

Article ID: 1004-4213(2011)08-1244-4

Entanglement Concentration of Entangled Coherent States

CAI Xin-hua, PENG Guang-han, QIAO Nao-sheng

(College of Physics and Electronics, Hunan University of Arts and Science, Changde, Hunan 415000, China)

Abstract: With linear optical components to manipulate quantum states of optical fields, the quantum entanglement control and quantum communication can be implemented remotely. By analyzing optical splitter and its use of coherent states, it was found that when the initial state was a direct product of two partially entangled coherent states; another two modes would collapse to a new entangled state using optical splitter and photon-detectors on two modes. Based on this feature, the scheme on entanglement concentration was presented. In this scheme, the two partially entangled coherent states were utilized as the quantum channel, when zero photon and odd photons were detected in two modes separately, and another two modes would collapse to the maximum entangled state, thereby completing the process of the entanglement concentration. It was proved that, no matter how small the initial entanglement is, to distill some maximally entangled states from partially entangled pure states is possible.

Key words: Entangled coherent state; Entanglement concentration; Beam-splitters; Detection of photon

CLCN: O431.2

Document Code: A

doi: 10.3788/gzxb20114008.1244

0 Introduction

Quantum entanglement is a fundamental resource in quantum information, ensuring security of key distribution and efficiency of quantum computing^[1-7]. Most of the quantum information protocols work best with maximally and pure entangled states, but entanglement is extremely sensitive to decoherence. Such difficulty imposes an extensive effort in order to recover a fairly good degree of entanglement after its interaction with noise, which is the main cause of the entanglement deterioration. In the last few years various schemes have been implemented in order to overcome this problem.

In an efficient realization of quantum information processing, highly entangled states play a key role. When an entangled state is initially prepared in a non-maximally entangled state, it needs to be distilled to a highly entangled state before using it for quantum information processing. To obtain highly entangled states from less entangled states, researchers have proposed the idea of entanglement concentration and purification, and proved that it is possible to convert two copies of a less entangled state into one copy of a more entangled state by using only

local operations and classical communication^[8]. Also, some entanglement concentration and purification protocols have been investigated^[9-11]. Here we show that the entanglement concentration of entangled coherent states can be realized in a very simple way using only linear optical elements and photon detectors.

1 Entangled coherent states

The coherent state can be used to encode quantum information on continuous variables^[12]. In quantum information processing, the entangled coherent state is normally categorized into a two mode continuous variable state. However, there was a suggestion to implement a logical qubit encoding through treating a coherent superposition state, which can make a single mode continuous variable state as a qubit in two dimensional Hilbert space^[13]. Thereupon, many schemes have been proposed for realizing quantum information processing by using the coherent state.

We first briefly review the bipartite entangled coherent states of the form

$$|\varphi\rangle_{12} = N_{\theta} (\cos\theta |\alpha\rangle_1 |\alpha\rangle_2 - \sin\theta |-\alpha\rangle_1 |-\alpha\rangle_2) \quad (1)$$

where $N_{\theta} = (1 - 2e^{-4|\alpha|^2} \cos\theta \sin\theta)^{-1/2}$ is the normalization factor and

Foundation item: The Natural Science Foundation of Hunan Province (No. 09JJ3004)

First author: CAI Xin-hua (1956-), male, professor, mainly focuses on quantum information. Email: xhcai2002@yahoo.com.cn

Received date: 2010-13-01 **Revised date:** 2011-04-07

$$|\alpha\rangle = \exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |0\rangle \quad (2)$$

is a coherent state. When $\theta = \pi/4$, Eq. (1) becomes

$$|\varphi\rangle_{12} = \frac{1}{\sqrt{2(1-e^{-4|\alpha|^2})}} (|\alpha\rangle_1 |\alpha\rangle_2 - |-\alpha\rangle_1 |-\alpha\rangle_2) \quad (3)$$

Now we introduce the even and odd coherent states

$$|\alpha\rangle^{\pm} = \frac{1}{\sqrt{2(1 \pm e^{-2|\alpha|^2})}} (|\alpha\rangle \pm |-\alpha\rangle) \quad (4)$$

The Eq. (3) can be rewritten as the form

$$|\varphi\rangle_{12} = \frac{1}{\sqrt{2}} (|\alpha\rangle_1^+ |\alpha\rangle_2^- - |-\alpha\rangle_1^- |-\alpha\rangle_2^+) \quad (5)$$

Because $|\alpha\rangle^+$ and $|\alpha\rangle^-$ are orthogonal, we can introduce them as a pair of orthogonal bases to span a two-dimensional subspace of the Hilbert space. Then Eq. (5) is a bipartite entangled pure state, its reduced density matrices is

$$\rho^{(1)} = \rho^{(2)} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \quad (6)$$

and it has the maximally partial entropy

$$S(\rho^{(1)}) = S(\rho^{(2)}) = -\text{tr}(\rho^{(1)} \log \rho^{(1)}) = 1 \quad (7)$$

It is a maximally entangled state.

Now we consider how to generate the entangled coherent state. The experimental setup consists of a lossless 50/50 beam-splitter which is described by

$$\hat{B}_{12} = \exp \left[i \frac{\pi}{4} (\hat{a}_1^+ \hat{a}_2 + \hat{a}_2^+ \hat{a}_1) \right] \quad (8)$$

here \hat{a}_i and \hat{a}_i^+ (i denote 1, 2) are the bosonic annihilation and creation operators of system i , respectively. It transforms the coherent state $|\alpha\rangle_1 |\beta\rangle_2$ as

$$\hat{B}_{12} |\alpha\rangle_1 |\beta\rangle_2 = \left| \frac{\alpha+i\beta}{\sqrt{2}} \right\rangle_1 \left| \frac{\beta+i\alpha}{\sqrt{2}} \right\rangle_2 \quad (9)$$

where the normalization factor is omitted on the right-hand side. Further equipping the beam splitter by a pair of $-\pi/2$ phase shifters described by the unitary operator $\hat{P}_2 = \exp(-i \frac{\pi}{2} \hat{a}_2^+ \hat{a}_2)$, we

can have the operator $BS_{12} = \hat{P}_2 \hat{B}_{12} \hat{P}_2$ that transforms the state $|\alpha\rangle_1 |\beta\rangle_2$ into

$$BS_{12} |\alpha\rangle_1 |\beta\rangle_2 = \left| \frac{\alpha+\beta}{\sqrt{2}} \right\rangle_1 \left| \frac{\alpha-\beta}{\sqrt{2}} \right\rangle_2 \quad (10)$$

This transformation plays an important role in the realization of quantum information processing which involves coherent states.

Suppose the input state of the beam splitter $BS(1)$ is a direct product of a coherent superposition states

$$|\Psi\rangle = \frac{1}{\sqrt{N_a}} (|\sqrt{2}\alpha\rangle_1 - |-\sqrt{2}\alpha\rangle_1) \quad (11)$$

and a vacuum state $|0\rangle_2$, where N_a is a normalization factor, and the superposition $|\Psi\rangle$ of two coherent states $|\sqrt{2}\alpha\rangle$ and $|-\sqrt{2}\alpha\rangle$ may be generated by exploiting a coherent state $|\sqrt{2}\alpha\rangle$ propagating through a nonlinear medium^[14]. Then the input state of the beam splitter $BS(1)$ is

$$|\Psi\rangle_1 \otimes |0\rangle_2 = \frac{1}{\sqrt{N_a}} (|\sqrt{2}\alpha\rangle_1 - |-\sqrt{2}\alpha\rangle_1) \otimes |0\rangle_2 \quad (12)$$

after interaction with the beam-splitter, the output state emerging from the beam splitter $BS(1)$ is given by

$$|\Phi\rangle_{12} = \frac{1}{\sqrt{N_a}} (|\alpha\rangle_1 |\alpha\rangle_2 - |-\alpha\rangle_1 |-\alpha\rangle_2) \quad (13)$$

This is a maximally entangled coherent state; we have proven its entanglement entropy is 1.

2 Entanglement concentration

In quantum information, researchers need maximally entangled states for an efficient realization of quantum information processing. If the initially prepared entangled coherent state is in a pure but not maximally entangled state, this state may be distilled to a maximally entangled state before using it for quantum information processing through entanglement concentration. Here we show that for an entangled coherent state the entanglement concentration may be realized by using a beam-splitter and photo-detectors.

Suppose two people, Alice and Bob, share a partially entangled coherent state

$$|\varphi\rangle_{12} = \frac{1}{\sqrt{N_\eta}} (\cos \eta |\alpha\rangle_1 |\alpha\rangle_2 - \sin \eta |-\alpha\rangle_1 |-\alpha\rangle_2) \quad (14)$$

Here, N_η is a normalization factor given by

$$N_\eta = 1 - \sin(2\eta) \exp(-4|\alpha|^2) \quad (15)$$

the real phase factor η , $0 < \eta < \pi/2$, determines the degree of entanglement for state (14), from which we want to distill the maximally entangled state.

After sharing the quantum channel with Bob, firstly Alice prepares another a pair of particles in state $|\varphi\rangle_{34}$ which is in the same entangled state as the quantum channel $|\varphi\rangle_{12}$. The initial state of the whole system consisting of the four subsystems is then given by

$$|\psi\rangle_{1234} = |\varphi\rangle_{12} \otimes |\varphi\rangle_{34} \quad (16)$$

For convenience, now we assume mode 1 on Bob's side and modes 2, 3, and 4 on Alice's side, then state $|\psi\rangle_{1234}$ can be explicitly written as

$$|\psi\rangle_{1234} = \frac{1}{N_\eta} [\cos^2 \eta |\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 |\alpha\rangle_4 - \frac{1}{2} \sin(2\eta) |\alpha\rangle_1 |\alpha\rangle_2 |-\alpha\rangle_3 |-\alpha\rangle_4 -$$

$$\frac{1}{2} \sin(2\eta) |-\alpha\rangle_1 |-\alpha\rangle_2 |\alpha\rangle_3 |\alpha\rangle_4 + \sin^2 \eta |-\alpha\rangle_1 |-\alpha\rangle_2 |-\alpha\rangle_3 |-\alpha\rangle_4 \quad (17)$$

Secondly Alice lets modes 2 and 3 enter the input ports of the beam-splitter $BS(2)$. After interacting with the beam-splitter $BS(2)$, the state of the whole system becomes

$$\begin{aligned} |\psi'\rangle_{1234} = & \frac{1}{N_\eta} [\cos^2 \eta |\alpha\rangle_1 |\sqrt{2}\alpha\rangle_2 |0\rangle_3 |\alpha\rangle_4 - \\ & \frac{1}{2} \sin(2\eta) |\alpha\rangle_1 |0\rangle_2 |\sqrt{2}\alpha\rangle_3 |-\alpha\rangle_4 - \\ & \frac{1}{2} \sin(2\eta) |-\alpha\rangle_1 |0\rangle_2 |-\sqrt{2}\alpha\rangle_3 |\alpha\rangle_4 + \\ & \sin^2 \eta |-\alpha\rangle_1 |-\sqrt{2}\alpha\rangle_2 |0\rangle_3 |-\alpha\rangle_4] \quad (18) \end{aligned}$$

Subsequently Alice makes the two-mode photon number measurement on modes 2 and 3 at her side to finish the entanglement concentration process. When zero photon is detected at 2 mode and n (nonzero) photons are detected at 3 mode simultaneously, the state consist of modes 1 and 4 collapses into

$$|\psi'\rangle_{14} \rightarrow |\alpha\rangle_1 |-\alpha\rangle_4 + (-1)^n |-\alpha\rangle_1 |\alpha\rangle_4 \quad (19)$$

If n being odd, Bob and Alice get an entangled coherent state which is defined by the following

$$|\psi''\rangle_{14} = \frac{1}{\sqrt{N_-}} (|\alpha\rangle_1 |-\alpha\rangle_4 - |-\alpha\rangle_1 |\alpha\rangle_4) \quad (20)$$

where $N_- = 2[1 - \exp(-4|\alpha|^2)]$ is normalization factor. For state (20), we have proved it is a maximally entangled state, its amount of entanglement is exactly one ebit and the entanglement is independent of the parameters involved. From Eq. (18) we can obtain the probability of finding zero photon in modes 2 and odd photon in modes 3 simultaneously

$$\begin{aligned} P(0, \text{odd}) = & |{}_2\langle 0 | {}_3\langle \text{odd} | \psi'\rangle_{1234}|^2 = \\ & \frac{\sin^2 2\eta e^{-2|\alpha|^2}}{4N_\eta^2} \frac{(2\alpha^2)^n}{n!} \quad (21) \end{aligned}$$

This leads the probability of obtaining the maximally entangled coherent states to be

$$\begin{aligned} P = \sum_{n=\text{odd}} P(0, \text{odd}) = & \frac{\sin^2 2\eta e^{-2|\alpha|^2}}{4N_\eta^2} \sum_{n=\text{odd}} \frac{(2\alpha^2)^n}{n!} = \\ & \frac{1}{4N_\eta^2} \sin^2 2\eta e^{-2|\alpha|^2} \sinh(2\alpha^2) \quad (22) \end{aligned}$$

which implies that no matter how small the initial entanglement is, it is possible to distill some maximally entangled states from partially entangled pure states.

3 Conclusion

In summary, we have presented a simple scheme to distill the maximally entangled coherent states from partially entangled pure states. In this

scheme, we exploited the important features of the beam splitter transforming entangled coherent state. According to this schemes, we have proved, no matter how small the initial entanglement is, to distill some maximally entangled states from partially entangled pure states is possible. In our entanglement concentration scheme, the key factor is photon number measurement which must be sensitive enough to measure the number of photons and determine whether the number is even or odd. In practice this is difficult, especially for large numbers of photons, but in principle this can be done. With the recent developments in experimental techniques for generating and manipulating the single photon, such possibility is increasing. Besides, in Ref. [15], Xiao-Guang Wang described one method to distinguish even and odd n by coupling the field to a two-level atom through dispersive interaction, which can enhance the possibility to realize this scheme.

Acknowledgement *The authors would like to thank professor Kuang Leman for his helpful discussions.*

References

- [1] DENG Xiao-ran, YANG Shuai, YAN Feng-li. Quantum secret sharing with n -particle entangled state [J]. *Acta Photonica Sinica*, 2010, **39**(11): 2083-2087.
- [2] LI Yuan-hua, NIE Yi-you, LIU Jin-chang. Quantum information splitting by using a genuinely entangled six-qubit state and bell-state measurements [J]. *Acta Photonica Sinica*, 2011, **40**(2): 307-310.
- [3] GIOVANNETTI V, LLOYD S, MACCONE L. Quantum-enhanced positioning and clock synchronization [J]. *Nature*, 2001, **412**(6845): 417-419.
- [4] CAI Xin-hua, PENG Guang-han, QIAO Nao-sheng. Entanglement diversion of entangled squeezed vacuum state [J]. *Acta Photonica Sinica*, 2008, **37**(4): 829-832.
- [5] XIA Jian-ping, REN Xue-zao, CONG Hong-lu, *et al.* Quantum entanglement of the coherent state field interacting with a λ type three-level atom without rotating wave approximation [J]. *Acta Photonica Sinica*, 2010, **39**(9): 1621-1626.
- [6] HARIBALA, SACHUERFU, YANG Rui-fang, *et al.* Quantum entanglement of the squeezed coherent state interacting with a λ -type three-level atom [J]. *Acta Photonica Sinica*, 2009, **38**(7): 1846-1851.
- [7] CAI Xin-hua, GUO Jie-rong, NIE Jian-jun, *et al.* Entanglement diversion and quantum teleportation of entangled coherent state [J]. *Chinese Physics Letters*, 2006, **15**(3): 488-491.
- [8] BENNETT C H, POPESCU S, SCHUMACHER B, *et al.* Purification of noisy entanglement and faithful teleportation via noisy channels [J]. *Physical Review Letters*, 1996, **76**(5): 722-725.
- [9] CAI Xin-hua, KUANG Le-man. Entanglement concentration of entangled squeezed vacuum states [J]. *Chinese Physics Letters*, 2002, **19**(10): 1407-1409.
- [10] SHENG Y B, DENG F D, ZHOU H Y. Nonlocal

- entanglement concentration scheme for partially entangled multipartite systems with nonlinear optics [J]. *Physical Review A*, 2008, **77**(6): 062324.
- [11] CAI Xin-hua, QIAO Nao-sheng, PENG Guang-han. Entanglement concentration of single-photon entangled states [J]. *Acta Photonica Sinica*, 2010, **39**(1): 153-155.
- [12] LLOYD S, BRAUNSTEIN S L. Quantum computation over continuous variables[J]. *Physical Review Letters*, 1999, **82**(8): 1784-1787.
- [13] COCHRANE P T, MILBURN G J, MUNRO W J. Macroscopically distinct quantum-superposition states as a bosonic code for amplitude damping[J]. *Physical Review A*, 1999, **59**(4): 2631-2634.
- [14] YURKE B, STOLER D. Generating quantum mechanical superpositions of macroscopically distinguishable states via amplitude dispersion[J]. *Physical Review Letters*, 1986, **57**(1): 13-16.
- [15] WANG Xiao-guang. Quantum teleportation of entangled coherent states [J]. *Physical Review A*, 2001, **64**(2): 22302.

纠缠相干态的纠缠浓缩

蔡新华, 彭光含, 乔闹生

(湖南文理学院 物理与电子科学学院, 湖南 常德 415000)

摘要: 利用线性光学元器件对光场量子态进行操纵, 可以实现远程的量子纠缠调控和量子通讯. 通过分析光学分束器对相干态光场的作用, 发现当初始光场态是两个两部分纠缠态的直乘时, 让其中的两模通过光学分束器作用后再对其进行光子计数, 另外两模将会塌缩到新的纠缠态. 基于这个特点, 提出了一个实现部分纠缠相干态纠缠浓缩的方案. 在这个方案中, 两个部分纠缠相干态被用来作为量子信道, 通过光学分束器作用后对光场进行光子数探测时, 如果测量到光场的两模分别处于奇光子数态和零光子数态, 则光场另外的两模将塌缩到最大纠缠态, 从而完成纠缠浓缩的过程. 计算结果表明, 对于纠缠相干态, 无论其初始的纠缠是多么微弱, 利用这种方法总有一定的几率可以从中提纯出最大纠缠态.

关键词: 纠缠相干态; 纠缠浓缩; 光束分离器; 光子探测