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Characteristics of Ytterbium-doped Double-cladding Pulsed Fiber Amplifiers

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Abstract：Output characteristics of Yb-doped double-cladding pulsed fiber amplifiers were discussed under conditions of low and high repetition rates using the time-dependent rate equations. Because there are several different initial conditions used in the finite-difference method with different repetition rates，some methods were presented to determine the initial conditions. The relationship of the peak powers of the high repetition rates with their equivalent continuous wave powers was also discussed when the signal light was in Gaussian shape. The change of the average output power with the pump power was obtained under the conditions of the pulsed signals with low repetition rates and high repetition rates.

Key words：Fiber amplifier；Ytterbium-doped；Equivalent CW power；Finite-difference method
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0 Introduction

The fiber amplifiers have been used in a wide range of applications because of its advantages (high conversion efficiency，high stability，high energy，excellent beam quality，good heat dissipation，and compact configuration). Recent years，Yb-doped fiber amplifiers have significant development. By Yb-doped fiber amplifiers，the chirped-pulse amplification and recompression of the 232 fs 45 pJ/pulse oscillator output yields a final direct fiber-end delivery of 7.3 nJ energy pulses of around 297 fs duration[1]. Using Yb-doped fiber amplifiers，K. K. Chen et. al. get the pulse energy of 1.7 μJ at a repetition rate of 56 MHz[2]. A three-stage Yb-fiber amplifier emitted 1.43 kW of single-mode power when seeded with a 25 GHz line width master oscillator[3].

Theoretical model is important to design and optimize the fiber amplifiers. Some papers solve the time-dependent rate equations with low repetition rate[4-7]. They suppose that the input signal pulse train is coming after the amplifier reaches its steady state (the steady state without input signals but pump light)，consider this steady state as the initial conditions，and the output pulse energy becomes stable soon after several pulses[5]. If the pulses repetition rate is very high，the interval of pulses is too short that we can not get the right results by the method above.

In this paper，we discuss a method to solve the time-dependent rate equations with low or high repetition rate signal pulses about Yb-doped double-cladding fiber amplifiers，and analyze their available range. As we know，boundary conditions and initial conditions should be determined before using the finite-difference method. We present a method to determine the initial conditions with high repetition rate signal pulses. With the initial conditions determined by this method，we can get the steady output power of fiber amplifier quickly. By finite-difference method，we get the relationship of the peak power of high repetition rates signal pulses with their equivalent CW power when the signal light is in Gaussian shape in time domain. The changes of the average output power with the pump power are obtained under the conditions of the signal pulses with low repetition rate and high repetition rate.

1 Theoretical models of fiber amplifiers

The experimental setup of fiber amplifier is shown in Fig. 1，while the theoretical model of the

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fiber amplifier is shown in Fig. 2. The pump power $P_p^+(0, t)$ (injected in $z=0$) is a constant, which is represented as P_0. The input signal $P^-(L, t, \lambda_s)$ (injected in $z=L$) is a pulse train, which is denoted as $P_1(t)$.

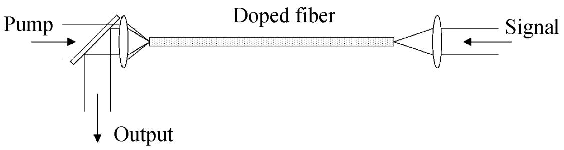


Fig. 1 Experimental setup of the fiber amplifier

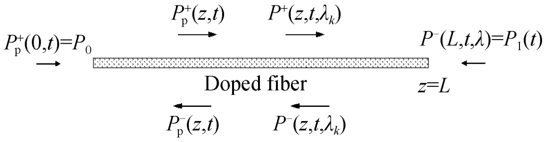


Fig. 2 Theoretical model of the fiber amplifier

The simplified two-level rate equations for ytterbium-doped double-cladding (YDDC) fiber amplifiers are given by[5]

$$\begin{aligned}
&N=N_1(z,t)+N_2(z,t)\\
&\frac{\partial N_2(z,t)}{\partial t}=\frac{\Gamma_p\lambda_p}{hcA}[\sigma_a(\lambda_p)N_1(z,t)-\sigma_e(\lambda_p)N_2(z,t)]\cdot\\
&[P_p^+(z,t)+P_p^-(z,t)]-\frac{N_2(z,t)}{\tau}+\frac{\Gamma}{hcA}\cdot\\
&\sum_{k=1}^{K}\lambda_k[\sigma_a(\lambda_k)N_1(z,t)-\sigma_e(\lambda_k)N_2(z,t)]\times\\
&[P^+(z,t,\lambda_k)+P^-(z,t,\lambda_k)]\\
&\pm\frac{\partial P_p^\pm(z,t)}{\partial z}+\frac{1}{V_p}\frac{\partial P_p^\pm(z,t)}{\partial t}=-\Gamma_p[\sigma_a(\lambda_p)\cdot\\
&N_1(z,t)-\sigma_e(\lambda_p)N_2(z,t)]\times P_p^\pm(z,t)-\\
&\alpha(\lambda_p)P_p^\pm(z,t)\\
&\pm\frac{\partial P^\pm(z,t,\lambda_k)}{\partial z}+\frac{1}{V}\frac{\partial P^\pm(z,t,\lambda_k)}{\partial t}=\Gamma[\sigma_e(\lambda_k)\cdot\\
&N_2(z,t)-\sigma_a(\lambda_k)N_1(z,t)]\times P^\pm(z,t,\lambda_k)-\\
&\alpha(\lambda_k)P^\pm(z,t,\lambda_k)+2\sigma_e(\lambda_k)N_2(z,t)\frac{hc^2}{\lambda_k^3}\Delta\lambda+\\
&S\alpha_{RS}(\lambda_k)P^\mp(z,t,\lambda_k) \qquad (1)
\end{aligned}$$

The ASE spectrum is thus divided into K channels with central wavelength of λ_k, $k=1,2\cdots K$, and an even spacing of $\Delta\lambda$. Since the practical signal bandwidth $\Delta\lambda_s$ is about 0～2 nm, we can let $\Delta\lambda=\Delta\lambda_s$. Then, the expressions of rate equations for the ASE and the signal have the same format.

N is the ytterbium dopant concentration and assumed to be uniform along the fiber. N_1 and N_2, as functions of the time t and coordinate z, are ground and upper-level populations, respectively. $P_p(z, t)$ is the pump power, and $P(z, t, \lambda_k)$ is the ASE power of each channel (± corresponds to forward and backward propagations, respectively). In the presence of the signal, $P(z, t, \lambda_s)$ thus represents the signal power. This processing is reasonable, because the signal power is stronger than the ASE power in corresponding to spectral channel ($k=s$) by at least three orders of magnitude, and the latter can be regarded as a weak background noise. V_p and V are the group velocities of the pump and ASE in the fiber, c is the light velocity in the vacuum. For our YDDC fiber amplifiers, the chromatic dispersion effect can be neglected, thus V is independent on wavelength. σ_a and σ_e are the absorption and emission cross sections of ytterbium ions, respectively[8]. A is the doped area of the YDDC fiber, and $\Gamma_p(\Gamma)$ is the overlapping factor between the pump (ASE) and the fiber doped area. α_{RS} is the Rayleigh scattering coefficient, and S is the capture factor of the fiber core[9-10]. $\alpha(\lambda)$ is the fiber attenuation coefficient.

The boundary conditions associated with the above partial differential equations are written as

$$\begin{cases}
P_p^+(0,t)=\eta P_0\\
P_p^-(0,t)=0\\
P^+(0,t,\lambda_k)=0\\
P^-(L,t,\lambda_k)=0,(k=1,2\cdots s-1,s+1\cdots K)\\
P^-(L,t,\lambda_s)=\eta P_1(t)
\end{cases} \qquad (2)$$

were P_0 is forward CW pump power injected into the YDDC fiber. η is the pump launch efficiency, and $P_1(t)$ is the input signal. The output signal power is $P^-(0, t, \lambda_s)$. The above initial-boundary-value problem can be numerically solved by the finite-difference method.

2 Low repetition rate condition

The energy stored in fiber is defined as $\frac{hcA}{\lambda_s}\cdot\int_0^L N_2(z,t)dz$. Fig. 3 shows the evolution of the stored energy with the pump powers of 25 W, 30 W, 35 W, when the fiber lengths are 3 m and 4 m. The energy stored in fibers could reach steady state when pump power is injected into fiber

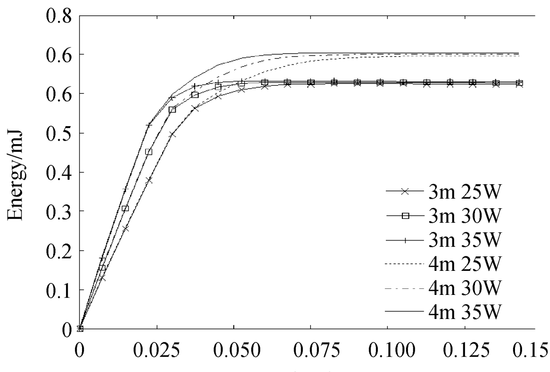


Fig. 3 Evolution of the stored energy under the pump power of 25 W, 30 W, 35 W (fiber length of 3 m and 4 m)

without signals. If the intervals of signal pulses are longer than the recovery time of the fiber energy, there is enough time for fiber to recover to the steady state before the next pulse comes. In other words, the previous pulse has no influence on the next one. When the pump power is 30 W and fiber length is 3 m, the recovery time is 0.05 ms. If we would use the low repetition rate method mentioned above , we should ensure the pulse repetition rate is smaller than 20 kHz (1/0.05 ms).

To solve the Eq. (1), we should set the initial conditions (when $t=0$) as the steady state that pump power is injected into fiber without signals. Hence we let Eq. (1) under $\partial/\partial t=0$ to get this initial conditions (the pump power distribution $P_p(z)$, ASE power distribution $P(z, \lambda_k)$, and upper-level population distribution $N_2(z)$ along fiber). This could be solved by relaxation method. Then we could solve Eq. (1) by finite-difference method [4-7] with the initial conditions given above and the boundary conditions of Eq. (2).

3 High repetition rate condition

We would set a bench mark to make a distinction between the high repetition rate and the low repetition rate to determine which method to calculate the initial conditions. If the pulse intervals are shorter than $\tau/100$, which means the repetition rate is higher than $100/\tau$, the high repetition rate pulses could be treated as quasi-CW light. For the Yb-doped fiber, the life time of the upper energy lever τ is 8.4×10^{-4} s, then the value of $100/\tau$ is 119 kHz.

The repetition rate of the signal pulses we used is 100 MHz, the signal pulse is Gaussian shape which pulse width is 1ns, peak power is 0.6 W, wavelength λ_s is 1 064 nm. Signal pulses are shown in Fig. 4. The length of the fiber is 3 m.

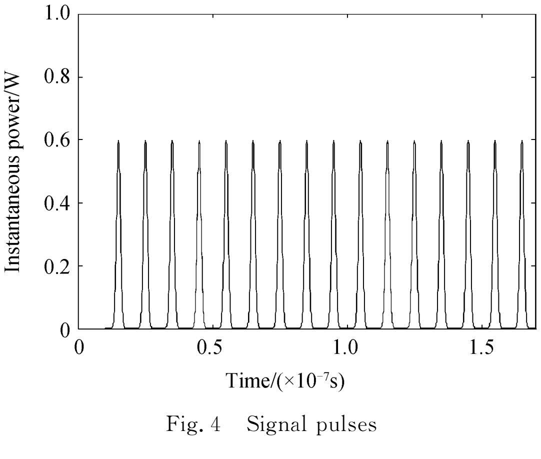


Fig. 4 Signal pulses

When there is no input signal but the pump light was injected into the fiber, this steady state (upper-level population distribution $N_2(z)$, pump power distribution $P_p(z)$, and ASE power distribution $P(z, \lambda_k)$) was set as the initial condition. The power evolution of the output pulses train is shown in Fig. 5. Compared with time step $\Delta t=5\times10^{-11}$ s used in the finite-difference method , the change of output pulse peaks is very slow, which means it will take an uncertain time to get the results. Even two hours are not long enough for a dual-core computer to get the result that output pulse peaks change hardly. In this case, we can not use the method of the low repetition rate condition into the high repetition rate condition, so we provide a new method to calculate the characteristics of the high repetition rate condition in fiber amplifier.

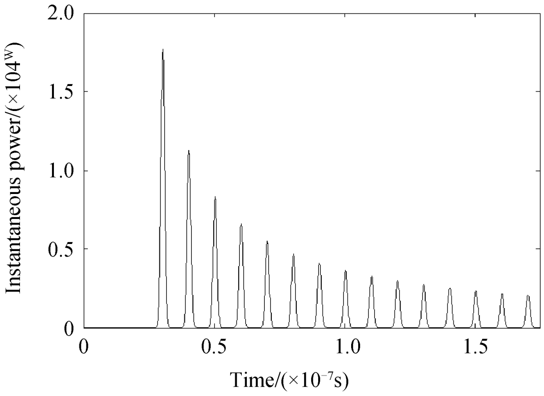
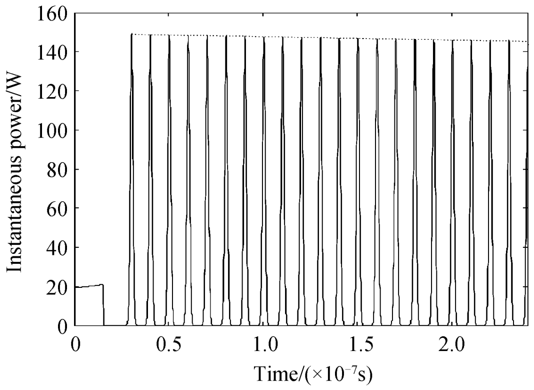


Fig. 5 The power evolution of the output pulses train

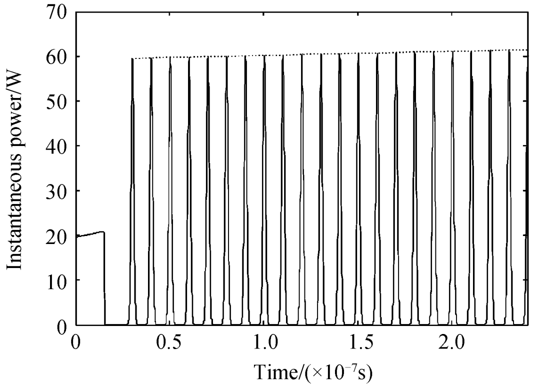
Because of the characteristics of the amplifier, a new method for the high repetition rate signal pulses is based on three factors: 1) The interval of pulses is far shorter than the life time of Yb (8.4×10^{-4} s), so the upper-level population distribution along fiber is stable after enough time; 2) When high repetition rate pulses are amplified, we could use an equivalent CW power P_{ep} to present the power of the pulses train to simplify the process of the calculation. In these two conditions, they should have the same upper-level population distribution $N_2(z)$, the same pump power distribution $P_p(z)$, and the same ASE power distribution $P(z, \lambda_k)$ under the same pump power when high repetition rate pulses and this equivalent CW power P_{ep} are amplified by the same amplifier separately; 3) If we do not change the parameters of the amplifier (e. g. pump power, fiber length), no matter what initial conditions we set, the stable output pulse peaks would reach the same values.

We would ensure an upper limitation and a lower limitation of P_{ep} when setting the initial conditions (upper-level population distribution $N_2(z,0)$, pump power distribution $P_p(z,0)$, ASE power distribution $P(z,0,\lambda_k)$). These two initial conditions correspond to two steady states that input CW signals of 0.06 W and 0.15 W are amplified separately. The output pulses trains are shown in Fig. 6. The envelope of output pulse peaks is a dotted line. According to the factor 3), the limitations of the envelopes should be the same and they would reach the stable output pulse peaks.

The envelope of output pulses peak in Fig. 6 (a) is decreasing. Because the P_{ep} is larger than 0.06 W, the upper-level population distribution $N_2(z,0)$ in initial condition is higher than that in steady situation. Then the previous pulse can take more energy than the later one, and the upper-level population distribution becomes lower and lower. The later pulses can only take less and less energy until the energy that the pump source provides equals the energy the signals take away at the same time. Then the amplifier reaches the stable situation.



(a) Input CW signal of 0.06W is amplified



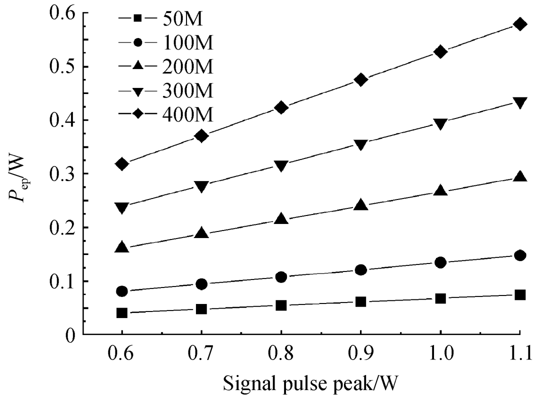
(b) Input CW signal of 0.15W is amplified

Fig. 6 Output pulses train(dotted line is the envelope of output pulse peak)

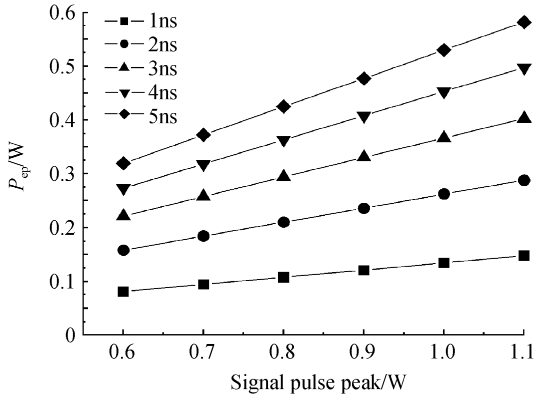
For the same reason, the P_{ep} is smaller than 0.15W by analyzing Fig. 6(b), so we could say that P_{ep} is between 0.06 W and 0.15 W.

Similarly we can reduce the range of P_{ep} by dichotomy. The steps are as follows: 1) Find the upper limit and the lower limit of P_{ep}; 2) Calculate the average of the upper limit and lower limit, then set the initial conditions ($N_2(z,0)$, $P_p(z,0)$, $P(z,0,\lambda_k)$) as the steady state that CW power this average is amplified, and get the output pulses by finite-difference method; 3) If the envelope of output amplified pulses is increasing, set this average as the new upper limit, then return to step 2). If the envelope of output amplified pulses is decreasing, set the average as the new lower limit, then return to step 2). Continue calculating until the range of the upper limit and lower limit of P_{ep} is short enough.

Fig. 7 (a) shows that P_{ep} versus the peak powers of Gaussian pulses in high repetition rates at 50 M, 100 M, 200 M, 300 M, 400 M respectively when the pulse width is 1ns. The change of the P_{ep} is linear with the peak powers of Gaussian pulses in high repetition rates. Under the same signal pulses peak, the larger the repetition rate is, the higher the equivalent power is.



(a) When repetition rate is 50M, 100M, 200M, 300M, 400M



(b) When repetition rate is 10M and pulse width is 1ns, 2ns, 3ns, 4ns, 5ns

Fig. 7 P_{ep} versus the peak power of high repetition Gaussian pulses as signal light

Fig. 7 (b) shows that P_{ep} versus the peak power of Gaussian pulses in high repetition rates when repetition rate is 100M and pulse widths are 1ns, 2ns, 3ns, 4ns and 5ns. The change of the P_{ep} is linear with the peak power of Gaussian pulses in high repetition rates. Under the same signal pulses peak, the larger the pulse width is, the higher the equivalent power is.

Fig. 8 show that the average output power versus pump power under high repetition rate signal pulses and low repetition rate signal pulses. The average output power is linear with pump power under high repetition rate signal pulse. This relationship is logical[11]. However, the average output power raise rarely when the pump power increase under low repetition rate signal pulse, because its repetition rate is too low to make use of pump power. In order to get larger average output power, we should increase the energy stored in the fiber by using a longer fiber.

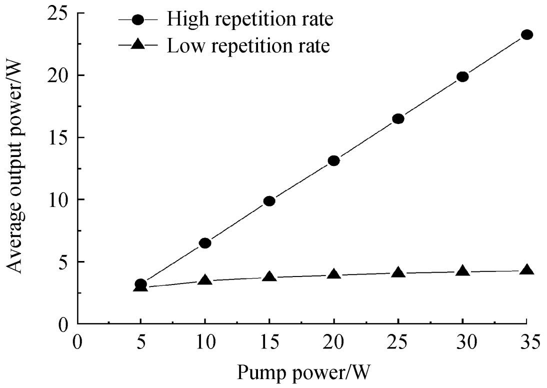


Fig. 8　Average output power versus pump power under high repetition rate signal pulses and low repetition rate signal pulses

4　Conclusion

We discussed output characteristics of the Yb-doped double-cladding pulsed fiber amplifiers under the conditions of low or high repetition rates and provided the methods to determine the initial conditions in the finite-difference method.

The relationship of the peak powers of the high repetition rates Gaussian pulses and their equivalent powers was obtained. The change of the equivalent power with the peak power of the high repetition Gaussian pulses is linear. With the same pulse peak power, the larger the repetition rate is, the higher the equivalent power is. Similarly, the larger the pulse width is, the higher the equivalent power is.

At last, the relationship of the average output powers and the pump powers was obtained under the high and low repetition rates. The change of the average output power with the pump power is linear under high repetition rate, while the average output power raises rarely when the pump power increasing under the low repetition rates.

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掺镱双包层脉冲光纤放大器的特性研究

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摘　要：在高频及低频脉冲种子光的情况下讨论了掺镱双包层光纤放大器的输出特性．在有限差分法中，对于不同重复频率的脉冲种子光，需要设置不同的初始条件以快速计算得到稳定输出．本文给出了在高频和低频情况下设置初始条件的方法，并讨论了作为种子光的准连续高频高斯脉冲峰值功率与其等效连续光功率之间的关系，最后计算了种子光为高频及低频脉冲情况下的平均输出功率与泵浦功率的关系．

关键词：光纤放大器；掺镱离子；等效连续功率；有限差分法