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# Nonlinear Compensation Based on Manakov Equation for Coherent 40 Gbit/s Polarization-Multiplexed QPSK Transmission

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**Abstract**: A digital signal proccessing (DSP) algorithm, nonlinear compensation based on Manakov equation, was applied in coherent 40Gbit/s polarization-division multiplexing quadrature phase-shift keying (PDM-QPSK) transmission systems to compensate the nonlinear impairment in optical fiber. The back propagation algorithm based on Manakov equation can reduce the interaction between nonlinear impairment caused by Kerr effect and polarization mode dispersion (PMD) while the traditional back propagation based on scalar nonlinear Schrödinger equation (NLSE) neglects the PMD and cannot compensate the impairment caused by PMD in the polarization-division multiplexing transmission systems. The perfomance of nonlinear compensation based on Manakov equation was verified. Both simulation and experimental results show that the Manakov equation based algorithm exhibits better performance of 400 km-long QPSK transmission comparing with NLSE, with a Q-factor improvement of approximately 3 dB at an OSNR of 18 dB.

**Key words**: Quadrature phase-shift keying; Nonlinear compensation; Manakov equation

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## 0 Introduction

Recently, coherent detection has attracted great interest. Not only it enhances optical noise sensitivity and tolerance to linear impairment[1-2], but coherent detection of the optical signal also allows digital pre/post compensation of impairments in optical fiber transmission systems and polarization-division multiplexing (PDM), which can double the data rate. Back propagation is a promising digital signal processing (DSP) method that can compensate any deterministic impairment effect, given the channel characteristics. Dispersion and nonlinearity in fibers can be described by propagation equations. The impairments in fibers can be compensated by solving the reversed propagation equations. Thus, the launched power can be increased and then better transmission performance can be achieved by higher optical signal-to-noise ratio (OSNR). Back propagation was first proposed in OOK transmission systems[3-5]. It was also used in single-channel and WDM systems using other modulation formats, such as BPSK, QPSK and DB[6-8].

The propagation equations the most common used are the invertible nonlinear Schrödinger equation (NLSE) and its modifications. In single polarization systems scalar NLSE is used, and in PDM systems it can be rewritten as coupled-NLSE (or vector NLSE) when considering the interaction between the two polarizations[9]. Back propagation using a simpler version of coupled-NLSE, Manakov equation, in PDM-DPSK WDM systems has been presented[6].

In this paper, we extended the back propagation using Manakov equation to single-channel coherent 40 Gbit/s PDM-QPSK systems

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and carried out a comparison with NLSE. Section I describes how Manakov equation is derived and applied in back propagation in Section 1. In Section 2, the simulation and experimental setups are stated in detail. The discussion of both simulation and experimental results follows in Section 3.

# 1 Back propagation using Manakov equation

### 1.1 Back propagation

In the absence of noise, exact information about transmitted signal can be obtained at the receiver end by solving back propagation equation. If the propagation equation is defined as

$$\frac{\partial A}{\partial z}=(\hat{D}+\hat{N})A \tag{1}$$

the back propagation equation will be

$$\frac{\partial A}{\partial z}=-(\hat{D}+\hat{N})A \tag{2}$$

where $\hat{D}$ and $\hat{N}$ are the linear and nonlinear operators. In this study, we compare the different cases where NLSE and its variation (Manakov equation) are used as propagation equations. Because the analytical solution of nonlinear Schrödinger equation is difficult, symmetrically split step Fourier solution is employed to solve it.

### 1.2 Manakov equation

Signal propagation in fiber can be described by scalar nonlinear Schrödinger equation (or NLSE)[9-10]

$$\frac{\partial \boldsymbol{A}}{\partial z}=-\frac{\alpha}{2}\boldsymbol{A}+\frac{\mathrm{i}\beta_2}{2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2}+\mathrm{i}\gamma|\boldsymbol{A}|^2\boldsymbol{A} \tag{3}$$

where $\boldsymbol{A}=[\boldsymbol{A}_x\ \ \boldsymbol{A}_y]^{\mathrm{T}}$ is the electric field. The parameters $\alpha$, $\beta_2$ and $\gamma$ are the attenuation, chromatic dispersion (CD) coefficients, and the nonlinear parameter of fiber. Corresponding to the Eq. (1), the linear and nonlinear operators are given as

$$\hat{D}=-\frac{\alpha}{2}+\frac{\mathrm{i}\beta_2}{2}\frac{\partial^2}{\partial t^2}$$

$$\hat{N}=\mathrm{i}\gamma|A|^2 \tag{4}$$

The Eq. (3) describes chromatic dispersion while neglecting another linear impairment, polarization mode dispersion (PMD). When considering the interaction between PMD and nonlinear impairment, we have the coupled-NLSE[11]

$$\frac{\partial A_x}{\partial z}=-\frac{\alpha}{2}A_x+\frac{\mathrm{i}\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2}+\mathrm{i}\gamma\Big(|A_x|^2+\frac{2}{3}|A_y|^2\Big)A_x+\frac{\mathrm{i}\gamma}{3}A_x^*A_y^2$$

$$\frac{\partial A_y}{\partial z}=-\frac{\alpha}{2}A_y+\frac{\mathrm{i}\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2}+i\gamma\Big(|A_y|^2+\frac{2}{3}|A_x|^2\Big)A_y+\frac{\mathrm{i}\gamma}{3}A_y^*A_x^2 \tag{5}$$

In Eq. (5) we can see that propagation in one polarization direction is affected by signal pulse in another polarization. The Eq. (5) can be simplified in some specific cases. In channels where nonlinear interaction length is much greater than the length of random polarization rotations, Manakov equation can be derived by averaging Eq. (5) of the nonlinear operator over fast polarization changes on Poincaré sphere[6]

$$\frac{\partial A_x}{\partial z}=-\frac{\alpha}{2}A_x+\frac{\mathrm{i}\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2}+\frac{8\mathrm{i}\gamma}{9}(|A_x|^2+|A_y|^2)A_x$$

$$\frac{\partial A_y}{\partial z}=-\frac{\alpha}{2}A_y+\frac{\mathrm{i}\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2}+\frac{8\mathrm{i}\gamma}{9}(|A_y|^2+|A_x|^2)A_y \tag{6}$$

The nonlinear operator is

$$\hat{N}=\frac{8\mathrm{i}\gamma}{9}(|A_x|^2+|A_y|^2)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{7}$$

# 2 Simulation and experimental setups

Both computer simulation and experiment are conducted in order to show the performance of back propagation using Manakov equation.

The simulation scheme for QPSK transmission is established in VPI transmission maker and the back propagation algorithm is implemented in MATLAB. The symbol rate is 10 Gsym/s corresponding to 40 Gbit/s with polarization multiplexing. To induce significant nonlinear impairment, the transmitted signal power should be large enough. So we set the signal power to 10dBm. A 400-km-long fiber link is simulated using 5 spans of 80-km-long fiber. The attenuation, CD coefficient, the nonlinear index parameter, and PMD of fiber are respectively set to 0.2 dB/km, $16\times10^{-6}$ s/m$^2$, $2.6\times10^{-20}$ m$^2$/W and 0.1 ps/$\sqrt{\text{km}}$. The digital signal processing at the receiver end includes synchronization and back propagation algorithm followed by phase estimation.

Fig. 1 illustrates experimental setup for 40 Gbit/s PDM-QPSK transmission. A distributed-feedback laser emits an optical carrier with wavelength of 1 553 nm and a linewidth of about 100 kHz. The light is modulated by an IQ modulator with a data stream of $2^{10}$-1 pseudorandom binary sequence (PRBS) generated by Arbitrary Waveform Generator (AWG) at

10 Gsym/s to produce NRZ-QPSK pulses. The operating point of the DC bias applied to IQ modulator is adjusted by an auto bias controller. By using a polarization beam splitter (PBS), the signal is split and one polarization component is delayed by one symbol time. Afterwards, both polarization components are combined in a polarization beam combiner (PBC). The light is amplified by an erbium doped fiber amplifiers (EDFA) and launched in the fiber link. EDFA is also used to control the transmitted signal power. The transmission fiber link consists of 5 fiber spans of 80 km standard single mode fiber without optical dispersion compensation, which has a dispersion of about $16\times10^{-6}$ s/m$^2$ and a nonlinear index coefficient of about $2\times10^{-20}$ m$^2$/W. EDFAs are used to compensate the loss of fiber in each span, which is about 16dB for 80 km-long fiber. After propagation over 400 km, the transmitted signal is amplified by EDFA and filtered by a filter with a bandwidth of 50 GHz in order to reduce the out-of-band noise. Then the signal is fed into a homodyne receiver. Received signal and local oscillator with linewidth of 100 kHz are combined at $2\times8$ 90° hybrid. The in-phase and quadrature components of two polarizations are obtained by four balanced detectors. Digitized by a 50 Gsa/s digital storage oscilloscope, the sampled signals are processed offline using MATLAB. The 50 Gsa/s digital signal is resampled to give 2 samples per symbol. And then we apply the algorithms mentioned above to demodulate QPSK signals.

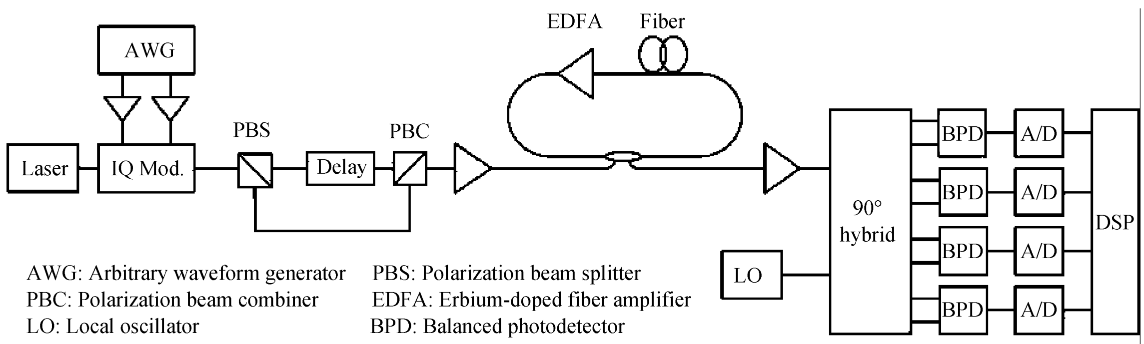


Fig. 1 The experimental setup of QPSK transmission system

## 3 Results and discussion

We compare the performance of back propagation based on the (coupled-) NLSE and Manakov equation in simulation and experiment.

Since Manakov equation focuses on interaction between PMD and nonlinear impairment effects, the transmitted signal power is set to be 10 dBm to simulate large nonlinear effect in VPI, which leads to an average nonlinear phase shift of about 1.5 rad. We obtain the bit error rate (BER) of QPSK by counting the errors, and further calculate $Q$-factor from BER by numerical approximation given as: $Q=\sqrt{2}\,\mathrm{erf}(2\cdot\mathrm{BER})$.

The calculated $Q$-factor is related to the number of steps in each span. The split step Fourier method we apply to solve the back propagation equation is a numerical solution. Every span is divided into several steps, and in each step the signal is propagated backward[9-10]. The step number of each span can be controlled in two ways. The first is to divide the span into steps by length. The step number is length of span divided by a fixed step size. For instance, a step size of 20 km means a step number of 4 for 80 km-long span. We apply the other method to divide fiber span by nonlinear phase shift. Since average nonlinear phase shift is related to the transmission power and the peak signal power declines during propagation due to pulse expansion caused by dispersion, the step size should be calculated carefully to introduce the same nonlinear phase shift. The step number is related to the value of selected nonlinear phase shift. Small phase shift leads to a larger step number and a higher computational complexity of DSP algorithms.

Fig. 2 shows that the effectiveness of back propagation using Manakov equation depends on the number of steps per span. Manakov equation (one step per span, corresponding to 80 km of step size and about 50° of nonlinear phase shift) presents an 8.7 dB $Q$-factor. To obtain a $Q$-factor of 8.9 dB, step number per span for NLSE should be 5 or more. It can be seen that Manakov equation can provide similar performance with lower computational complexity. But there seems to be a limitation in $Q$-factor as step number increases. The maximum $Q$-factor for Manakov equation that

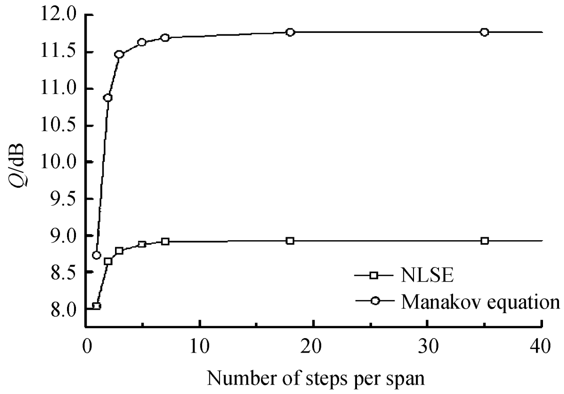


Fig. 2 Q-factors as function of step number

can be achieved is 11.8 dB, which is 2.9 dB higher as compared with that for NLSE.

As shown in Fig. 3, the constellation for back propagation with Manakov equation exhibits better performance than NLSE. The nonlinear phase shift we choose is 3°, corresponding to a step number of 7. Because of the PMD information contained in the equation, Manakov equation based algorithm provides better DSP compensation performance.

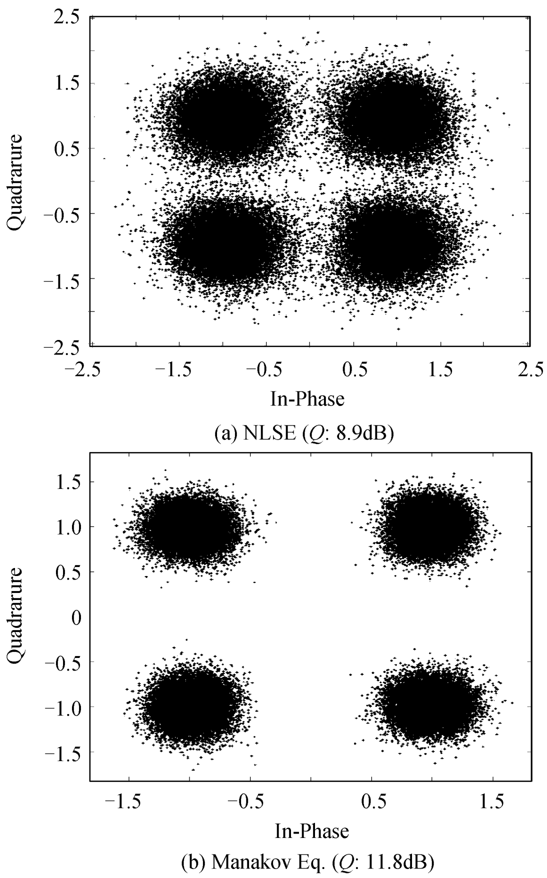


Fig. 3 Constellations for QPSK signal after NLSE-based and Manakov equation-based DSP

The Manakov equation gives better BER results than NLSE not only in simulation but also in a practical system. The launched power is adjusted to 10 dBm, including signal and noise. Before transmission of 400 km, the optical signal-to-noise ratio (OSNR) is changed by an attenuator and an EDFA and monitored by an optical spectrum analyzer. Q-factor calculated as a function of the optical signal-to-noise ratio (OSNR) at a launched power of 10dBm is presented in Fig. 4. At a low OSNR of 8 dB, the interaction of noise and nonlinearity in fiber can not be neglected, so (coupled-) NLSE and Manakov equation all perform badly. They attain nearly the same Q-factor of about 3.5 dB. As OSNR grows, signal power rises in launched power. For NLSE, the Q-factor increases at the OSNR from 8 dB to 14 dB, but it stays about 6.8 dB when OSNR gets 14 dB or above. When the signal power is large enough, the performance of algorithms is constrained by nonlinearity in fiber because it now becomes the most important limiting factor rather than noise. Coupled-NLSE gives similar results with NLSE when signal power is small and OSNR is low. However, once OSNR gets higher, coupled-NLSE obtains higher Q-factor due to its PMD term while NLSE reaches its limitation. The coupled-NLSE and its simplified version, Manakov equation, improve the nonlinear tolerance. The disparity between NLSE and coupled-NLSE increases when OSNR grows from 14 dB to 18 dB and gets to 1.2 dB at 18 dB OSNR. Taking advantage from both the PMD term and the averaging over polarization shift, Manakov equation shows the best results. As OSNR grows, there is an increase in Q-factor improvement between (coupled-) NLSE and Manakov equation based back propagation. The Q-factor enhancement is up to about 3dB for Manakov equation as compared with NLSE at OSNR of 18 dB.

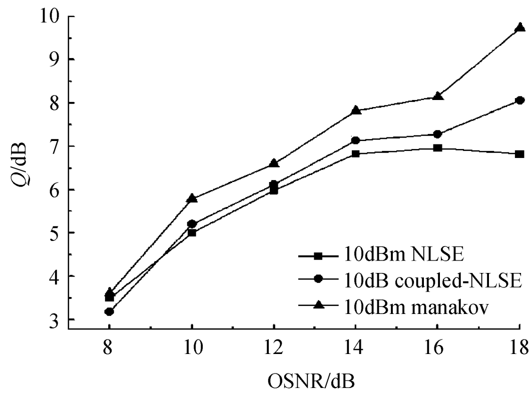


Fig. 4 Q-factor as function of OSNR after 400 km transmission

## 4 Conclusion

In conclusion, we studied the digital back propagation using Manakov equation in coherent 40 Gbit/s PDM-QPSK transmission systems. As shown in our 400 km transmission experiment, back propagation using Manakov equation improves the nonlinear tolerance and provides better performance comparing with NLSE-based back propagation, with a *Q*-factor improvement of approximate. 3 dB at an OSNR of 18 dB.

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# 40 Gbit/s 相干偏振复用 QPSK 传输中基于 Manakov 方程的非线性补偿

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**摘　要**:在 40 Gbit/s 相干偏振复用正交相移键控(QPSK)传输系统中,为了补偿由于光纤中非线性效应引起的传输信号损伤,采用了基于 Manakov 方程的反向传播非线性补偿算法.传统的基于标量非线性薛定谔方程(NLSE)的反向传播算法忽略了偏振模色散(PMD)的作用,因此在偏振复用系统中不能补偿由于 PMD 引起的信号损伤;而基于 Manakov 方程的数字信号处理方法能够对 PMD 与克尔非线性效应的耦合作用进行补偿.从仿真与实验两个方面对此方法在 40 Gbit/s 相干偏振复用 QPSK 传输系统中的补偿效果进行了验证.结果均表明,与 NLSE 相比,基于 Manakov 方程的反向传播算法在 400 km 长距离 QPSK 传输中显示出更好的性能.在光信噪比(OSNR)为 18 dB 时,基于 Manakov 方程的反向传播算法得到的 *Q* 值与 NLSE 相比提高约 3dB.

**关键词**:正交相移键控;非线性补偿;Manakov 方程