$)

model when the initial field is in coherent state which has been discussed in Refs. [12-15, 26]. Figs. 1(b), (c) and (d) illustrate the dynamical properties of the linear entropy when atomic motion is taken into account. From these figures we can observe the atomic motion leads to the periodic evolution of the linear entropy and an increase in parameter p results in not only shortening of the evolution period of the linear entropy but also decreasing in the amplitude of the linear entropy. Remarkably, these results are very similar to those discussed by Fang in Ref. [26]. This can be understood by looking at the photon statistics of the initial fields.

In Fig. 2 we illustrate the behavior of the linear entropy when the initial field is in even coherent state ($\varphi=0$). Fig. 2(a) displays the case when the atomic motion is neglected, this corresponds to the results obtained in Ref. [23]. Figs. 2(b), (c) and (d) illustrate the dynamical properties of the linear entropy when atomic motion is taken into account. From these figures, we can conclude that: an increase of the parameter p leads to the shortening of the evolution periodicity of the linear entropy, but has no effect on the amplitude of the linear entropy. This result is quite interesting and this feature gives the opposite to the case of the standard Jaynes-Cummings model which has been observed in Ref. [26].



(a) The atomic motion is neglected　　(b) The atomic motion is considered where p=1

(c) The atomic motion is considered where p=2　　(d) The atomic motion is considered where p=4

Fig. 2　The time evolution of the linear entropy (the field is initially in even coherent state with mean photon number $\bar{n}=25$)

2.2　The initial field is in a squeezed coherent state

We initially prepare the field in a squeezed coherent state. Squeezed coherent state is purely quantum state since it has less uncertainty in one quadrature than the vacuum noise level[37-40]. It can be generated via a degenerate parametric amplifier[41]. It is defined by the action of squeezing operator on the coherent state having the form[39]

$$|\gamma\rangle = S(r)|\alpha\rangle \tag{16}$$

Here we consider a squeezing operator

$$S(r) = \exp\left[\frac{r(a^2 - a^{+2})}{2}\right] \tag{17}$$

thus the quantity p_n takes the form[39]

$$p_n = \frac{\tanh\,(r)^n}{n!\ 2^n \cosh\,(r)} \exp\left[-|\alpha|^2 (1 - \tanh\,(r))\right] H_n\left(\frac{\alpha}{[\sinh\,(2r)]^{1/2}}\right)^2 \tag{18}$$

In Figs. 3～5 we illustrate the time evolution of the linear entropy for different squeezing parameters (r=0.75, 1, 1.5) which are similar to those of Ref. [33].

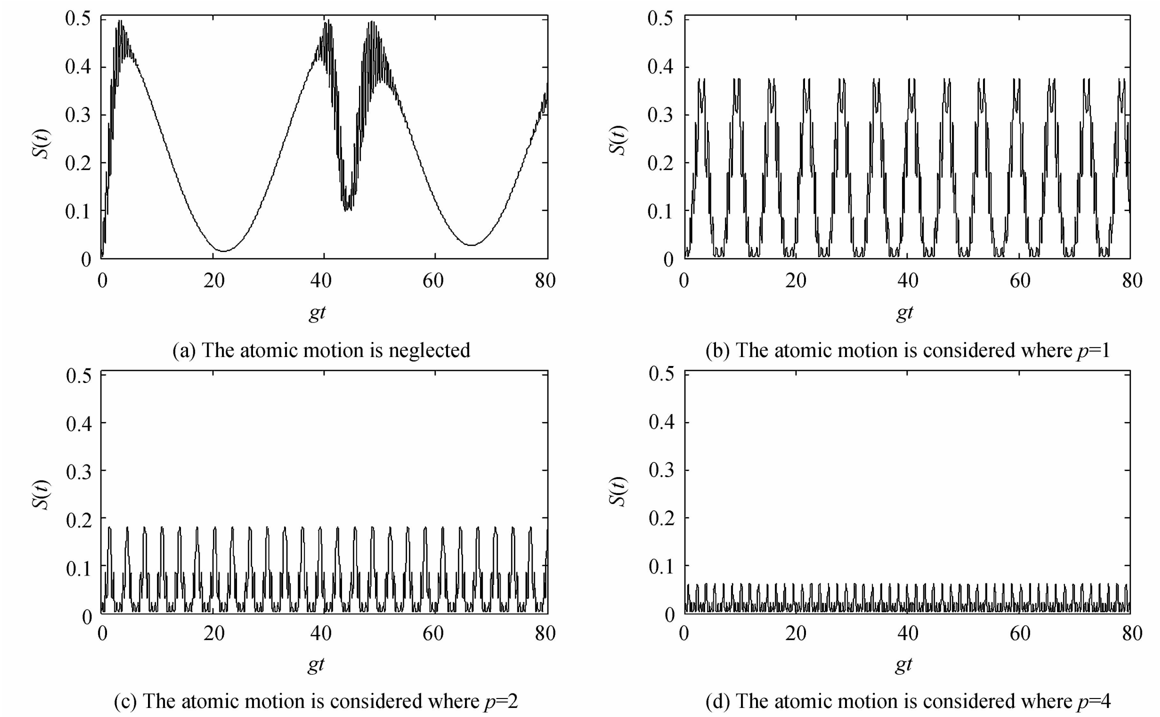


Fig. 3 The time evolution of the linear entropy (the field is in a squeezed coherent state with r=0.75 and α(≈14.72))

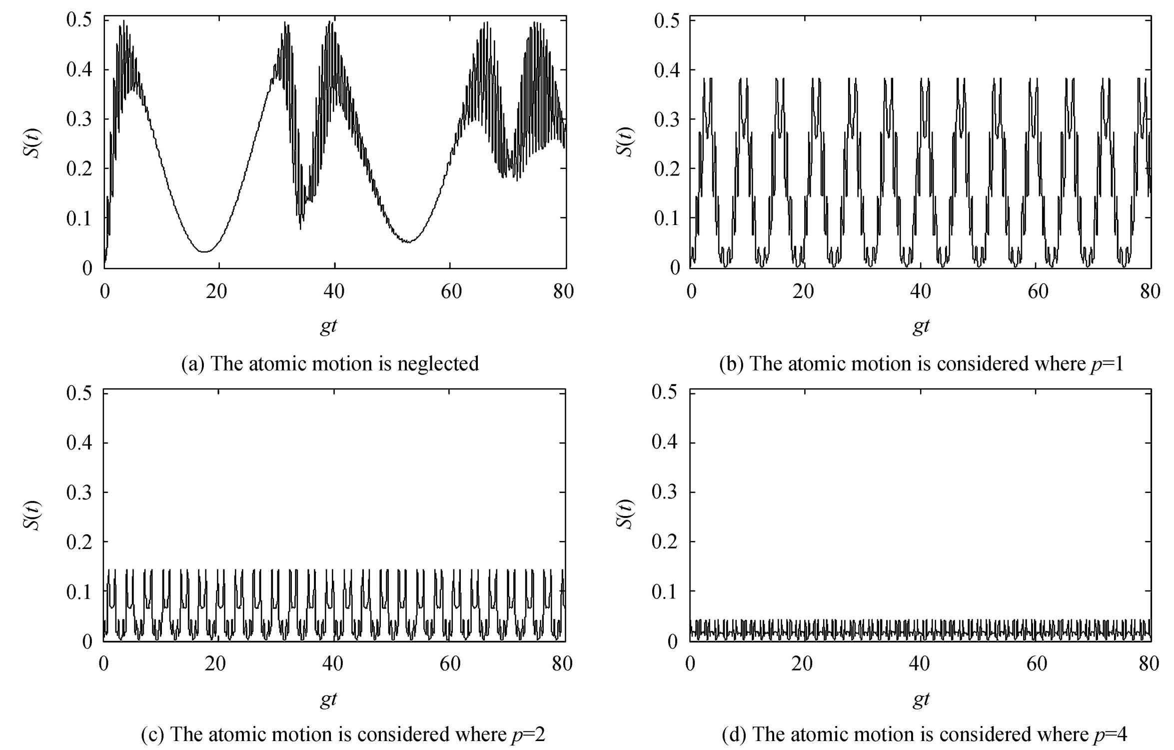


Fig. 4 The time evolution of the linear entropy (the field is in a squeezed coherent state with r=1.0 and α(≈14.72))

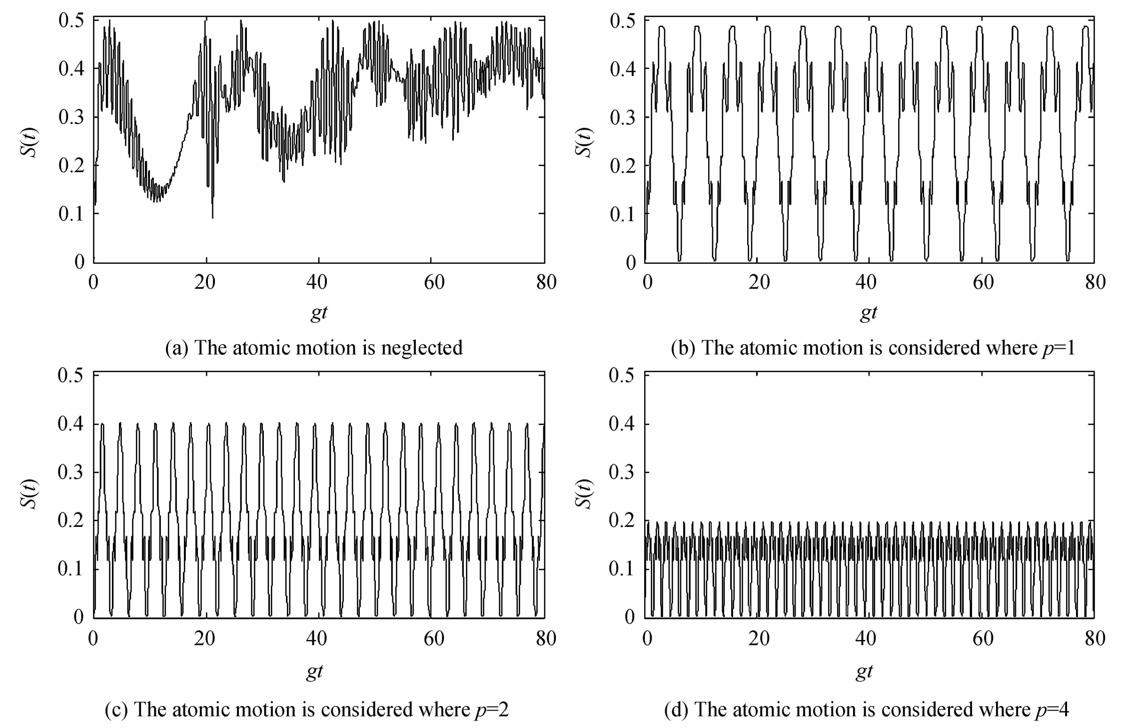
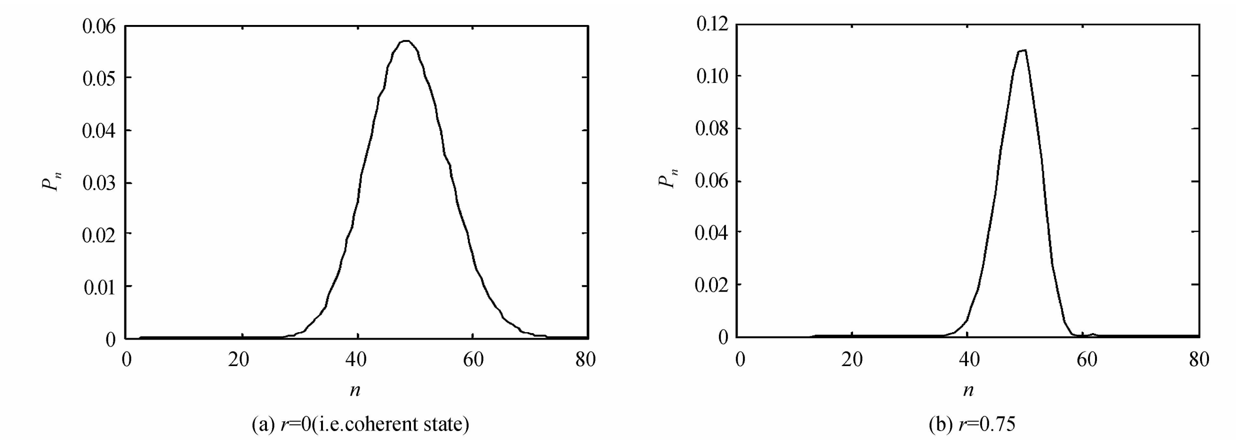


Fig. 5 The time evolution of the linear entropy(The field is in a squeezed coherent state with $r=1.5$ and $\alpha(\approx 14.72)$)

Figs. 3(a), 4(a) and 5(a) display the case when the atomic motion is neglected, It is observed that the linear entropy reaches a minimum at approximately the revival time corresponding to an initial coherent state with amplitude α and the value of the entropy is almost the same at the revival time and at half of the revival time. Furthermore, because they corresponding to minima, the states of the field are less mixed at these times. This is due to the nature of the squeezed coherent states [24-25].

Figs. 3(b)~(d), 4(b)~(d) and 5(b)~(d) illustrate the dynamical properties of the linear entropy when atomic motion is considered. From these figures it is observed that the atomic motion leads to the periodic evolution of the linear entropy and an increase in parameter p results in not only shortening of the evolution period of the linear entropy but also decreasing in the amplitude of the linear entropy. The comparison of the curves of Figs. 3(c), 4(c) and 5(c) at $p=2$ shows that the linear entropy is not monotone for the squeezing parameter r. This can be understood by looking at the photon number distribution of the initial fields (see Fig. 6).

In Fig. 6 we show the photon number distribution of the initial field for different squeezing parameters ($r=0, 0.75, 1, 1.5$). From the Fig. 6(b) the photon number distribution of the squeezed state ($r=0.75$) is sub-Poissonian, i. e. narrower than the Poissonian distribution for a coherent state (see Fig. 6(a)). Moreover, the oscillations in the photon number distribution are present for some squeezed states (see Figs. 6(c)



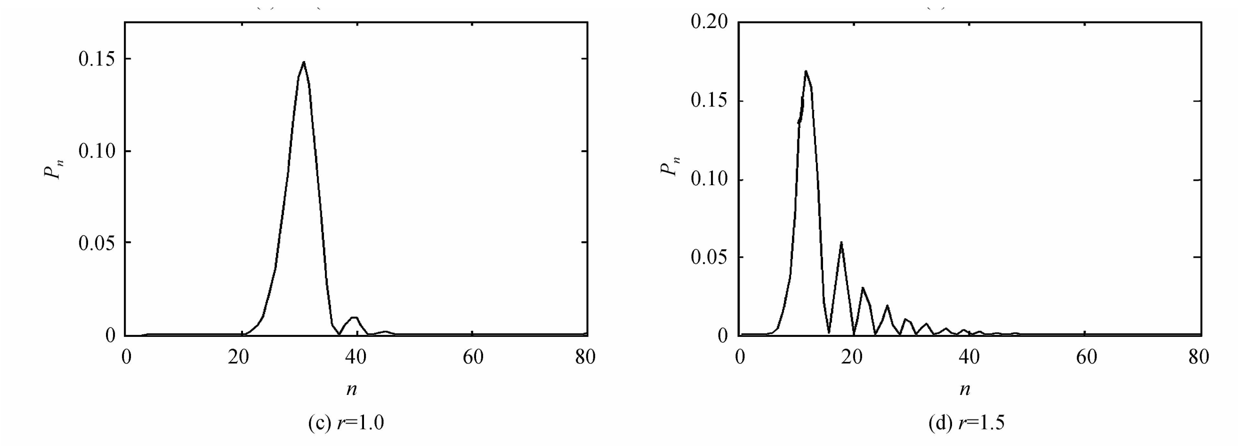


Fig. 6 Photon number distribution of the initial field for different squeezing parameters

and (d)). The physics behind these oscillations has been explained in terms of the concept of interference in phase space by Wheeler and Schleich [42-45].

3 Conclusion

In this paper, we have discussed the dynamical properties of time evolution of the linear entropy of the atom in the Jaynes-Cummings model, taking into account the atomic motion. The influences of the atomic motion and the field-mode structure on the temporal evolution of the linear entropy are examined. Furthermore, we examine the effects of Schrödinger cat state and squeezed coherent state as the initial states on the linear entropy and entanglement. The detailed study is of some interest for the following results. The atomic motion leads to the periodic evolution of the linear entropy and an increase in parameter p results in not only shortening of the evolution period of the linear entropy but also decreasing in the amplitude of the linear entropy when the initial state of the field is in Yurke-Stoler state. Second, an increase of the parameter p leads to the shortening of the evolution periodicity of the linear entropy, but has no effect on the amplitude of the linear entropy when the field is initially prepared in even coherent state. It is also shown that the degree of entanglement between the atom and the field is very sensitive to the squeezing parameter.

References

[1] PERES A. Quantum theory: concepts and methods[M]. The Netherlands: Kluwer Academic, Dordrecht, 1993.

[2] ALBER G, BETH T, HORODECKI M, *et al*. Quantum information: an introduction to basic concepts and experiments [M]. Berlin: Springer, 2001.

[3] EKERT A. Quantum cryptography based on Bell's theorem [J]. *Physical Review Letters*, 1991, **67**(6): 661-663.

[4] BENNETT C H, BRASSARD G, CREPEAU C, *et al*. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels [J]. *Physical Review Letters*, 1993, **70**(13): 1895-1899.

[5] BENNETT C H, WIESNER S J. Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states[J]. *Physical Review Letters*, 1993, **69**(16): 2881-2884.

[6] MOZES S, OPPENHEIM J, REZNIK B. Deterministic dense coding with partially entangled states[J]. *Physical Review A*, 2005, **71**(1): 012311-1-7.

[7] LIEB E H, SEIRINGER R. Coherent control of trapped ions using off-resonant lasers[J]. *Physical Review A*, 2005, **71**(6): 062309-1-13.

[8] BISWAS A, AGARWAL G S. Strong subadditivity inequality for quantum entropies and four-particle entanglement [J]. *Physical Review A*, 2003, **68**(5): 054303-1-4.

[9] GLÖCKL O, LORENZ S, MARQUARDT C, *et al*. Experiment towards continuous-variable entanglement swapping: Highly correlated four-partite quantum state[J]. *Physical Review A*, 2003, **68**(1): 012319-1-8.

[10] YANG M, SONG W, CAO Z L. Entanglement swapping without joint measurement[J]. *Physical Review A*, 2005, **71**(3): 034312-1-3.

[11] LI H R, LI F L, YANG Y, *et al*. Entanglement swapping of two-mode Gaussian states in a thermal environment [J]. *Physical Review A*, 2005, **71**(2): 022314-1-7.

[12] PHOENIX S J D, KNIGHT P L. Fluctuations and entropy in models of quantum optical resonance[J]. *Annals of Physics*, 1988, **186**(2): 381-407.

[13] PHOENIX S J D, KNIGHT P L. Establishment of an entangled atom-field state in the Jaynes-Cummings model[J]. *Physical Review A*, 1991, **44**(9): 6023-6029.

[14] GEA-BANACLOCHE J. Collapse and revival of the state vector in the Jaynes-Cummings model: An example of state preparation by a quantum apparatus[J]. *Physical Review Letters*, 1990, **65**(27): 3385-3388.

[15] GEA-BANACLOCHE J. Atom- and field-state evolution in the Jaynes-Cummings model for large initial fields [J]. *Physical Review A*, 1991, **44**(9): 5913-5931.

[16] BUŽEK V, MOYA-CESSA H, KNIGHT P L. Schrödinger-cat states in the resonant Jaynes-Cummings model: Collapse and revival of oscillations of the photon-number distribution [J]. *Physical Review A*, 1992, **45**(11):8190-8203.

[17] BUŽEK V, HLADKY B. Macroscopic superposition states of light via 2-photon resonant interaction of atoms with cavity field[J]. *Journal of Modern Optics*, 1993, **40**(7): 1309-1324.

[18] FANG M F, ZHOU G H. Influence of atomic coherence on the evolution of field entropy in multiphoton processes[J].

Physics Letters A, 1994, **184**(6): 397-402.

[19] FANG M F, LIU H E. Properties of entropy and phase of the field in the two-photon Jaynes-Cummings model with an added Kerr medium[J]. *Physics Letters A*, 1995, **200**(3-4): 250-256.

[20] FANG M F. Entropy evolution of the bimodal field interacting with an effective 2-level atom via the Raman transition[J]. *Journal of Modern Optics*, 1994, **41**(7): 1319-1326.

[21] FANG M F, ZHOU P. Entropy propertites of the field in the two-photon Jaynes-Cummings model with an additional Kerr medium[J]. *Acta Physica Sinica*, 1994, **43**(4): 570-579.

[22] FANG M F, ZHOU P. Quantum entropy and entanglement in the Jaynes-Cummings model without the rotating-wave approximation[J]. *Physica A*, 1996, **234**(1-2): 571-580.

[23] OBADA A S F, MOHAMMED F A, HESSIAN H A, *et al*. Entropies and entanglement for initial mixed state in the multi-quanta JC model with the stark shift and Kerr-like medium[J]. *International Journal of Theoretical Physics*, 2007, **46**(4): 1027-1044.

[24] MOYA-CESSAT H, VIDIELLA-BARRANCO A. Interaction of superpositions of coherent states of light with two-level atoms[J]. *Journal of Modern Optics*, 1992, **39**(7): 2481-2499.

[25] MAHMOUD A A, ABD AL-KADER G M, OBADA A S F. Entropy and entanglement of an effective two-level atom interacting with two quantized field modes in squeezed displaced fock states[J]. *Chaos, Solitons and Fractals*, 2001, **12**(13): 2455-2470.

[26] FANG M F. Effects of atomic motion and field mode structure on the field entropy and Schrödinger-cat states in the Jaynes-Cummings model[J]. *Physica A*, 1998, **259**(1-2): 193-204.

[27] LIAO X P, FANG M F, ZHOU Q P. Quantum entanglement in the SU(1, 1)-related coherent fields interacting with a moving atom[J]. *Physica A*, 2006, **365**(2): 351-359.

[28] AN X Q, SHAO B, ZOU J. Entropy exchange and entanglement of an in motion two-level atom with a quantized field[J]. *Chaos, Solitons and Fractals*, 2008, **37**(3): 835-841.

[29] ABDEL-KHALEK S. The effect of atomic motion and two-quanta JCM on the information entropy[J]. *Physica A*, 2008, **387**(4): 779-786.

[30] SCHLICHER R R. Jaynes-Cummings model with atomic motion[J]. *Optics Communications*, 1989, **70**(2): 97-102.

[31] JOSHI A, LAWANDE S V. Effect of atomic motion on Rydberg atoms undergoing two-photon transitions in a lossless cavity[J]. *Physical Review A*, 1990, **42**(3): 1752-1756.

[32] SARGENT M III, SCULLY M O, LAMB W E. Laser physics[M]. Addison-Wesley, Reading, MA, 1974.

[33] REKDAL P K, SKAGERSTAMSS B K, KNIGHT P L. On the preparation of pure states in resonant microcavities[J]. *Journal of Modern Optics*, 2004, **51**(1): 75-84.

[34] EL-ORANY F A A. Relationship between the linear entropy, the von Neumann entropy and the atomic Wehrl entropy for the Jaynes-Cummings model[DB/OL]. (2007-05-30)[2013-04-11]. http://arxir.org/abs/0705.4373.

[35] GERRY C C, KNIGHT P L. Quantum superpositions and Schrödinger cat states in quantum optics[J]. *American Journal of Physics*, 1997, **65**(10): 964-974.

[36] MONROE C, MEEKHOF D M, KING B E, *et al*. A Schrödinger cat superposition state of an atom[J]. *Science*, 1996, **272**(5265): 1131-1136.

[37] STOLER D. Equivalence classes of minimum uncertainty packets[J]. *Physical Review D*, 1970, **1**(12): 3217-3219.

[38] STOLER D. Equivalence classes of minimum-uncertainty packets. II[J]. *Physical Review D*, 1970, **4**(6): 1925-1926.

[39] YUEN H P. Two-photon coherent states of the radiation field[J]. *Physical Review A*, 1976, **13**(6): 2226-2243.

[40] LOUDON R, KNIGHT P L. Squeezed light[J]. *Journal of Modern Optics*, 1987, **34**(6): 709-759.

[41] WU L A, KIMBLE H J, HALL J L, *et al*. Generation of squeezed states by parametric down conversion[J]. *Physical Review Letters*, 1986, **57**(20): 2520-2523.

[42] WHEELER J A. Franck-condon effect and squeezed-state physics as double-source interference phenomena[J]. *Letters in Mathematical Physics*, 1985, **10**(2-3): 201-206.

[43] SCHLEICH W, WHEELER J A. Oscillations in photon distribution of squeezed states[J]. *JOSA B, Optical physics*, 1987, **4**(10): 1715-1722.

[44] SCHLEICH W, WHEELER J A. Oscillations in photon distribution of squeezed states and interference in phase space [J]. *Nature*, 1987, **326**: 574-577.

[45] SCHLEICH W, WALLS D F, WHEELER J A. Area of overlap and interference in phase space versus Wigner pseudoprobabilities[J]. *Physical Review A*, 1988, **38**(3): 1177-1186.