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Preparation of Entangled Four-photon Polarization State and Its Application in Teleportation via Cross-Phase Modulation

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Abstract: A scheme is proposed for generating a four-photon polarized Dicke state, Greenberger-Horne-Zeilinger state, and W-type state. The scheme only uses Kerr medium, polarization beam splitters, half wave plates and homodyne measurement on the coherent light field, which can be efficiently achieved in quantum optical laboratories. Strong probe mode interacts successively with multiple signal-mode photons, each causing a conditional phase rotation in the probe mode. Subsequent homodyne measurement of the probe mode will project the signal mode photons into the desired entangled polarization-photon state. In addition, in order to show the power of prepared entanglement as a resource, we further propose an experimental scheme for teleporting an entangled three-photon polarization state, based on cross-phase modulation.

Key words: Cross-phase modulation; Homodyne measurement; Quantum teleportation

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0 Introduction

Entanglement in bipartite quantum systems is well understood and can be easily quantified. In contrast, multipartite quantum systems offer a much richer structure of entanglement with various types. Multipartite entanglement is a vital resource for numerous quantum information applications such as quantum computation, quantum communication, and quantum metrology. Therefore, different classifications of multipartite entanglement have been developed [1]. Further, quantum states with promising properties and applications have been identified and studied experimentally[2-4]. So far, many experimental schemes have focused on the observation of graph states[5], the Greenberger-Horne-Zeilinger (GHZ) states or the cluster states, which are, e. g., useful for one-way quantum computation[6]. Dicke states form another important group of states, which were first investigated with respect to light emission from a cloud of atoms[7] and have now come into the focus of both experimental realizations [2, 8] and theoretical studies [9-10]. W states, a subgroup of the Dicke states, first received attention triggered by the seminal work on three-qubit classification based on stochastic local operations and classical communication (SLOCC) by Ref. [1]. Particularly, by applying projective measurements on a few of their qubits, states of different SLOCC entanglement classes are obtained[8, 10]. These Dicke states can act as a rich resource of multipartite entanglement as required for quantum information applications.

Thus, crucial questions are how strongly and, in particular, in which way a quantum state is entangled. The implementation of choice for long-distance quantum communication will almost certainly be optical in practice, while quantum communication can be implemented with atoms or electrons. The advantage of using photons is that single-qubit operations can be implemented with high precision. In addition, photons have an intrinsic advantage in that they are better suited for communication over long distances and more robust against decoherence because of their fast speed and weak interaction with environment[11]. Therefore, people always choose their entangled states in the polarization degree of freedom to fulfill the tasks of quantum computation. As there

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is no efficient way of creating entanglement between photons by direct interaction, entangled photonic states are generally observed by supplementing the linear-optical approach with weak cross-Kerr nonlinearity and quantum non-demolition measurement[12] (QNDM). Two photonic fields undergo cross-phase modulation (XPM) when they are sent to a medium with cross-Kerr nonlinearity. Recently, there has been considerable interest in quantum information processing based on weak cross-Kerr nonlinearities[13-16].

1　Preparation of the entangled states

The essential component used in our scheme is a weak nonlinear interaction between a photon qubit and a probe coherent field. Let briefly review the useful weak cross-Kerr nonlinearity which has been used in Refs. [12, 17]. Suppose a nonlinear cross-Kerr interaction between a signal mode initially in a superposition of photon-number states $|\varphi\rangle_s = c_0|0\rangle_s + c_1|1\rangle_s$ and a *probe* mode initially in a coherent state $|\alpha\rangle_p$. The cross-Kerr interaction causes the combined signal-probe system to evolve as

$$|\varphi\rangle_s \otimes |\alpha\rangle_p \rightarrow c_0|0\rangle_s|\alpha\rangle_p + c_1|1\rangle_s|\alpha e^{i\theta}\rangle_p \tag{1}$$

where θ is induced by the nonlinearity. Conditioned on the results of QNDM, the signal state will be projected into a definite number state or superposition of number states with high fidelity.

Now let us study the generation of the polarization-entangled states among four modes using the weak-nonlinearity-based method. To illustrate the essential features of our scheme that allows a flexible observation of entanglement, it is sufficient to first consider how to produce symmetric four-photon Dicke state, GHZ states and W states. The schematic setup of our proposal is depicted in Fig. 1. The photon in the signal mode is prepared in a superposition state of horizontal and vertical polarizations while the probe beam is initially set in a strong coherent

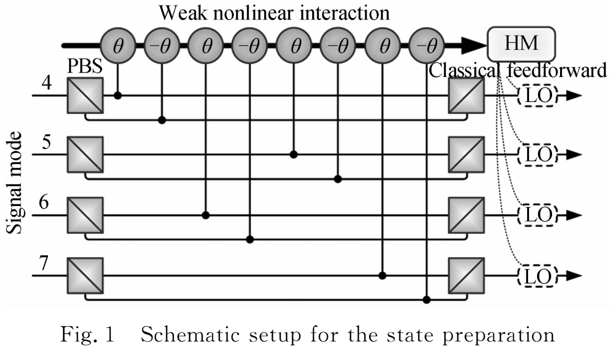

Fig. 1　Schematic setup for the state preparation

state. Without loss of generality, we have assumed the amplitude α is real and positive. Then the initial state of the system that consists of four photons in the signal modes and a coherent probe beam is

$$|\Psi\rangle_{in} = \bigotimes_{j=4}^{7} |+\rangle_j \otimes |\alpha\rangle \tag{2}$$

where $|+\rangle \equiv (|H\rangle + |V\rangle)/\sqrt{2}$. H and V denote respectively horizontal and vertical linear polarizations. The four-photon qubits in the signal modes are split individually on the PBSs into two spatial modes which transmit $|H\rangle$ and reflect $|V\rangle$. Horizontal and vertical modes then interact with probe mode in the Kerr nonlinear medium. After recombined by the last PBS array, the whole system evolves into the following state

$$\begin{aligned}|\Psi'\rangle = \frac{1}{4}(&\sqrt{6}|D_4^{(2)}\rangle|\alpha\rangle + 2|\overline{W_4^{(1)}}\rangle|\alpha e^{i2\theta}\rangle + \\ &2|W_4^{(1)}\rangle|\alpha e^{-i2\theta}\rangle + |HHHH\rangle_{4567}|\alpha e^{i4\theta}\rangle + \\ &|VVVV\rangle_{4567}|\alpha e^{-i4\theta}\rangle)\end{aligned} \tag{3}$$

where

$$|D_4^{(2)}\rangle \equiv (C_4^2)^{-1/2}\sum_l P_l|HHVV\rangle_{4567}$$

$$|W_4^{(1)}\rangle \equiv \frac{1}{2}\sum_l P_l|HVVV\rangle_{4567}$$

$$|W_4^{(3)}\rangle \equiv \frac{1}{2}\sum_l P_l|HHHV\rangle_{4567} = |\overline{W_4^{(1)}}\rangle \tag{4}$$

in which $\sum_l P_l(\cdots)$ means the sum over all permutations in the usual notation of polarization encoded logical photonic qubits, and $(C_n^k)^{-1/2}$ is a normalization factor with C_n^k as binomial coefficient. $|D_4^{(2)}\rangle$ is the four-photon Dicke state with two excitations that is symmetric under all permutations of qubits. Kiesel *et al* [8] experimentally observed the polarization-entangled four-photon Dicke state using the second-order emission process of collinear type-II spontaneous parametric down conversion. Generally, a symmetric N-qubit Dicke state [7, 18] with M excitations is the equally weighted superposition of all permutations of N-qubit product states with M logical H's and $(N-M)$ logical V's, here denoted by $|D_N^{(M)}\rangle$, and well-known examples are the N-qubit W states $|D_N^{(1)}\rangle$ (in the present notation $|W_N^{(1)}\rangle$) [2].

Strong probe mode interacts successively with multiple signal-mode photons, each causing a conditional phase rotation in the probe mode. Subsequent X-quadrature (X: position) homodyne measurement of the probe mode, as quantum scissors, will project the photons in the signal mode into the desired entangled states. Very convenient measurement tools in this case are the quadrature amplitudes of the optical field, which are continuous variables analogous to the position and momentum of a harmonic oscillator. For the convenience of the analysis of the X-quadrature homodyne measurement with α real, the state in Eq. (3) can be expanded in terms of the eigenstates of the $\hat{x}$ operator

$$|\Psi'\rangle_x \equiv \langle x|\Psi'\rangle = N_x[\sqrt{6}f(x,\alpha_0)|D_4^{(2)}\rangle + 2f(x,\alpha_0\cos 2\theta)|G_1\rangle + f(x,\alpha_0\cos 4\theta)|G_2\rangle] \quad (5)$$

where

$$\begin{aligned}|G_1\rangle &\equiv e^{i\varphi_2(x)}|\overline{W_4^{(1)}}\rangle + e^{-i\varphi_2(x)}|W_4^{(1)}\rangle \\ |G_2\rangle &\equiv e^{i\varphi_4(x)}|H^{\otimes 4}\rangle_{4567} + e^{-i\varphi_4(x)}|V^{\otimes 4}\rangle_{4567}\end{aligned} \quad (6)$$

Here the coefficients[15] are

$$\begin{aligned}f(x,\beta) &= (2\pi)^{-1/4}\exp[-(x-2\beta)^2/4] \\ \varphi_n(x) &= \alpha(x-2\alpha\cos n\theta)\sin n\theta\end{aligned} \quad (7)$$

and N_x is a normalization factor. Here $n=2,4$.

In Fig. 2, we plot the Gaussian functions of the homodyne measurement result x with $\alpha = 30\ 000$ and $\theta=0.01$, such as $f(x,\alpha)$, $f(x,\alpha\cos 2\theta)$ and $f(x,\alpha\cos 4\theta)$ which correspond to probability amplitudes associated with one of the three states $|D_4{}^{(2)}\rangle$, $|G_1\rangle$ and $|G_2\rangle$ in Eq. (5), respectively. We observe that $f(x,\beta)$ are three Gaussian curves with the peaks located at $2\alpha\cos n\theta$ where $n=0,2,4$, respectively. The neighboring peaks are separated by the distances $2d_j = 2\alpha\{\cos[(j-2)\theta] - \cos(j\theta)\}$ $(j=2,4)$, which are referred to as the distinguishabilities of the measurement. The distinguishabilities are approximately in proportion to $\alpha\theta^2$.

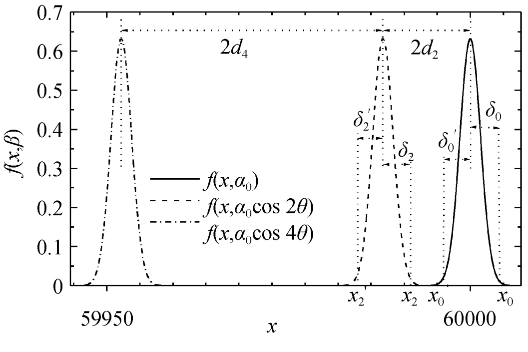

Fig. 2 Curves of the homodyne measurement result x

The X-quadrature homodyne measurement, which is near the center of one of the peaks, is subsequently implemented on the probe beam. Suppose, for example, the result is $x_2' = 2\alpha\cos(2\theta) - \delta_2'$, which is near the peak of $f(x,\alpha\cos 2\theta)$ as seen from Fig. 2. In this case, the polarization photon state in Eq. (5) is very close to the state $|G_1\rangle$. The fidelity[19] of the resulting state with respect to $|G_1\rangle$ is

$$F_{|G_1\rangle}(x_2') = |\langle G_1|\Psi'\rangle_x| \approx \left[1+\frac{1}{4}e^{-2d_4(d_4-\delta_2')}\right]^{-\frac{1}{2}} \quad (8)$$

For given α and θ, we can decide δ_2' from the needed high fidelity. Assuming that $\theta = 0.01$ and $\alpha = 30\ 000$, the fidelity in Eq. (8) will be 0.999 9 with $\theta_2' \approx 0.989d_4$. Similarly, if the measurement result is $x_2 = 2\alpha\cos(2\theta) + \delta_2$, the resulting fidelity with respect to $|G_1\rangle$ will be

$$F_{|G_1\rangle}(x_2) \approx [1+1.5e^{-2d_2(d_2-\delta_2)}]^{-1/2} \quad (9)$$

Assume the minimal acceptable fidelity of the resulting state is $F_{\min}$. By appropriately choosing δ_2 and δ_2', we can have $F_{|G_1\rangle}(x_2') = F_{|G_1\rangle}(x_2) = F_{\min}$. Then, as long as the measurement result x is in the regime of $x_2' < x < x_2$, we can get the polarization photon state $|G_1\rangle$ with the fidelity higher than $F_{\min}$. The success probability of such an event that is close to the state $|G_1\rangle$ is

$$P = \int_{x_2'}^{x_2}\langle x|\mathrm{tr}_{si}(|\Psi'\rangle\langle\Psi'|)|x\rangle dx \approx [\mathrm{erf}(\delta_2) + \mathrm{erf}(\delta'_2)]/4 \quad (10)$$

As the distinguishabilities increase, this probability of obtaining the state will approach to 1/2.

The phase shift $\varphi_2(x)$ in the state $|G_1\rangle$, associated with the components depends on the outcome of the homodyne measurement result x, is unwanted but can be simply removed by dynamic phase shifter combined with classical feed-forward process to transform the state $|G_1\rangle$ to

$$|\mathrm{GHZ}\rangle' \equiv (|W_4^{(1)}\rangle + |\overline{W_4^{(1)}}\rangle)/\sqrt{2} \quad (11)$$

which is independent of x. This does mean our homodyne measurement must be accurate enough such that we can determine $\varphi_2(x)$ precisely, otherwise this unwanted phase factor cannot be undone. That is to say, the success probability of obtaining the GHZ state given by Eq. (11) can be near 1/2.

Similar analysis can be made to obtain the state $|D_4{}^{(2)}\rangle$ and $|G_2\rangle$ with the desired fidelity when the measurement result x is near the centers of the $f(x,\alpha)$ and $f(x,\alpha\cos 4\theta)$ in Fig. 2, respectively. With regard to the state $|G_2\rangle$, the phase shift $\varphi_4(x)$ can be simply removed by dynamic phase shifter combined with classical feed-forward process to transform the state to

$$|\mathrm{GHZ}\rangle \equiv (|H^{\otimes 4}\rangle_{4567} + |V^{\otimes 4}\rangle_{4567})/\sqrt{2} \quad (12)$$

The success probability of obtaining the states $|D_4{}^{(2)}\rangle$ and $|\mathrm{GHZ}\rangle$ can be near 3/8 and 1/8, respectively. The entanglement of the symmetric Dicke states[7] is known to be very robust against photon loss. Eventually, one of the states $\{|D_4{}^{(2)}\rangle, |\mathrm{GHZ}\rangle', |\mathrm{GHZ}\rangle\}$ can be prepared by using X-quadrature homodyne measurement on the probe coherent field $|\alpha\rangle$ in the present scheme.

Now consider the other case that P-quadrature homodyne measurement is subsequently implemented on the probe beam in our scheme. The resulting four-photon state in the signal mode is then

$$|\Psi'\rangle_p = N(g_{4-}e^{i\tau_{4-}}|VVVV\rangle + 2g_{2-}e^{i\tau_{2-}}|W_4^{(1)}\rangle + \sqrt{6}g_0e^{i\tau_0}|D_4^{(2)}\rangle + 2g_{2+}e^{i\tau_{2+}}|\overline{W_4^{(1)}}\rangle + g_{4+}e^{i\tau_{4+}}|HHHH\rangle) \quad (13)$$

Here the coefficients are

$$g_0 = \pi^{-1/4}e^{-(1/2)p^2},\ \tau_0 = \sqrt{2}\alpha p$$
$$g_{j\pm} = \pi^{-1/4}e^{-(p\mp\sqrt{2}\alpha\sin j\theta)^2/2} \quad (14)$$
$$\tau_{j\pm} = \sqrt{2}\alpha(p \mp \frac{1}{\sqrt{2}}\sin j\theta)\cos j\theta$$

where $j=2,4$ and N is a normalization factor.

Next analysis is similar to previous one of X-quadrature homodyne measurement. We observe that g_0 and $g_{j\pm}$ are five Gaussian curves with the peaks located at $\sqrt{2}\alpha\sin(j\theta)$, respectively. The neighboring peaks are separated by the distances $2d_j = \sqrt{2}\alpha\ \sin\{j\theta) - \sin[(j-1)\theta]\}$, which are referred to as the distinguishabilities of the measurement. The distinguishabilities are approximately in proportion to $\alpha\theta$ in the case of P-quadrature homodyne measurement. However, the distinguishabilities of X-quadrature homodyne measurement above are approximately in proportion to $\alpha\theta^2$ As we have shown, as long as the probe beam has a sufficient amplitude α, we can work with much smaller phase shifts. This makes our distinguishability rather easier to implement. The following discussion will be made on the assumption that good distinguishability is already achieved, e. g. , with the realistic pumps. Analysis can be made to obtain the state $|W_4^{(1)}\rangle$ and $\overline{|W_4^{(1)}\rangle}$ with the desired fidelity when the measurement result p is near the centers of the g_{2-} and g_{2+}, respectively. Under P-quadrature homodyne measurement on the probe beam, the total success probability of obtaining the $|W_4{}^{(1)}\rangle$ state with a high fidelity is near 1/2 because a simple local polarization rotation would be performed to transform $\overline{|W_4^{(1)}\rangle}$ to $|W_4{}^{(1)}\rangle$. Of course, symmetric Dicke state, $|D_4{}^{(2)}\rangle$, can also be obtained when the measurement result p is near the centers of the g_0. In Ref. [20], Stockton *et al*. considered that Dicke state is difficult to reliably produce from an initially unentangled state. However, here we can propose the efficient scheme, in which it′s relatively easy to realize highly symmetric Dicke state of light fields.

In a word, GHZ-type, W-type states and Dicke state can all be prepared in the present scheme. The cascaded scheme given above can be generalized expediently in order to produce maximally entangled states of five or more photons. Multipartite entangled states, such as Dicke states and W states, can be useful for important quantum information processing (QIP) tasks. Maximally entangled states of three or more particles, socalled GHZ states, have been fascinating quantum systems to reveal the nonlocality of the quantum world[21]. GHZ entanglement between several particles is the most important feature of many such quantum communication and computation protocols[22]. For Dicke states, quantum telecloning, quantum secret sharing, open-destination teleportation[23] and quantum games[24] have been mentioned. For W-class states, quantum teleportation[25], dense coding[26], quantum telecloning[27], quantum key distribution[28], have been proposed. Besides, entanglement in the Dicke states is highly resilient against external perturbations and measurements on individual qubits[18, 29]. Gühne *et al*.[30] provided a versatile method to determine the decay of multiparticle entanglement for quantum states under the influence of decoherence. Gühne *et al*.'s study revealed that the Dicke state is the most robust state while the GHZ state is the most fragile state among the four-qubit states: GHZ, W, Cluster, Dicke. W states possess a robust two-party entanglement even after all the other $N2$ parties are traced out[1].

2 Teleportation based cross-Kerr nonlinearity

In order to show the power of prepared entanglement as a resource, we further propose an experimental scheme for teleporting an entangled three-photon polarization state with prepared four-photon GHZ state, *i. e.* given by Eq. (12), as quantum channel, based on the weak cross-Kerr nonlinearities, where photon 7 belongs to the sender Alice and the other three photons 4, 5 and 6 belong to the receiver Bob. Let us therefore consider an entangled three-photon polarization state

$$|\Psi\rangle_{123} = \alpha|HHH\rangle_{123} + \beta|VVV\rangle_{123} \quad (15)$$

which is given to Alice and has to be teleported to Bob. In Eq. (15), α and β are unknown parameters, and $|\alpha|^2+|\beta|^2=1$. Initially, the total state of the system can be expressed as

$$|\Psi\rangle=|\Psi\rangle_{123}\otimes|\mathrm{GHZ}\rangle_{4567} \quad (16)$$

Before proceeding with a detailed description of teleportation, it is worth to note that Bell-state measurement (BSM) as shown in Fig. 3 is employed so as to realize the teleportation scheme. The module '$-\theta$' denotes phase-shifter, N-PG denotes normal parity gate, and 45-PG denotes a parity gate with 45-PBS. The idea to BSM is rooted in the coalition of Ref. [16] and Ref. [31]. The whole chain of transformations of the present BSM setup in Fig. 3 can be summarized by the Table 1.

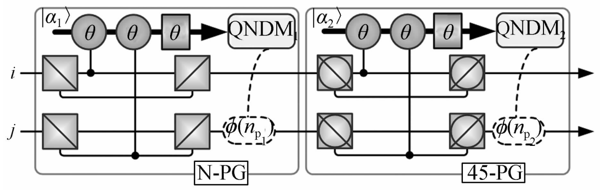

Fig. 3 Bell state measurement (BSM)

Table 1 Bell states corresponding to the results of QNDM on the probe beams 1 and 2

The state of modes i and j at the input of BSM	Result of QNDM_1	Result of QNDM_2
$\|\Psi^+\rangle$	$n_{p1}>0$	$n_{p2}=0$
$\|\Psi^-\rangle$	$n_{p1}>0$	$n_{p2}>0$
$\|\Psi^+\rangle$	$n_{p1}=0$	$n_{p2}=0$
$\|\Psi^-\rangle$	$n_{p1}=0$	$n_{p2}>0$

Note: n_p denotes the photon number via QNDM on the probe beam 1 or 2. The first thing to notice is that after a QNDM result $n_p>0$ we remove the unwanted phase factors that have arisen via classical feed-forward.

The table 1 illustrates the unambiguous discrimination of the four Bell states. Here, the four Bell states can be written as

$$\begin{aligned}|\Psi^{\pm}\rangle&\equiv(|HV\rangle\pm|VH\rangle)/\sqrt{2}\\|\Phi^{\pm}\rangle&\equiv(|HH\rangle\pm|VV\rangle)/\sqrt{2}\end{aligned} \quad (17)$$

To realize the teleportation, joint BSM on photons (2, 3) and photons (1, 7) are made by Alice at the first step, which will project photons 4, 5 and 6 into the following states

$$\begin{aligned}{}_{17}\langle\Phi^{\pm}|_{23}\langle\Phi^{\pm}|\Psi\rangle&=\frac{1}{2\sqrt{2}}(\alpha|HHH\rangle\pm^{23}\\&\pm^{17}\beta|VVV\rangle)_{456}\\{}_{17}\langle\Psi|_{23}\langle\Phi^{-}|Y\rangle&=\frac{1}{2\sqrt{2}}(\alpha|VVV\rangle\pm^{23}\\&\pm^{17}\beta|HHH\rangle)_{456}\end{aligned} \quad (18)$$

where $|\Psi^{\pm}\rangle_{ij}$ and $|\Phi^{\pm}\rangle_{ij}$ are the Bell states of the photon pair (i, j) described above, the superscripts 23 and 17 denote the Bell-state composed of photons (2, 3) and (1, 7), respectively. In order to complete the teleportation, Alice sends the results through the classical channel to Bob when the photons are detected. And after knowing the classical information of four bits coming from Alice, Bob can perform relevant unitary transformation on photons 4, 5 and 6 to obtain the original polarized state prepared on photons 1, 2 and 3 shown in Eq. (15). Table 2 gives Alice′s all different measurement results and Bob′s corresponding optical unitary operations. Thus we achieve the teleportation procedure successfully. In the ideal case, the total successful probability $P_{\text{total}}=(2\sqrt{2})^{-2}\times 8=1$, and the fidelity of the output state is 1.0.

Table 2 Bob′s unitary transformations corresponding to Alice′s measurement results

Alice′s result of joint BSM				Bob′s unitary transformations
BSM_{23}		BSM_{17}		
N-PG	45-PG	N-PG	45-PG	
$n_{p1}^{23}=0$	$n_{p2}^{23}=0$	$n_{p1}^{17}=0$	$n_{p2}^{17}=0$	$I_4\otimes I_5\otimes I_6$
$n_{p1}^{23}=0$	$n_{p2}^{23}>0$	$n_{p1}^{17}=0$	$n_{p2}^{17}=0$	$I_4\otimes I_5\otimes\sigma_6{}^z$
$n_{p1}^{23}=0$	$n_{p2}^{23}=0$	$n_{p1}^{17}=0$	$n_{p2}^{17}>0$	$I_4\otimes I_5\otimes\sigma_6{}^z$
$n_{p1}^{23}=0$	$n_{p2}^{23}>0$	$n_{p1}^{17}=0$	$n_{p2}^{17}>0$	$I_4\otimes I_5\otimes I_6$
$n_{p1}^{23}=0$	$n_{p2}^{23}=0$	$n_{p1}^{17}>0$	$n_{p2}^{17}=0$	$\sigma_4{}^x\otimes\sigma_5{}^x\otimes\sigma_6{}^x$
$n_{p1}^{23}=0$	$n_{p2}^{23}=0$	$n_{p1}^{17}>0$	$n_{p2}^{17}>0$	$i\sigma_4{}^y\otimes\sigma_5{}^y\otimes\sigma_6{}^y$
$n_{p1}^{23}=0$	$n_{p2}^{23}>0$	$n_{p1}^{17}>0$	$n_{p2}^{17}=0$	$i\sigma_4{}^y\otimes\sigma_5{}^y\otimes\sigma_6{}^y$
$n_{p1}^{23}=0$	$n_{p2}^{23}>0$	$n_{p1}^{17}>0$	$n_{p2}^{17}>0$	$\sigma_4{}^x\otimes\sigma 5^x\otimes\sigma_6{}^x$

3 Discussion

It is clear that the feasibility of the present schemes depends on the veracity in the homodyne measurement and the implementation of weak cross-Kerr nonlinearity. Analysis of the veracity in the X-quadrature homodyne measurement is presented[15]. The scheme of our present paper has some similar features with that of Ref. [15] in using such small-but-not-tiny Kerr nonlinearities. Such nonlinearities are potentially available today by using doped optical fibers and electromagnetically induced transparency (EIT). Strong nonlinearities are not a prerequisite to be able to perform quantum computation. Besides, phases shift, such as $\varphi_2(x)$ in connection with X-quadrature homodyne measurement, can then simply be eliminated via a classical feed-forward operation. This does mean our homodyne measurement must be accurate enough such that we can determine $\varphi_2(x)$ precisely, otherwise this unwanted phase factor cannot be undone. This is a technological challenge. Compared the paper with Refs. [32-34], our scheme does not the requirement that optical paths should be stable to subwave-length order for interferometric stability. It is also not necessary to have an ancilla photon, which was needed in Ref. [35]. This success probability is more than head and shoulders above one in the scheme of Ref. [8] based on linear optical elements.

4 Conclusion

In conclusion, the cascaded scheme is presented for preparing the entangled polarization-photon states, e. g., GHZ-type, W-type states and Dicke state, with the position or momentum quadrature homodyne measurement. Multipartite entangled states, such as Dicke state, GHZ and W states, can be useful for important QIP tasks. This scheme has the following distinct advantages: first, the scheme *only* involves weak cross-Kerr nonlinear interaction between probe coherent state and signal modes followed by the homodyne detection which can be made much more efficient than the single-photon detection. It is not necessary that the cross-Kerr nonlinearity is very large, as long as the coherent light is bright enough in order to amplify the effect of the weak cross-Kerr nonlinearities. For instance, calculations for EIT systems in NV-diamond have shown potential phase shifts of order of magnitude of $\theta=0.01$. With $\theta=0.01$ the probe beam must have an amplitude of at least 10^5, which is physically reasonable with current technology. So far, this scheme uses only the basic tools in quantum optical laboratories and can be implemented in the regime of the weak cross-Kerr nonlinearity. Second, we employ homodyne measurement which a highly efficient non-absorbing single photon number resolving detector that does not absorb the photon from the signal mode. Third, the preparation scheme takes advantage of the higher success probability and near perfect fidelity. As the measurement distinguishability increases, the total success probability achieving W state will approach to 1/2 due to using P-quadrature homodyne measurement, and the success probability obtaining Dicke state is near 3/8 with either X- or P-quadrature homodyne measurement. Finally, in addition to preparing the GHZ and W class states among four modes, plentiful new kinds of entangled state can be generated with this scheme, e. g. highly symmetric Dicke state. These Dicke states can act as a rich resource of multipartite entanglement as required for quantum information applications. GHZ and W class states cannot be transformed into one another via SLOCC and not even by entanglement catalysis. However, the Dicke state can be projected into both classes by a local operation. In order to show the power of prepared entanglement as a resource, we further propose an experimental scheme for teleporting an entangled three-photon polarization state with prepared four-photon GHZ state as quantum channel, based on the weak cross-Kerr nonlinearities. More importantly, the employed BSM-based weak cross-Kerr nonlinearity is extremely simple and also robust against detector inefficiencies, and provides a reliable method to unambiguously discriminate among the four polarization-entangled photon pairs. The successful probability and fidelity of this teleportation scheme both reach theoretically 1.0.

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基于交叉相位调制制备四光子偏振纠缠态及其在隐形传态上的运用

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摘 要:利用克尔介质、偏振分束器、半波片和对强相干探测场的零拍探测,呈现了一个关于制备四光子偏振Diche态、GHZ态和W态的方案,当前量子光学实验技术条件均能有效满足该方案的要求. 强的探测模相继和多个信号模光子相互作用,每次对于探测模而言,都会产生一个相位旋转. 接下来,对探测模利用零拍探测,信号模可以投影得到想要的光子偏振纠缠态. 此外,为了展现所制备的纠缠态作为重要的量子信息资源的价值,基于交叉相位调制进一步提出了一个隐形传送三光子偏振纠缠态的实验方案.

关键词:交叉相位调制;零拍探测;量子隐形传态