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Entanglement Properties in the System of Two Atoms Trapped in Two Distant Cavities Connected by an Optical Fiber

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Abstract: Two two-level atoms are separated in two initially empty cavities that are connected by an optical fiber. The temporal evolution in the entanglement between the cavities as well as between the atom and the local cavity mode were investigated. The influence of the state-selective measurement on the entanglement and that of atom-cavity coupling coefficient on the entanglement were discussed. The results show that the entanglement between the cavities as well as between the atom and the local cavity mode can be strengthened through the state-selective measurement on the atom.

Key words: Quantum optics; Atom-cavity-fiber compound system; Selective atomic measurement; Quantum entanglement

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0 Introduction

Quantum entanglement is one of the most striking features of quantum mechanics, and is the crucial ingredient in quantum information processing, such as quantum computing, teleportation, cryptography and precision measurements. Fair attention has been paid to studying and characterizing the entanglement. For example, Shan Chuan-Jia et al investigated the entanglement character of two entangled atoms in Tavis-Cummings mode[1]. Xiang Shao-Hua et. al. studied time evolution of two-atom entanglement and thermal entanglement in a generalized Jaynes-Cummings model[2]. Yang Xiong et. al. discussed entanglement and thermal entanglement in two-photon Jaynes-Cummings model[3]. On the other hand, Since Gerry and Ghosh showed that the squeezing can be greatly enhanced via selective atomic measurements[4]. The method of selective atomic measurement has been widely employed in quantum state engineering. For example, Yang and Guo have considered a pair of two-level atoms initially in the EPR single state[5], and put one of two atoms into a cavity. They drew a conclusion that the emission properties of the atom inside the cavity are much affected by the manipulation of the atom outside the cavity. Wu and Su studied nonclassical properties in the resonant interaction of a three level Λ-type atom with two-mode field in coherent state[6]. Ye has shown enhancement of squeezing in two-photon Jaynes-Cummings model with atomic measurement[7]. I investigated remote control entanglement properties of two-atom inside cavities[8]. So far, the method of selective atomic measurement is only applied in the system of atom interacting with uncoupled cavity. Recently, the atom-cavity-fiber system shown in Fig. 1 has attracted some interest. For exemple, Yin et. al. put forward a multiatom and resonant interaction scheme for quantum state transfer and logical gates between two remote cavities via an optical fiber[9], Zheng et. al. proposed generation of two-mode squeezed states for two separated atomic ensembles via coupled cavities[10], Zhang investigated entanglement between two atoms in two distant cavities connected by an optical fiber beyond strong fiber-cavity coupling[11], and so on[12-15]. But, until now the entanglement between atom and cavity as well as between two cavities has not discussed. This paper considers the situation of two identical two-level atoms trapped in two distant single-mode optical cavities, which are coupled by an optical fiber. We investigate the entanglement between atom and cavity as well as the entanglement between two cavities, and discuss the influence of the state-selective measurement on the entanglement.

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1 Model

We introduce the atom-cavity-fiber system as shown in Fig. 1. We consider the case where two identical two-level atoms (atom 1 and atom 2) are trapped in two distant single-mode optical cavities, which are coupled by an optical fiber and initially in vacuum state. The atoms resonantly interact with the local cavity fields. In the rotating-wave approximation, the interaction Hamiltonian of the atom-cavity system can be written as (set $\hbar=1$)

$$H_{ac}=\sum_{i=1}^{2}g_i(a_i^+s_i^-+a_is_i^+) \tag{1}$$

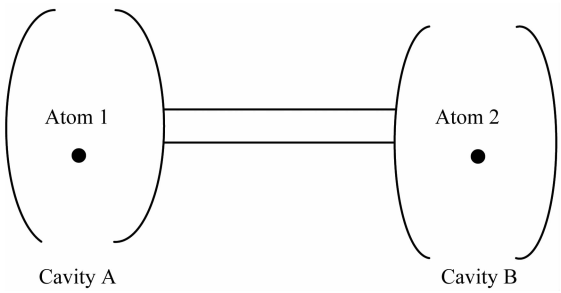

Fig. 1 Sketch of the set-up

where a_i^+ and a_i ($i=1$ or 2) are the creation and annihilation operator of the field. s_i^+ and s_i^- ($i=1$ or 2) are the atomic rising and loweing operators. g_i ($i=1$ or 2) is the coupling coefficient between an atom and local cavity field. For the sake of simplicity we put here $g_1=g_2=g$, and suppose that g is a real number.

On the other hand, the interaction Hamiltonian for the cavity mode coupled to the fiber mode may be written as[9]

$$H_{cf}=\sum_{j=1}^{\infty}f_j(b_j(a_1^++(-1)^j e^{j\theta}a_2^+)+H.C) \tag{2}$$

Where b_j is the annihilation operator for the mode j of the fiber, f_j is the coupling coefficient between the fiber mode j and the cavity mode, the phase θ is due to the propagation of the field through the fiber of length l: $\theta=2\pi\omega l/c$, where ω is the frequency of the cavities. Let $\tilde{v}$ be the decay rate of the cavities' field into a continuum of fiber modes. In the short fiber limit $2l\tilde{v}/(2\pi c)\leqslant 1$, only one resonant mode b of the fiber interacts with the cavity modes. The Hamiltonian H_{cf} may be approximated to[9]

$$H_{cf}=f(b(a_1^++a_2^+)+H.C) \tag{3}$$

where f is the cavity-fiber coupling coefficient. In the interaction picture, the total Hamiltonian of the atom-cavity-fiber combined system is

$$H=H_{ac}+H_{cf} \tag{4}$$

We introduce the total excitation operator $\hat{N}=|e_1\rangle\langle e_1|+|e_2\rangle\langle e_2|+a_A^+a_A+a_B^+a_B+b^+b$, where $|e_i\rangle$ is the excited state of atom and $|g_i\rangle$ is the ground state of atom. Because the excitation operator commutes with the Hamiltonian (4), the total excitation number is a conserved quantity. In the subspace spanned by the basis state vectors $|\varphi_1\rangle=|g_1\rangle|e_2\rangle|0_A\rangle|0_B\rangle|0\rangle_f$, $|\varphi_2\rangle=|e_1\rangle|g_2\rangle|0_A\rangle|0_B\rangle|0\rangle_f$, $|\varphi_3\rangle=|g_1\rangle|g_2\rangle|0_A\rangle|1_B\rangle|0\rangle_f$, $|\varphi_4\rangle=|g_1\rangle|g_2\rangle|0_A\rangle|0_B\rangle|1\rangle_f$, $|\varphi_5\rangle=|g_1\rangle|g_2\rangle|1_A\rangle|0_B\rangle|0\rangle_f$, where $|n_I\rangle$ denotes photons in the cavity mode I, the Hamiltonian matrix is

$$H=\begin{bmatrix}0&0&g&0&0\\0&0&0&0&g\\g&0&0&\lambda&0\\0&0&\lambda&0&\lambda\\0&g&0&\lambda&0\end{bmatrix} \tag{5}$$

The time evolution of the total system is governed by the Schrödinger equation ($\hbar=1$)

$$i\frac{\partial|\varphi(t)\rangle}{\partial t}=H|\varphi(t)\rangle \tag{6}$$

Assume that the total system is initially in state $|\varphi_1\rangle$. After an interaction time t, the total system evolves to the state

$$|\varphi(t)\rangle=A|\varphi_1\rangle+B|\varphi_2\rangle+C|\varphi_3\rangle+D|\varphi_4\rangle+E|\varphi_5\rangle \tag{7}$$

where

$$A=\frac{1}{2}(\cos gt+\frac{g^2}{\alpha^2}\cos\alpha t+\frac{2f^2}{\alpha^2}),$$
$$B=\frac{1}{2}(-\cos gt+\frac{g^2}{\alpha^2}\cos\alpha t+\frac{2f^2}{\alpha^2}),$$
$$C=-\frac{i}{2}(\sin gt+\frac{g}{\alpha}\sin\alpha t),$$
$$D=\frac{gf}{\alpha^2}(\cos\alpha t-1),$$
$$E=\frac{i}{2}(\sin gt-\frac{g}{\alpha}\sin\alpha t),$$
$$\alpha=\sqrt{2f^2+g^2} \tag{8}$$

2 Atom-cavity entanglement evolution

In order to discuss the entanglement dynamics in the above system, we adopt the negative eigenvalues of the partial transposition of density matrix ρ to quantify the degree of entanglement. The state is separable if the eigenvalues of the partial transposition of density operator ρ are positive. However, the state is entangled if one of eigenvalues of ρ^T (which ρ^T is the partial transposition of density matrix ρ) is negative. For a two-qubit system described by the density operator, the negativity is defined by[16]

$$N=-2\sum_i\mu_i \tag{9}$$

where μ_i are the negative eigenvalues of ρ^T. When $N=0$, the atom-cavity are separable, and $N=1$

indicates maximal entanglement between the atom and cavity.

Using formula (7), through tracing over the state of fiber mode, the state of cavity B and the state of atom 2, the density matrix of atom 1 and cavity A can be written as

$$\rho_{1A}=\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & |B|^2 & B^*E & 0\\ 0 & BE^* & |E|^2 & 0\\ 0 & 0 & 0 & |A|^2+|C|^2+|D|^2\end{pmatrix} \tag{10}$$

Using formulae (9) and (10), the degree of entanglement between atom 1 and cavity A can be written as

$$N_{1A}=\sqrt{(|A|^2+|C|^2+|D|^2)^2+4|BE|^2}-(|A|^2+|C|^2+|D|^2) \tag{11}$$

Fig. 2 shows the degree of entanglement between atom 1 and cavity A versus the scaled time ft. In Figs. 2, the ordinates represent the degree of entanglement between atom 1 and cavity A N_{1A}, the x-coordinates represent the scaled time ft, there in (a) $g=0.2f$, (b) $g=0.5f$, (c) $g=1f$ and (d) $f=2f$. As shown in Fig. 2, the degree of entanglement between atom 1 and cavity A N_{1A} displays irregular oscillation, and its oscillation frequency increases as g increases. When g is larger than a fixed value, the degree of entanglement N_{1A} displays the collapse-revival effect.

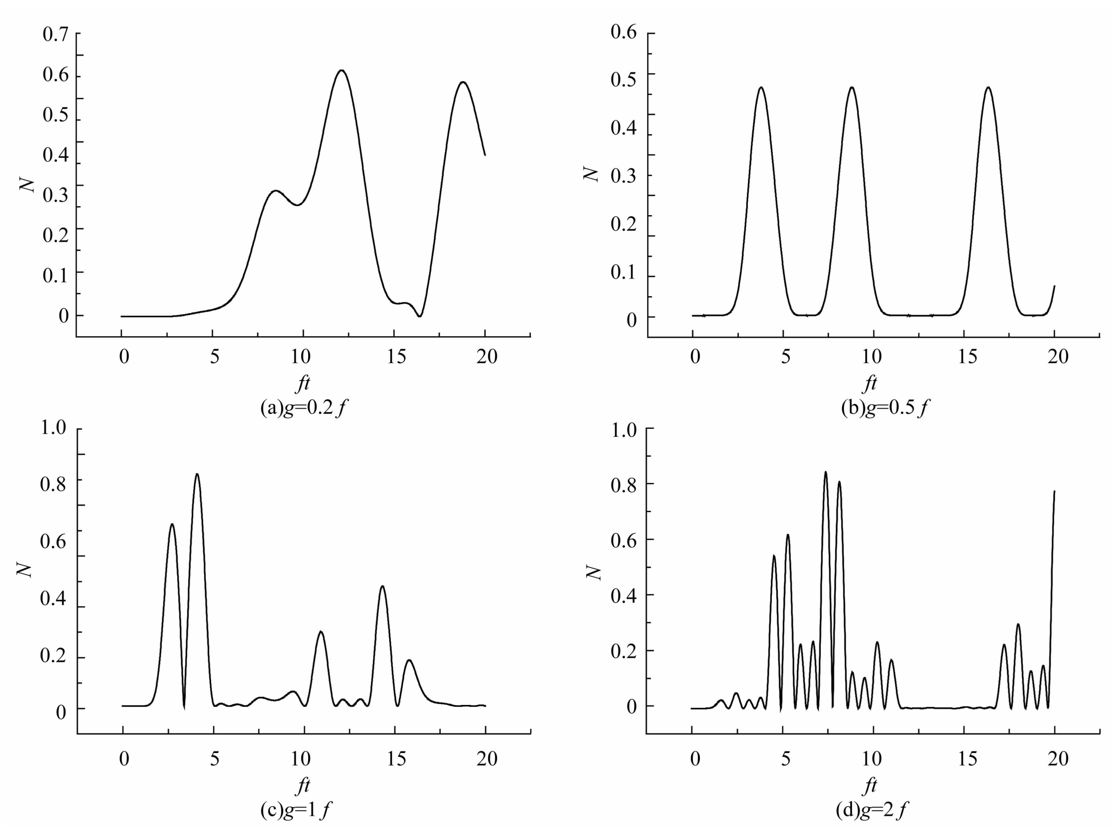

Fig. 2 Time evolution of atom-cavity entanglement N_{1A}

On the other hand, if the atom 2 is selectively measured in state $|g\rangle$, the system collapses onto the state

$$|\varphi(t)\rangle=F\{[B|e_1\rangle|0_B\rangle+C|g_1\rangle|1_B\rangle]|0_A\rangle|0\rangle_f+[D|0_A\rangle|1\rangle_f+E|1_A\rangle|0\rangle_f]|g_1\rangle|0_B\rangle\} \tag{12}$$

where $F^{-2}=|B|^2+|C|^2+|D|^2+|E|^2$. Using formula (12), through tracing over the state of fiber mode, the state of cavity B and the state of atom 2, the density matrix of atom 1 and cavity A can be written as

$$\rho'_{1A}=F^2\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & |B|^2 & B^*E & 0\\ 0 & BE^* & |E|^2 & 0\\ 0 & 0 & 0 & |C|^2+|D|^2\end{pmatrix} \tag{13}$$

Using formula (13), the degree of entanglement between atom 1 and cavity A can be written as

$$N'_{1A}=F^2[\sqrt{(|C|^2+|D|^2)^2+4|BE|^2}-(|C|^2+|D|^2)] \tag{14}$$

Using formula (14) we plot a figure of the atom-cavity entanglement evolution versus the scaled time f_t for different atom-cavity coupling coefficient, there in (a) $g=0.2f$, (b) $g=0.5f$, (c) $g=1f$ and (d) $g=2f$. In Fig. 3, the degree of entanglement between atom 1 and cavity A N_{1A} also displays irregular oscillation, and its oscillation frequency also increases as g increases. Comparing Fig. 3 with Fig. 2, we can find that the oscillation amplitude and average of the degree of entanglement N_{1A} in Fig. 3 are larger than that in Fig. 2, and the atom-cavity entanglement is strengthened through the state-selective measurement on the atom 2.

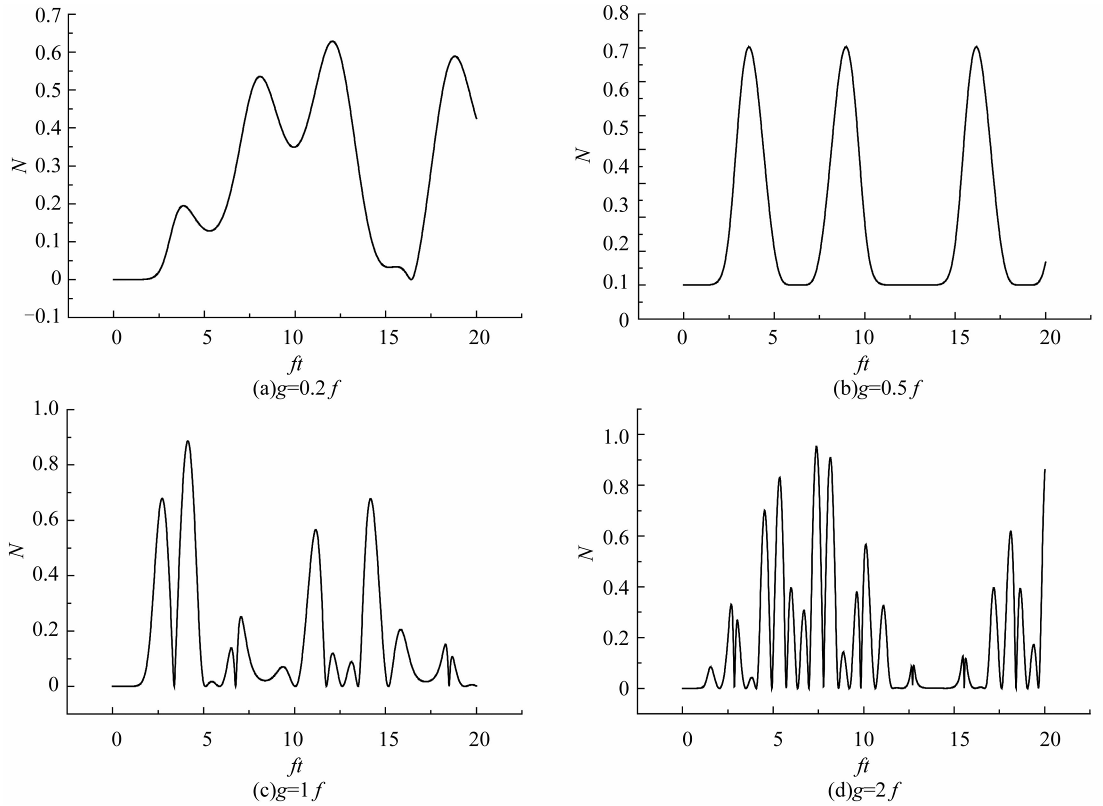

Fig. 3 Time evolution of atom-cavity entanglement N_{1A} when direct selective measurement is performed. The atom 2 is detected in state $|g\rangle$

3 Cavity-cavity entanglement evolution

In the subspace spanned by the basis state vectors $|1_A\rangle|1_B\rangle$, $|1_A\rangle|0_B\rangle$, $|0_A\rangle|1_B\rangle$ and $|0_A\rangle|0_B\rangle$, through tracing over the state of fiber mode, the state of atom 1 and the state of atom 2 the density matrix of the cavity A and cavity B can be written as

$$\rho_{AB}=\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & |E|^2 & E^*C & 0\\ 0 & EC^* & |C|^2 & 0\\ 0 & 0 & 0 & |A|^2+|B|^2+|D|^2\end{pmatrix} \quad (15)$$

Using formula (15), the degree of entanglement between cavity A and cavity B can be written as

$$N_{AB}=[\sqrt{(|A|^2+|B|^2+|D|^2)^2+4|CE|^2}-(|A|^2+|B|^2+|D|^2)] \quad (16)$$

On the other hand, if the atom 2 is selectively measured in state , the system collapses onto the state of Eq. (12). Through tracing over the state of fiber mode, the state of atom 1 and the state of atom 2 the density matrix of the cavity A and cavity B can be written as

$$\rho'_{AB}=F^2\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & |E|^2 & E^*C & 0\\ 0 & EC^* & |C|^2 & 0\\ 0 & 0 & 0 & |B|^2+|D|^2\end{pmatrix} \quad (17)$$

The degree of cavity-cavity entanglement can be written as

$$N_{AB}=F^2[\sqrt{(|B|^2+|D|^2)^2+4|CE|^2}-(|B|^2+|D|^2)] \quad (18)$$

Fig. 4 shows the entanglement evolution between cavity A and cavity B with different atom-cavity coupling coefficient, there in (a) $g=0.2f$, (b) $g=0.5f$, (c) $g=1f$ and (d) $g=2f$. In Fig. 4, the dashed lines represent the curves of the degree of entanglement N_{AB} when the atom 2 is detected in state $|g\rangle$, and solid lines represent the curves of N_{AB} when no selective measurement is performed. Comparing the solid line with the dashed one, we can find that the dashed lines are higher than the solid lines. It shows that the cavity-cavity entanglement is strengthened through the state-selective measurement of the atom.

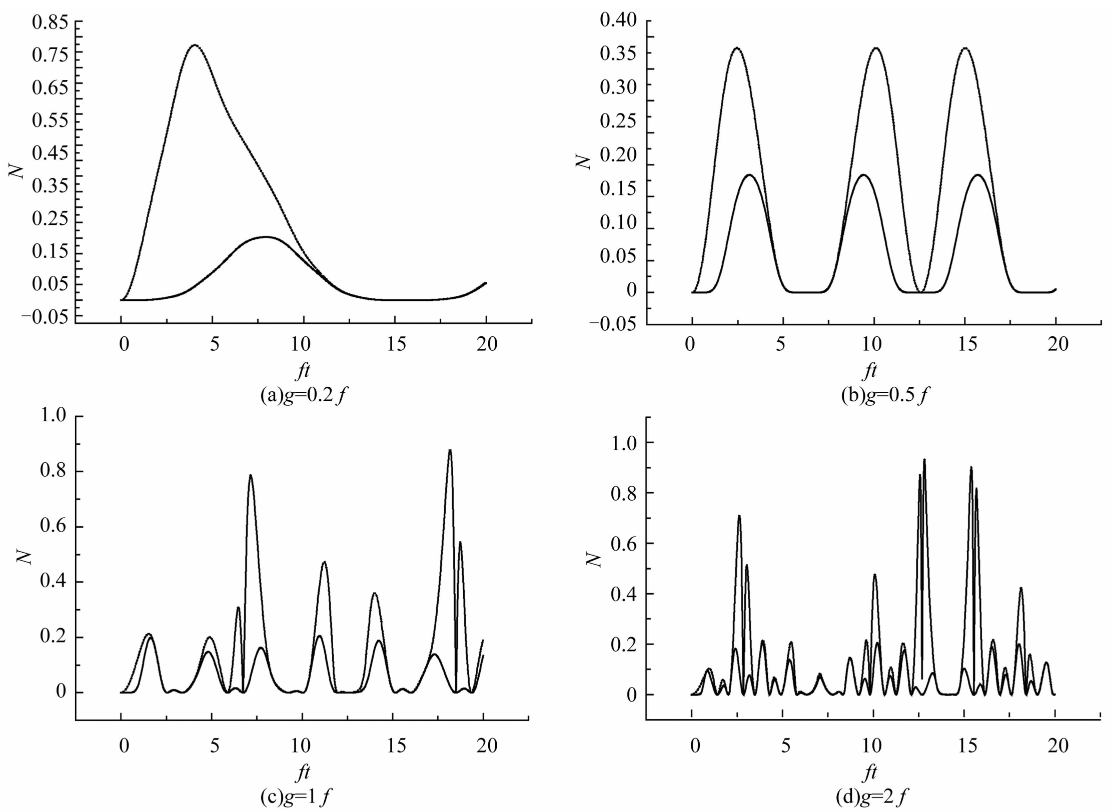

Fig. 4 Time evolution of cavity-cavity entanglement N_{AB}

4 Conclusion

We consider the case where two identical two-level atoms are trapped in two distant single-mode optical cavities, which are coupled by an optical fiber and initially in vacuum state. The atoms resonantly interact with the local cavity fields. We have investigated the evolution of the atom-cavity entanglement and the cavity-cavity entanglement by using the negative eigenvalues of the partial transposition of density matrix. The influences of state-selective measurement on the atom on the entanglement are discussed. The results obtained using the numerical method show that the cavity-atom entanglement and cavity-cavity entanglement are strengthened by state-selective measurement on the atom. On the other hand, the degree of entanglement between atom 1 and cavity A as well as the degree of entanglement between cavities displays irregular oscillation, and their oscillation frequency increases as atom-cavity coupling coefficient increases. Our investigation results will be helpful for the control of quantum entanglement in quantum computation and quantum information.

References

[1] SHAN Chuan-jia, XIA Yun-jie. The entanglement character of two entangled atoms in Tavis-Cummings model [J]. *Acta Physica Sinica*, 2006, **55**(4): 1585-1590.

[2] XIANG Shao-hua, YANG Xiong, SONG Ke-hui. Time evolution of two-atom entanglement and thermal entanglement in a generalized Jaynes-Cummings model [J]. *Acta Physica Sinica*, 2004, **53**(5): 1289-1292.

[3] YANG Xiong, XIANG Shao-hua, SONG Ke-hui. Entanglement and thermal entanglement in two-photon Jaynes-Cummings model[J]. *Journal of Atomic and Molecular Physics*, 2004, **21**(1): 68-72.

[4] GERRY C C, GHOSH H H. Squeezing enhancement in the Jaynes-Cummings model via selective atomic measurements [J]. *Phys Lett A*, 1997, **229**(1): 17-22.

[5] YANG C P, GUO G C. Controllable emission properties of an atom inside a cavity by manipulating the atom outside the cavity[J]. *Phys Lett A*, 1999, **255**(3): 129-132.

[6] WU Huai-zhi, SU Wan-jun. Nonclassical properties in the resonant interaction of a three level -type atom with two-mode field in coherent state[J]. *Chinese Physics*, 2007, **16**(1): 106-110.

[7] YE Sai-yun. Enhancement of squeezing in two-photon Jaynes-Cummings model with atomic measurement[J]. *Commun Theor Phys*, 2006, **45**(1): 147-150.

[8] LU Dao-ming. Remote control entanglement properties of two-atom inside cavities[J]. *Acta photonica Sinica*, 2010, **39**(11): 2088-2092.

[9] YIN Z Q, LI F L. multiatom and resonant interaction scheme for quantum state transfer and logical gates between two remote cavities via an optical fiber[J]. *Phys Rev A*, 2007, **75**(1): 012324.

[10] ZHENG S B, YANG Z B, XIA Y. Generation of two-mode squeezed states for two separated atomic ensembles via coupled cavities[J]. *Phys Rev A*, 2010, **81**: 015804.

[11] ZHANG B. Entanglement between two atoms in two distant cavities connected by an optical fiber beyond strong fiber-cavity coupling[J]. *Optics Communications*, 2010, **283**(1): 196-199.

[12] HARTMANN M J, BRANDAO F G S L, PLENIO M B. Effective spin systems in coupled microcavities[J]. *Phys Rev*

Lett, 2007, **99**(16): 160501.

[13] ZHENG S B, YANG C P, NORI F. Arbitrary control of coherent dynamics for distant qubits in a quantum network [J]. *Phys Rev A*, 2010, **82**(4): 042327.

[14] ZHENG S B. Quantum communication and entanglement between two distant atoms via vacuum field[J]. *Chin Phys B*, 2010, **19**(6): 064204.

[15] YANG Z B, XIA Y, ZHENG S B. Resonant scheme for realizing quantum phase gates for two separate atoms via coupled cavities[J]. *Optics Communications*, 2010, **283**(5): 3052-3057.

[16] AKHTARSHENAS S J, FARSI M. Negativity as entanglement degree of the Jaynes-Cummings model[OL]. 2007, quant-ph/0702101V1.

原子-腔-光纤复合系统中的纠缠特性

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摘　要: 采用 Negativity 熵来描述两子系统间的纠缠, 研究了由两个二能级原子与光纤联接的单模腔构成的系统的纠缠特性. 利用数值计算方法讨论了原子与腔场的耦合强度和原子选择性测量对纠缠特性的影响. 研究结果表明: 对一个原子的选择性测量, 可增强原子间与腔场间和腔场与腔场间的纠缠.

关键词: 量子光学; 原子-腔-光纤复合系统; 选择原子测量; 量子纠缠