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Quantum Entanglement Properties in Multimode Degenerate Multi-photon Tavis-Cummings Model

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Abstract: The evolution properties of quantum entanglement in a system of multimode coherent field interacting with two identical two-level atoms through degenerating multi-photon process are investigated using the Von Neumann quantum reduced entropy theory, and the analytical expression of quantum entanglement and the numerical calculation results for two-mode field interacting with the atoms are obtained. The results show that: the quantum entanglement will strengthen with the enhancement of the photon degeneracy; the periodicity of the quantum entanglement will become more and more apparent with the increasing of the average photon number; when the field and the atoms are far from resonance, the quantum entanglement will decrease with the increase of the frequency detuning; when the frequency detuning is large enough, the field and the atoms are nearly always in entangled states. These results are useful for the preparation of the entangled states or pure states and for quantum information in optics systems.

Key words: Quantum entanglement; Multimode field; Degenerate multi-photon process; Tavis-Cummings model

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0 Introduction

Quantum entanglement is one of the most characteristic properties that make quantum theory distinct from classical theory. It not only serves to demonstrate theoretically fundamental quantum properties far beyond the conceptual framework defined by classical physics, but also forms a fundamental resource for quantum information processing and has promising practical applications in quantum computation[1], quantum dense coding[2], quantum cryptography and quantum teleportation[3] and so on. Most of the research in quantum information processing is based on the entanglement generation of quantum two-level systems. Tavis-Cummings Model (TCM)[4] (or Dicke model[5]) describes the simplest fundamental interaction between a single mode quantized field and a collection of N atoms under the usual two-level and rotating wave approximations. The quantum properties in TCM have been extensively studied. Entanglement properties of two entangled atoms without rotating wave approximation have been studied by JIANG Dao-lai et al[6]. The entanglement between two atoms in an overdamped cavity injected with squeezed vacuum was investigated by LI Gao-xiang et al[7-8]. CAI Xun-ming et al. studied the population inversion of two moving atoms in a cavity[9]. The quantum entanglement of two-photon[10] and multi-photon[11] TCM also have been studied. We have studied the quantum entanglement in the Bell state entangled atoms[12] and the multi-mode (q-mode) coherent light field multi-photon Jaynes-Cummings Model (JCM)[13] and considered the influence of frequency detuning on the quantum entanglement[14]. However, up to now, less attention has been paid to the study of the quantum entanglement of the multi-mode field multi-photon TCM.

In this paper, we investigate the properties of quantum entanglement in the system of multi-mode coherent light field interacting with two identical two-level atoms through any N_j-degree degenerate multi-photon process by utilizing the quantum entropy theory. Our attention focuses on the discussion of the influences of the photon degeneracy, the initial average photon number and the frequency detuning on the evolution properties of quantum entanglement.

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1 The model and its solution

The system considered here consists of two identical two-level atoms interacting with multimode field through any N_j-degenerate multi-photon transition process, and the dipole-dipole interaction between the atoms is neglected and there is the same coupling of the two atoms interacting with the q-mode field. Under these conditions, the Hamiltonian can be written in the rotating wave approximation (RWA) and the dipole approximation as ($\hbar=1$)

$$H=\sum_{j=1}^{q}\omega_j a_j^+ a_j+\omega_0 S_z+g[S_+(\prod_{j=1}^{q}a_j^{N_j})+(\prod_{j=1}^{q}a_j^{+N_j})S_-] \quad (1)$$

where $S_z=\sum_{i=1}^{2}S_z^i$, $S_+=S_+^1+S_+^2 e^{i\xi}$, $S_-=S_-^1+S_-^2 e^{-i\xi}$, a_j^+ (a_j) is the photon creation (annihilation) operator of the jth field mode of frequency ω_j ($j=1,2,\cdots,q$), ω_0 is the atomic transition frequency, S_z^i ($i=1,2$) is the atomic inversion operator for the ith atom, $S_\pm^i$ are the atomic "spin-flip" operators for the ith atom, ξ is the relative phase of the atomic transition, N_j is the degree of degenerate of the jth field mode, and g is the coupling constant between the atom and the q-mode light fields.

The Hamiltonian (1) can be decomposed as

$$H=H_0+H_I \quad (2a)$$

$$H_0=\sum_{j=1}^{q}(\omega_j a_j^+ a_j)+\sum_{j=1}^{q}(\omega_j N_j)S_z \quad (2b)$$

$$H_I=[\omega_0-(\sum_{j=1}^{q}\omega_j N_j)]S_z+g[S_+(\prod_{j=1}^{q}a_j^{N_j})+(\prod_{j=1}^{q}a_j^{+N_j})S_-] \quad (2c)$$

set $\Delta_q=\omega_0-\sum_{j=1}^{q}\omega_j N_j$ denotes the detuning of the field mode from the two-level atom. Using the standard techniques in Ref. [15], we can obtain the following time-evolution operator in the interaction picture

$$U_I(t)=\begin{pmatrix} C & -iD & 0 & 0 \\ -iS & T & 0 & 0 \\ 0 & 0 & C & -iDe^{i\xi} \\ 0 & 0 & -iSe^{-i\xi} & T \end{pmatrix} \quad (3)$$

Here we have written the operators as

$$\begin{cases} C=\cos(At)-i\left(\frac{\Delta_q}{2}\right)\frac{\sin(At)}{A} \\ S=g\prod_{j=1}^{q}a_j^{+N_j}\ \frac{\sin(At)}{A} \\ D=g\prod_{j=1}^{q}a_j^{N_j}\ \frac{\sin(Bt)}{B} \\ T=\cos(Bt)+i\left(\frac{\Delta_q}{2}\right)\frac{\sin(Bt)}{B} \\ A=\sqrt{\left(\frac{\Delta_q}{2}\right)^2+g^2\prod_{j=1}^{q}a_j^{N_j}\prod_{j=1}^{q}a_j^{+N_j}} \\ B=\sqrt{\left(\frac{\Delta_q}{2}\right)^2+g^2\prod_{j=1}^{q}a_j^{+N_j}\prod_{j=1}^{q}a_j^{N_j}} \end{cases} \quad (4)$$

We chose the atoms be prepared in coherent superposition state of the excited state and the ground state and the q-mode field be prepared in coherent state at the initial time, that is

$$|\psi_a(0)\rangle=\cos\frac{\theta}{2}|e,e\rangle+e^{-i\phi}\sin\frac{\theta}{2}|g,g\rangle \quad (5)$$

$$|\psi_f(0)\rangle=\sum_{n_1,n_2,\cdots,n_q=0}^{\infty}P(n_1)P(n_2)\cdots P(n_q)|n_1,n_2,\cdots,n_q\rangle \quad (6)$$

where $P(n_j)=\exp\left(-\frac{\bar{n}_j}{2}\right)\frac{\alpha_j^{n_j}}{\sqrt{n_j!}}$, the subscript f and a stand for field and atom respectively, $0<\theta<\pi$ denotes the atomic distribution, $0<\phi<2\pi$ is the phase of the atom dipole, $\alpha_j=\sqrt{\bar{n}_j}\exp(i\varphi_j)$, $\bar{n}_j$ and φ_j ($0\leqslant\varphi_j<2\pi$) represent the initial average photon number and the direction angle of the excitation for j mode field , respectively. Then the initial combined density operator of the system is

$$\rho(0)=|\psi_a(0)\rangle\langle\psi_a(0)|\otimes|\psi_f(0)\rangle\langle\psi_f(0)| \quad (7)$$

along with the time evolution, the combined density operator of the system at any time $t>0$ is given by

$$\rho(t)=\begin{pmatrix} |C\rangle\langle C| & |C\rangle\langle S| & |C\rangle\langle D| & |C\rangle\langle T| \\ |S\rangle\langle C| & |S\rangle\langle S| & |S\rangle\langle D| & |S\rangle\langle T| \\ |D\rangle\langle C| & |D\rangle\langle S| & |D\rangle\langle D| & |D\rangle\langle T| \\ |T\rangle\langle C| & |T\rangle\langle S| & |T\rangle\langle D| & |T\rangle\langle T| \end{pmatrix} \quad (8)$$

The corresponding coefficients are given by

$$\begin{cases} |C\rangle=\cos\frac{\theta}{2}\sum_{n_1,n_2,\cdots,n_q=0}^{\infty}[\prod_{j=1}^{q}q(n_j)]\exp[i(\sum_{j=1}^{q}n_j\varphi_j)]\left[\cos(\Omega^+t)-i\left(\frac{\Delta_q}{2}\right)\frac{\sin(\Omega^+t)}{\Omega^+}\right]|n_1,n_2,\cdots,n_q\rangle \\ |S\rangle=-ig\cos\frac{\theta}{2}\sum_{n_1,n_2,\cdots,n_q=0}^{\infty}[\prod_{j=1}^{q}q(n_j)]\exp[i(\sum_{j=1}^{q}n_j\varphi_j)]\prod_{j=1}^{q}[(n_j+1)(n_j+2)\cdots(n_j+N_j)]^{1/2}\frac{\sin(\Omega^+t)}{\Omega^+}\cdot |n_1+N_1,n_2+N_2,\cdots,n_q+N_q\rangle \\ |D\rangle=-ige^{i(\xi-\phi)}\sin\frac{\theta}{2}\sum_{n_1,n_2,\cdots,n_q=0}^{\infty}[\prod_{j=1}^{q}q(n_j)]e^{i(\sum_{j=1}^{q}n_j\varphi_j)}\prod_{j=1}^{q}[(n_j-N_j+1)(n_j-N_j+2)\cdots(n_j-1)n_j]^{1/2}\cdot \frac{\sin(\Omega^-t)}{\Omega^-}|n_1-N_1,n_2-N_2,\cdots,n_q-N_q\rangle \\ |T\rangle=e^{-i\phi}\sin\frac{\theta}{2}\sum_{n_1,n_2,\cdots,n_q=0}^{\infty}[\prod_{j=1}^{q}q(n_j)]\exp[i(\sum_{j=1}^{q}n_j\varphi_j)][\cos(\Omega^-t)+i\left(\frac{\Delta_q}{2}\right)\frac{\sin(\Omega^-t)}{\Omega^-}|n_1,n_2,\cdots,n_q\rangle \end{cases} \quad (9)$$

With

$$q(n_j)=\exp\left(-\frac{\bar{n}_j}{2}\right)\frac{\bar{n}_j^{(n_j/2)}}{\sqrt{n_j}},\Omega^+=\left\{\left(\frac{\Delta_q}{2}\right)^2+g^2\left[\prod_{j=1}^{q}(n_j+1)(n_j+2)(n_j+3)\cdots(n_j+N_j)\right]\right\}^{1/2},$$

$$\Omega^-=\left\{\left(\frac{\Delta_q}{2}\right)^2+g^2\left[\prod_{j=1}^{q}(n_j-N_j+1)\cdot(n_j-N_j+2)\cdots(n_j-1)n_j\right]\right\}^{1/2}$$

So we can get the reduced density matrix of the coherent field

$$\rho_f(t)=|C\rangle\langle C|+|S\rangle\langle S|+|D\rangle\langle D|+|T\rangle\langle T| \quad (10)$$

In order to derive a calculation formalism of the field entropy, we must obtain the eigenvalues of the reduced density matrix given by Eqs. (10). Knight and his co-workers (see Ref. [15]) have developed a general method to calculate the various field eigenstates and eigenvalues in a simple way; here we will use their method to calculate the eigenvalues. Assume the eigenstate of the reduced density operator can be written in the following form

$$|\pi_f(t)\rangle=\lambda_1|C\rangle+\lambda_2|S\rangle+\lambda_3|D\rangle+\lambda_4|T\rangle \quad (11)$$

and we write the eigen equation as

$$\rho_f(t)|\pi_f(t)\rangle=\pi_f(t)|\pi_f(t)\rangle \quad (12)$$

where $\pi_f(t)$ is an eigenvalue of the reduced density operator.

Applying the density matrix given by Eq. (10) to the eigenstate Eq. (12), we get the eigenvalues of the reduced density matrix of the field

$$\begin{cases}\pi_f^1(t)=-\frac{\theta_1}{4}+\frac{1}{2}(k_1+k_2+k_3)\\ \pi_f^2(t)=-\frac{\theta_1}{4}+\frac{1}{2}(k_1-k_2-k_3)\\ \pi_f^3(t)=-\frac{\theta_1}{4}+\frac{1}{2}(-k_1+k_2-k_3)\\ \pi_f^4(t)=-\frac{\theta_1}{4}+\frac{1}{2}(-k_1-k_2+k_3)\end{cases} \quad (13)$$

where the coefficients are given by

$$k_1=\left[\frac{2}{3}\sqrt{a^2+12c}\cos\varphi-\frac{2a}{3}\right]^{1/2}$$

$$k_2=\left[\frac{2}{3}\sqrt{a^2+12c}\cos\left(\varphi+\frac{2\pi}{3}\right)-\frac{2a}{3}\right]^{1/2}$$

$$k_3=\left[\frac{2}{3}\sqrt{a^2+12c}\cos\left(\varphi+\frac{4\pi}{3}\right)-\frac{2a}{3}\right]^{1/2}$$

$$\varphi=\frac{1}{3}\arccos\left(\frac{\xi}{\eta}\right)$$

$$\xi=2a^3-72ac+27b^2$$

$$\eta=2(a^2+12c)^{\frac{3}{2}}$$

$$a=-\frac{3\theta_1^2}{8}+\theta_2$$

$$b=\frac{\theta_1^3}{8}-\frac{\theta_1\theta_2}{2}+\theta_3$$

$$c=-\frac{3\theta_1^4}{256}+\frac{\theta_1^2\theta_2}{16}-\frac{\theta_1\theta_3}{4}+\theta_4$$

$$\theta_1=-(P_{11}+P_{22}+P_{33}+P_{44})$$

$$\theta_2=P_{11}(P_{22}+P_{33}+P_{44})+P_{22}(P_{33}+P_{44})+P_{33}P_{44}-(|P_{12}|^2+|P_{13}|^2+|P_{14}|^2+|P_{23}|^2+|P_{24}|^2+|P_{34}|^2)$$

$$\theta_3=-[P_{11}P_{22}(P_{33}+P_{44})+(P_{11}+P_{22})P_{33}P_{44}+P_{12}P_{23}P_{31}+P_{13}P_{34}P_{41}+P_{14}P_{43}P_{31}+P_{12}P_{24}\cdot P_{41}+P_{13}P_{32}P_{21}+P_{14}P_{42}P_{21}+P_{24}P_{43}P_{32}+P_{23}P_{34}P_{42}-P_{11}(|P_{23}|^2+|P_{34}|^2+|P_{42}|^2)-P_{22}(|P_{13}|^2+|P_{34}|^2+|P_{41}|^2)-P_{33}\cdot(|P_{12}|^2+|P_{24}|^2+|P_{41}|^2)-P_{44}(|P_{12}|^2+|P_{23}|^2+|P_{31}|^2)]$$

$$\theta_4=P_{11}P_{22}P_{33}P_{44}-P_{12}P_{23}P_{34}P_{41}-P_{13}P_{32}P_{24}\cdot P_{41}-P_{14}P_{43}P_{32}P_{21}-P_{12}P_{24}P_{43}P_{31}-P_{13}P_{34}\cdot P_{42}P_{21}-P_{14}P_{42}P_{23}P_{31}-P_{11}P_{22}|P_{34}|^2-P_{11}P_{33}|P_{24}|^2-P_{11}P_{44}|P_{23}|^2-P_{22}P_{33}\cdot|P_{14}|^2-P_{22}P_{44}|P_{13}|^2-P_{33}P_{44}|P_{12}|^2+|P_{12}|^2|P_{34}|^2+|P_{13}|^2|P_{24}|^2+|P_{14}|^2\cdot|P_{23}|^2+P_{11}(P_{23}P_{34}P_{42}+P_{24}P_{43}P_{32})+P_{22}\cdot(P_{34}P_{41}P_{13}+P_{31}P_{14}P_{43})+P_{33}(P_{42}P_{21}P_{14}+P_{41}P_{12}P_{24})+P_{44}(P_{12}P_{23}P_{31}+P_{13}P_{32}P_{21})$$

Where

$$\begin{pmatrix}P_{11}&P_{12}&P_{13}&P_{14}\\P_{21}&P_{22}&P_{23}&P_{24}\\P_{31}&P_{32}&P_{33}&P_{34}\\P_{41}&P_{42}&P_{43}&P_{44}\end{pmatrix}=\begin{pmatrix}\langle C|C\rangle&\langle C|S\rangle&\langle C|D\rangle&\langle C|T\rangle\\\langle S|C\rangle&\langle S|S\rangle&\langle S|D\rangle&\langle S|T\rangle\\\langle D|C\rangle&\langle D|S\rangle&\langle D|D\rangle&\langle D|T\rangle\\\langle T|C\rangle&\langle T|S\rangle&\langle T|D\rangle&\langle T|T\rangle\end{pmatrix}$$

2 The quantum entanglement between the *q*-mode coherent field and two identical atoms

Following the work done by P-K in Ref. [15], we can express the q-mode coherent field quantum entropy in terms of the eigenvalues of the field reduced density matrix as

$$S_f(t)=-\{\pi_f^1(t)\ln[\pi_f^1(t)]+\pi_f^2(t)\ln[\pi_f^2(t)]+\pi_f^3(t)\ln[\pi_f^3(t)]+\pi_f^4(t)\ln[\pi_f^4(t)]\} \quad (14)$$

It seems to be impossible to express the sums in Eq. (14) in a closed form, but for a not too large $\bar{n}_j$, the direct numerical temporal evolution of the entropy can take place based on the analytical solution presented by Eq. (14). In what follows we shall consider the behaviors of the quantum entanglement of the system. For simplicity, here we have bimodal field as an example.

Firstly, we examine the influence of the photon degeneracy on the evolution of the quantum entanglement. There are two cases. One situation is the two modes have the same photon degeneracy, the other is the two modes have different photon degeneracy. Fig. 1 and Fig. 2 give

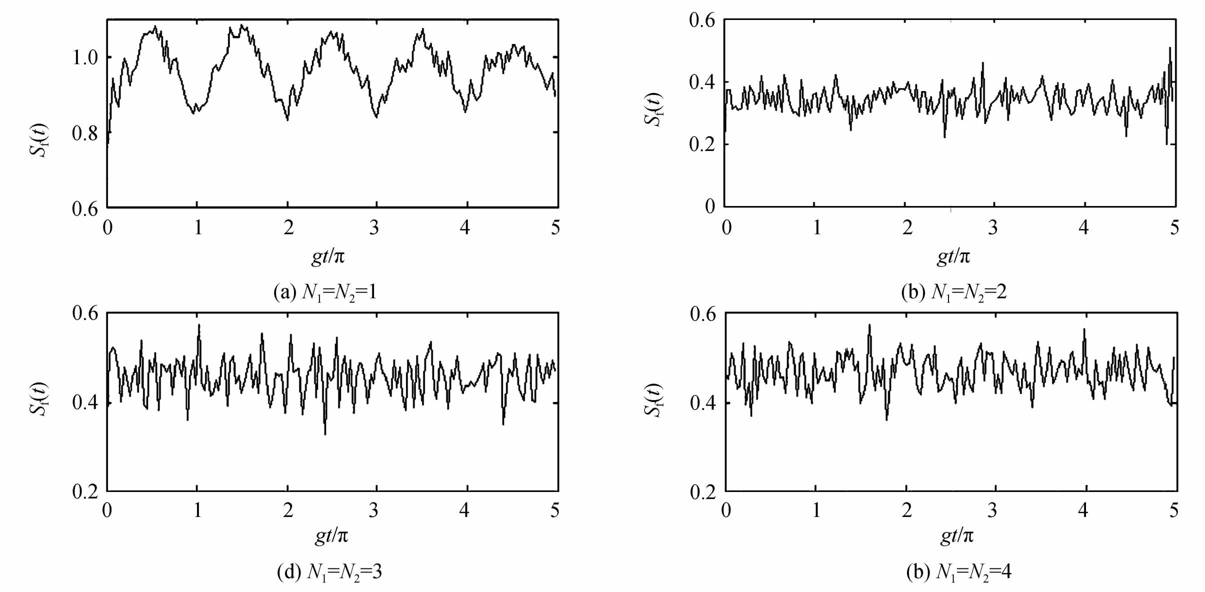

Fig. 1 The evolution curves of the field quantum entropy as a function of the scaled time gt/π for different values of photon degeneracy with the two modes have the same degeneracy

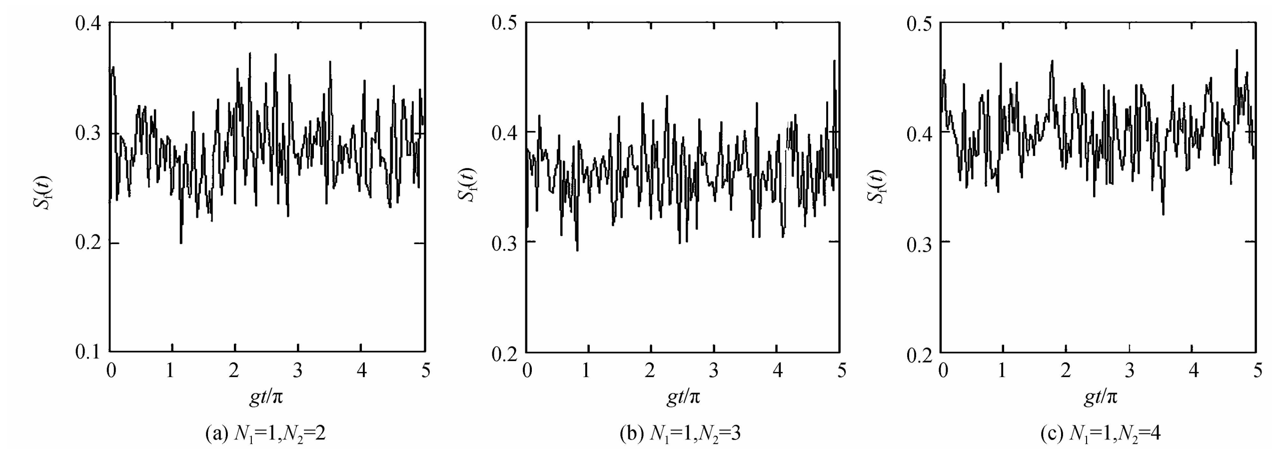

Fig. 2 The evolution curves of the field quantum entropy as a function of the scaled time gt/π for different values of photon degeneracy with the two modes have different degeneracy

the numerical results for the time evolution of the reduced field quantum entropy under the two conditions, where we have taken $\bar{n}_1=\bar{n}_2=5$, $\theta=\pi/2$, $\varphi=0$, $\varphi_1=\varphi_2=0$ and $\Delta_2=2g$.

According to the plots in Fig. 1 , one can see the non-degenerate double photon transition process is distinct completely with the degenerate multi-photon process, the maximum value of the quantum entanglement of the multi-photon transition less clearly than that of the non-degenerate biphoton process, and the mean value of the quantum entanglement increasing with the increase of the photon degeneracy, which indicates that the more strengthen the photon degeneracy is, the stronger the quantum entanglement between the field and the two atoms is.

Comparing the plots in Fig. 2, one can find the quantum entanglement is strongly dependent on the degeneracy difference of the double mode field. The larger the difference is, the bigger the maximum value and the mean value of the quantum entanglement are, which means that under these circumstances the quantum entanglement between the field and the atoms is stronger.

Secondly, we investigate the evolution properties of the quantum entanglement with the variation of the initial average photon number under the condition that $N_1=N_2=1$, $\theta=\pi/2$, $\phi=0$, $\varphi_1=\varphi_2=0$ and $\Delta_2=10g$.

From plots (a)～(c) in Fig. 3 one can see that when the two modes have the same initial average photon number, with the increasing of the average photon number, the periodicity of the quantum entanglement get more and more apparent and the periodicity is π approximatively, the mean value of it becomes smaller and smaller till drives to value zero, the oscillation amplitude of it changes also weaker and weaker until almost becomes a straight line (see Fig. 3(c)). Based on the plots (d)～(f) in Fig. 3, one can find when the two modes have different initial average photon number, the bigger the difference of the average photon number, the weaker the quantum entanglement between the multi-mode coherent field and the two-level atoms.

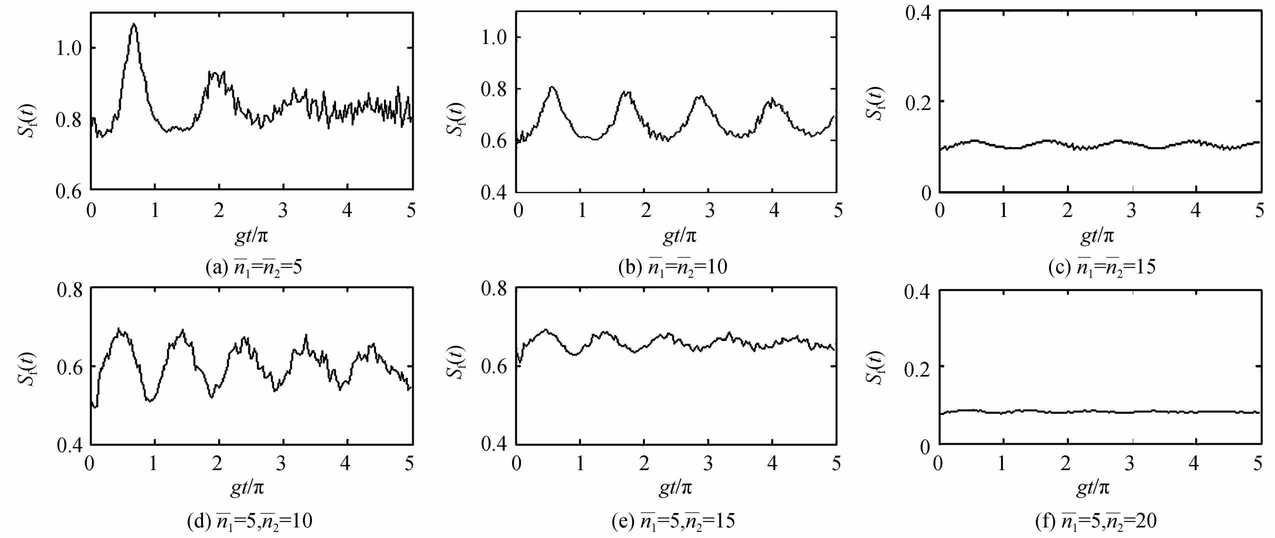

Fig. 3 The evolution curves of the field quantum entropy as a function of the scaled time gt/π for different values of initial average photon number

Finally, we see the influence of the frequency detuning on the evolution of the quantum entanglement with $N_1=N_2=1$, $\theta=\pi/2$, $\phi=0$, $\varphi_1=\varphi_2=0$ and $\bar{n}_1=\bar{n}_2=5$.

From the plots in Fig. 4, one can find the variation of the field quantum entanglement is less obvious when the frequency detuning is small, that is to say, when the field and the two atoms are near resonance, the frequency detuning has little effect on the quantum entanglement between the two-mode field and the two non-coupled two-level atoms. With the increasing of the frequency detuning, the evolution of the quantum entanglement has no longer periodic property and the mean value of it decreasing gradually, the oscillation amplitude of it is further weaker, which means that when the field and the atoms are far from resonance, the bigger the frequency detuning, the weaker the quantum entanglement between the multi-mode coherent field and the two-level atoms. More examination finds that the larger the frequency detuning, the shorter of the time for the quantum entanglement to reaches its mean value and then the quantum entanglement almost remains in that sustained mean value. Especially, when the frequency detuning is large enough (see plot (f)), the quantum entanglement almost remains in its mean value all the time, namely under that condition the field and the atoms are nearly always in entangled states.

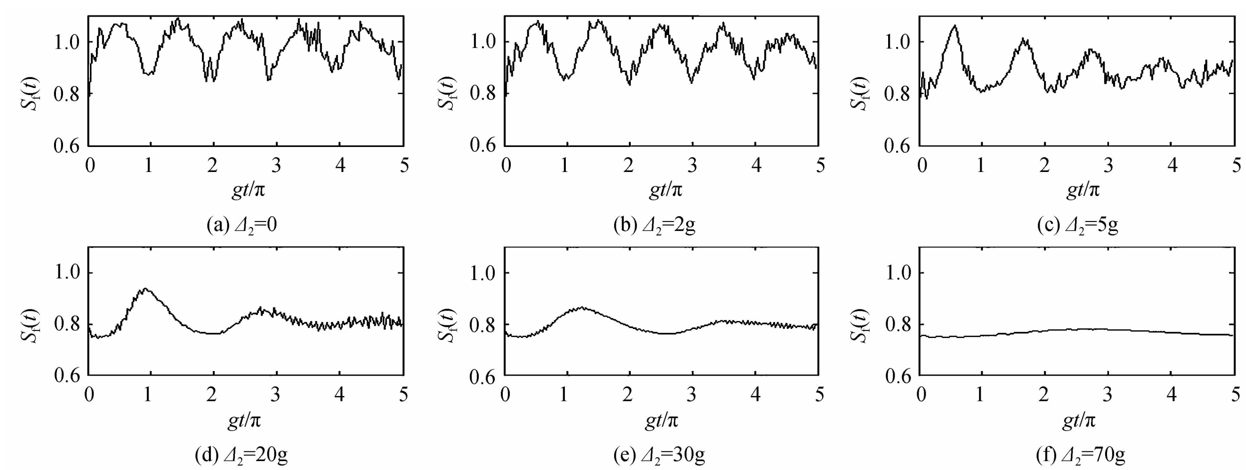

Fig. 4 The evolution curves of the quantum entanglement as a function of the scaled time gt/π with different values of frequency detuning

3 Conclusion

We have not only examined the evolution properties of the quantum entanglement of the multi-mode field interacting detuning with a pair of two-level atoms via any N_j-degree degenerate N^{Σ}-photon transition process, obtained the analytical expression of the quantum entanglement including the frequency detuning, but also presented the numerical results for the case of the bimodal coherent field interacting with the pair of two-level atoms and discussed the influences of the parameters N_j, $\bar{n}_j$ and Δ_2 on the evolution of the quantum entanglement between the field and the two atoms. Our numerical calculation results show that: 1) when the two modes have same degeneracy, the maximum value of the quantum entanglement of the multi-photon transition is less clearly than that of the non-degenerate biphoton process, and the mean value of the quantum entanglement will increase with the increase of the photon degeneracy; when the two modes have different degeneracy, the larger the difference is, the bigger the maximum value and the mean value of the quantum entanglement are; 2) when the two modes have the same initial average photon number, with the increasing of the average photon number, the periodicity of the field quantum entanglement will become more and more apparent, the mean value of it become smaller and smaller till drives to value zero, the oscillation amplitude of it changes also weaker and weaker until almost becomes a straight line; when the two modes have different initial average photon number, the bigger the difference of the average photon number is, the weaker the quantum entanglement between the multi-mode coherent field and the two-level atoms is; 3) when the field and the two atoms are near resonance, the frequency detuning has little effect on the quantum entanglement, when the field and the atoms are far from resonance, the bigger the frequency detuning is, the weaker the quantum entanglement is, and the shorter of the time for the field quantum entanglement to reaches its mean value is; when the frequency detuning is large enough, the field and the atoms are nearly always in entangled states.

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多模简并多光子 Tavis-Cummings 模型中的量子纠缠特性

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摘　要:利用冯·纽曼约化熵理论研究了多模相干态光场与两等同二能级原子简并多光子相互作用系统量子纠缠演化特性,得到了多模光场量子纠缠的解析表达式,并给出了双模光场与两原子相互作用时量子纠缠的数值计算结果.结果表明:量子纠缠随着光子简并度的增大而增强;随着初始平均光子数的增加,量子纠缠演化的周期性变得越来越明显;当场与原子远离共振时,量子纠缠随着频率失谐量的增大而减弱;当失谐量足够大时,场与原子几乎总是处于纠缠状态.这些结论对于纠缠态或纯态的制备及获取光学系统中的量子信息研究中有一定参考价值.

关键词:量子纠缠;多模光场;简并多光子过程;Tavis-Cummings 模型