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Remote State Preparation of Arbitrary Three-qubit State via Cluster State and Bell State

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Abstract: A novel scheme for remote preparation of an arbitrary three-qubit state using four-qubit cluster state and Bell state is proposed. The three-particle state can be perfectly prepared if the sender (Alice) performs the orthogonal complete measurement on her particles and the receiver (Bob) introduces an appropriate unitary transformation on his particles. For the two cases of remote state preparation, two different projective measurement bases are constructed at sender's side and the corresponding success probabilities are calculated. The probability of success regarding this preparation scheme is calculated in both general and some particular cases. It is that in general such remote state preparation can be realized with a probability of 1/8. But in several special cases, the probability of success can be improved to 1/4, 1/2 or even 1.

Key words: Remote state preparation; Orthogonal complete measure bases; Arbitrary three-qubit state

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0 Introduction

In the process of developing quantum information theory, entanglement plays a key role in most quantum information process. Quantum entanglement is a valuable resource for the implementation of quantum computation and quantum communication protocols, like quantum teleportation[1-3], quantum key distribution [4], quantum secure direct communication[5], dense coding[6-7], quantum computation, remote state preparation[8-15] and so on. If a sender (Alice) wants to transmit an unknown quantum state to a receiver (Bob), they may use teleportation. In remote state preparation (RSP), Alice is assumed to know fully the transmitted state to be prepared by Bob, while in the teleportation neither Alice nor Bob has knowledge of the transmitted state.

RSP[8] has been extended by many authors to various cases in recent years. In the theoretical aspect, Devetak and Berger have proposed a low entanglement RSP protocol using more classical bits but less entanglement bits. Berry and Sanders have proved that it is possible to remotely prepare ensembles of mixed states using communication equal to the Holevo information for this ensemble. The authors of refs have studied the oblivious RSP protocols in which no information about the state to be prepared can be retrieved from the classical message. Ye *et. al.* have focused on faithful RSP using finite classical bits and a shared non-maximally entangled state. Kurucz *et. al.* have discussed exact deterministic RSP with minimal classical communication in quantum systems of continuous variables corresponding to infinite dimensional Hilbert space. On the experimental side, over the past several years the implementation of RSP has been reported with liquid-state nuclear magnetic resonance technology and optical systems.

1 RSP of an arbitrary three-qubit state

Suppose that a sender Alice wants to help a remote receiver Bob prepare an arbitrary three-qubit state described as

$$\begin{aligned}|\chi\rangle = & a_0 e^{i\theta_0}|000\rangle + a_1 e^{i\theta_1}|001\rangle + a_2 e^{i\theta_2}|010\rangle + \\ & a_3 e^{i\theta_3}|011\rangle + a_4 e^{i\theta_4}|100\rangle + a_5 e^{i\theta_5}|101\rangle + \\ & a_6 e^{i\theta_6}|110\rangle + a_7 e^{i\theta_7}|111\rangle \end{aligned} \tag{1}$$

where the real coefficients $a_i \geqslant 0 (i=0,1,\cdots,7)$ and $\theta_j \in [0,2\pi]$ $(j=0,1,\cdots,7)$ with the normalization

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condition $\sum_{i=0}^{7} a_i = 1$. Here, the state $|\chi\rangle$ is known partially to Alice. The receiver does not have any knowledge about the original state $|\chi\rangle$ at all. In this situation, Alice can help Bob to reconstruct the original state.

The sender and the receiver initially share a four-qubit cluster state and a Bell state as the quantum channel, which can be written as

$$|\Omega\rangle = |C_4\rangle \otimes |\varphi^+\rangle = \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)_{A_1B_1A_2B_2} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_3B_3} \tag{2}$$

where particles (A_1, A_2, A_3) are in the possession of Alice, while particles (B_1, B_2, B_3) belong to the Bob.

In order to help Bob re-establish the original state in his location, Alice is required to perform a three-particle joint projective measurement on her own particles based on the knowledge of the original state they have. Alice must make a measurement on her three particles (A_1, A_2, A_3). The projective measurement basis chosen by Alice is a set of mutually orthogonal basis vectors $\{|\varphi^1\rangle, |\varphi^2\rangle, |\varphi^3\rangle, |\varphi^4\rangle, |\varphi^5\rangle, |\varphi^6\rangle, |\varphi^7\rangle, |\varphi^8\rangle\}$. We find that this basis can be related to the computation basis $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$ by the following relations

$$\begin{pmatrix} |\varphi^1\rangle \\ |\varphi^2\rangle \\ |\varphi^3\rangle \\ |\varphi^4\rangle \\ |\varphi^5\rangle \\ |\varphi^6\rangle \\ |\varphi^7\rangle \\ |\varphi^8\rangle \end{pmatrix} = \begin{pmatrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ -x_1 & x_0 & -x_3 & x_3 & x_5 & -x_4 & -x_7 & x_6 \\ x_2 & -x_3 & -x_0 & x_1 & x_6 & x_7 & -x_4 & -x_5 \\ x_3 & x_2 & -x_1 & -x_0 & x_7 & -x_6 & x_5 & -x_4 \\ x_4 & -x_5 & -x_6 & -x_7 & -x_0 & x_1 & x_2 & x_3 \\ x_5 & x_4 & -x_7 & x_6 & -x_1 & -x_0 & -x_3 & x_2 \\ x_6 & x_7 & x_4 & -x_5 & -x_2 & x_3 & -x_0 & -x_1 \\ x_7 & -x_6 & x_5 & x_4 & -x_3 & -x_2 & x_1 & -x_0 \end{pmatrix} \begin{pmatrix} e^{-i\theta_0}|000\rangle \\ e^{-i\theta_1}|001\rangle \\ e^{-i\theta_2}|010\rangle \\ e^{-i\theta_3}|011\rangle \\ e^{-i\theta_4}|100\rangle \\ e^{-i\theta_5}|101\rangle \\ e^{-i\theta_6}|110\rangle \\ e^{-i\theta_7}|111\rangle \end{pmatrix} \tag{3}$$

The set of basis vectors $|\varphi^m\rangle\{m \in (1, 2, \cdots, 8)\}$ form complete and orthogonal bases in eight-dimensional Hilbert space, respectively. Under these two sets of bases, the whole quantum system consisting of the channel can be rewritten as

$$\begin{aligned} |\Omega\rangle = {} & \frac{1}{2\sqrt{2}}(|000000\rangle + |000011\rangle + |001100\rangle + |001111\rangle + |110000\rangle + |110011\rangle - |111100\rangle - |111111\rangle)_{A_1B_1A_2B_2} = \\ & \frac{1}{2\sqrt{2}}[|\varphi^1\rangle_{A_1A_2A_3}(a_0 e^{i\theta_0}|000\rangle + a_1 e^{i\theta_1}|001\rangle + a_2 e^{i\theta_2}|010\rangle + a_3 e^{i\theta_3}|011\rangle + \\ & a_4 e^{i\theta_4}|100\rangle + a_5 e^{i\theta_5}|101\rangle - a_6 e^{i\theta_6}|110\rangle - a_7 e^{i\theta_7}|111\rangle)_{B_1B_2B_3} + |\varphi^2\rangle_{A_1A_2A_3}(-a_1 e^{i\theta_0}|000\rangle + \\ & a_0 e^{i\theta_1}|001\rangle - a_3 e^{i\theta_2}|010\rangle + a_2 e^{i\theta_3}|011\rangle + a_5 e^{i\theta_4}|100\rangle - a_4 e^{i\theta_5}|101\rangle + a_7 e^{i\theta_6}|110\rangle - \\ & a_6 e^{i\theta_7}|111\rangle)_{B_1B_2B_3} + \cdots + |\varphi^8\rangle_{A_1A_2A_3}(a_7 e^{i\theta_0}|000\rangle - a_6 e^{i\theta_1}|001\rangle + a_5 e^{i\theta_2}|010\rangle + a_4 e^{i\theta_3}|011\rangle - \\ & a_3 e^{i\theta_4}|100\rangle - a_2 e^{i\theta_5}|101\rangle - a_1 e^{i\theta_6}|110\rangle + a_0 e^{i\theta_7}|111\rangle)_{B_1B_2B_3}]) \end{aligned} \tag{4}$$

After the measurements, Alice transmits some classical information about measurement outcomes to the receiver Bob. Bob then needs to reconstruct the original state on his particles (B_1, B_2, B_3) conditioned on the classical information from Alice. According to Eq. (4), if the result of Alice's projective measurement is $|\varphi^1\rangle$, Bob knows that he has a chance to realize the RSP by implementing a suitable unitary operation U on his three particles. In this case, it is easy to verify that Bob can always re-establish the original state with the same probability when any one of the eight possible outcomes of Alice's projective measurement occurs. As an example, we consider the case that Alice's projective measurement result is $|\varphi^1\rangle$; the state of particles (B_1, B_2, B_3), as shown by Eq. (4) will collapse into

$$(a_0 e^{i\theta_0}|000\rangle + a_1 e^{i\theta_1}|001\rangle + a_2 e^{i\theta_2}|010\rangle + a_3 e^{i\theta_3}|011\rangle + a_4 e^{i\theta_4}|100\rangle + a_5 e^{i\theta_5}|101\rangle - a_6 e^{i\theta_6}|110\rangle - a_7 e^{i\theta_7}|111\rangle)_{B_1B_2B_3} \tag{5}$$

After that if Alice communicates to Bob of her actual measurement outcome via a classical channel, then, Bob will be able to apply the following unitary tra- nsformation on his particles (B_1, B_2, B_3). The resulting

$$U^1_{B_1B_2B_3} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{6}$$

state of Bob's particles will be the original state

$|\chi\rangle$. However, when the measurement other outcomes, the remote state preparation cannot be successful.

2 Improvement of the success probability

However, when Alice′s measurement outcome is $|\varphi^2\rangle, \cdots$, or $|\psi^8\rangle$, the remote state preparation cannot be successful. From the previous analyses, we can find that Bob can get the original state with the total probability of successful RSP is 1/8. But for some special values of the coefficients in Eq. (1) or in Eq. (4), the probability of successful RSP can be improved. We examine the special case as follows

Case1: $a_0^2=a_1^2, a_2^2=a_3^2, a_4^2=a_5^2, a_6^2=a_7^2$

In this case, if Alice′s measurement outcome is $|\varphi^2\rangle$, the remote state preparation can also be successful, because the state of particles (B_1, B_2, B_3), as shown by Eq. (4), will collapse into

$$\begin{aligned}(&-a_1 e^{i\theta_0}|000\rangle+a_0 e^{i\theta_1}|001\rangle-a_3 e^{i\theta_2}|010\rangle+\\ &a_2 e^{i\theta_3}|011\rangle+a_5 e^{i\theta_4}|100\rangle-a_4 e^{i\theta_5}|101\rangle+\\ &a_7 e^{i\theta_6}|110\rangle-a_6 e^{i\theta_7}|111\rangle)_{B_1B_2B_3}\end{aligned} \tag{7}$$

Therefore, Bob will be able to apply the following unitary transformation $U^2_{B_1B_2B_3}$ on his particles (B_1, B_2, B_3). The resulting state of Bob′s particles will be the original state $|\chi\rangle$. Thus, Bob can get the original state with a higher success probability of 1/4 in this case.

$$U^2_{B_1B_2B_3}=\begin{pmatrix}0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&1&0\end{pmatrix} \tag{8}$$

Case2: $a_0^2=a_1^2=a_2^2=a_3^2=a_4^2=a_5^2=a_6^2=a_7^2$

As in the first case, we can show that the remote state preparation can be successfully. In this case, it is also not difficult for Bob to prepare the three-particle state $|\chi\rangle$ by performing the suitable unitary operation determined by the outcome of Alice′s measurement. Hence, the remote state preparation can be implemented, and so the probability of success in this particular case will be 100%.

Here, we give anther orthogonal complete measure bases of Alice. It is a set of mutually orthogonal basis vectors $\{|\varphi^1\rangle, |\varphi^2\rangle, |\varphi^3\rangle, |\varphi^4\rangle, |\varphi^5\rangle, |\varphi^6\rangle, |\varphi^7\rangle, |\varphi^8\rangle\}$. We find the basis can be related to the compu-tation basis $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$ by the following relations

$$\begin{pmatrix}|\varphi^{1'}\rangle\\|\varphi^{2'}\rangle\\|\varphi^{3'}\rangle\\|\varphi^{4'}\rangle\\|\varphi^{5'}\rangle\\|\varphi^{6'}\rangle\\|\varphi^{7'}\rangle\\|\varphi^{8'}\rangle\end{pmatrix}=\begin{pmatrix}x_0&x_1&x_2&x_3&x_4&x_5&x_6&x_7\\-x_1&x_0&-x_3&x_2&-x_5&x_4&-x_7&x_6\\-x_2&x_3&x_0&-x_1&x_6&-x_7&-x_4&x_5\\-x_3&-x_2&x_1&x_0&-x_7&-x_6&x_5&x_4\\-x_4&x_5&-x_6&x_7&x_0&-x_1&x_2&-x_3\\-x_5&-x_4&x_7&x_6&x_1&x_0&-x_3&-x_2\\-x_6&x_7&x_4&-x_5&-x_2&x_3&x_0&-x_1\\-x_7&-x_6&-x_5&-x_4&x_3&x_2&x_1&x_0\end{pmatrix}\begin{pmatrix}e^{-i\theta_0}|000\rangle\\e^{-i\theta_1}|001\rangle\\e^{-i\theta_2}|010\rangle\\e^{-i\theta_3}|011\rangle\\e^{-i\theta_4}|100\rangle\\e^{-i\theta_5}|101\rangle\\e^{-i\theta_6}|110\rangle\\e^{-i\theta_7}|111\rangle\end{pmatrix} \tag{9}$$

The set of basis vectors $|\varphi^n\rangle\{n\in(1,2,\cdots,8)\}$ form complete and orthogonal bases in eight-dimensional Hilbert space, respectively. Under these two sets of bases, the whole quantum system consisting of the channel can be rewritten as

$$\begin{aligned}|\Omega'\rangle=&\frac{1}{2\sqrt{2}}(|000000\rangle+|000011\rangle+|001100\rangle+\\ &|001111\rangle+|110000\rangle+|110011\rangle-|111100\rangle-\\ &|111111\rangle)_{A_1B_1A_2B_2}+a_5 e^{i\theta_5}|101\rangle-a_6 e^{i\theta_6}|110\rangle-\\ &a_7 e^{i\theta_7}|111\rangle)_{B_1B_2B_3}+|\varphi^{2'}\rangle_{A_1A_2A_3}(-a_1 e^{i\theta_0}|000\rangle+\\ &a_0 e^{i\theta_1}|001\rangle-a_3 e^{i\theta_2}|010\rangle+a_2 e^{i\theta_3}|011\rangle-\\ &a_5 e^{i\theta_4}|100\rangle+a_4 e^{i\theta_5}|101\rangle+a_7 e^{i\theta_6}|110\rangle-\\ &a_6 e^{i\theta_7}|111\rangle)_{B_1B_2B_3}+\cdots+|\varphi^{8'}\rangle_{A_1A_2A_3}(-a_7 e^{i\theta_0}|000\rangle-\\ &a_6 e^{i\theta_1}|001\rangle-a_5 e^{i\theta_2}|010\rangle-a_4 e^{i\theta_3}|011\rangle+\\ &a_3 e^{i\theta_4}|100\rangle+a_2 e^{i\theta_5}|101\rangle-a_1 e^{i\theta_6}|110\rangle-\\ &a_0 e^{i\theta_7}|111\rangle)_{B_1B_2B_3}])\end{aligned} \tag{10}$$

If $a_0^2=a_1^2=a_2^2=a_3^2=a_4^2=a_5^2=a_6^2=a_7^2$, in this case, it is also not difficult for Bob to prepare the three-particle state $|\chi\rangle$ by performing the suitable unitary operation determined by the outcome of Alice′s measurement. Hence, the remote state preparation can be implemented, and so the probability of success in this particular case will be 100%.

3 Conclusion

In summary, we presented a scheme for remote preparation of a three-particle state using four-qubit cluster state and Bell state as the quantum channel. The three-particle state can be perfectly prepared if the sender performs the orthogonal complete measurement and the receiver introduces appropriate unitary transformation. We also calculated the probability of success of the RSP scheme in general and some particular cases. Although the probability of success for a general state is only 1/8, it has been shown that for certain states, the probability of success can be improved to 1/4, 1/2 or even 1. Furthermore, we pointed out that the probability of success may also depend on the sender′s choice of measurement basis. If some appropriate measurement is chosen, RSP can be achieved with the maximum probability, but the measurements of Alice should do to implement the protocol is not that simple. The main difficulties on its experimental realization lie in the measurement. Actually, this is the bottleneck of this protocol. The main difficulties on experimental realization lie in the measurements (3) and (9). Again, a clear and sincere statement of this fact is no drawback to the manuscript, it is just the truth. However, our study is only theoretical work. We hope that the scheme proposed here will be experimentally realized in the near future.

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基于Cluster态和Bell态的任意三粒子态远程制备

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摘 要:本文提出了一个新颖的基于四粒子cluster态和Bell态制备任意三粒子远程制备方案.在发送者(Alice)对自己手中的粒子做正交完备测量,接受者(Bob)对自己手中的粒子做适当的幺正变换后,任意三粒子远程制备成功.对于Alice的两种不同的正交完备基测量的情况,分别计算了远程制备成功的概率.另外,本方案成功制备的概率在一般情况和一些特殊情况下是可以计算的.分析结果表明:在一般情况下,远程态制备可以以1/8的概率实现;但在一些特殊情况下,成功的概率可以提高到1/4、1/2,甚至1.

关键词:远程制备;正交完备基;任意三粒子态