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The Effects of Radial and Positional Disorders on Two-dimensional Photonic Crystal Waveguides with Different Air-holes

ZHANG Xuan, TIAN Hui-ping, JI Yue-feng

(*Key Laboratory of Information Photonics and Optical Communications*, (*Ministry of Education*), *School of Information and Communication Engineering*, *Beijing University of Posts and Telecommunications*, *Beijing* 100876, *China*)

Abstract: The impact of positional and radial disorders on the transmission spectrum of photonic crystal waveguides with different shapes of air-holes, including circular, square and elliptic were numerically investigated. It was found that the disorder affects the anti-crossing point greatest. The flat transmission spectrum becomes narrower as disorder increasing. The transmission property performs more sensitive on the air-holes positions than radii. The square air-holes are more robust against the positional disorder. The circular air-holes are more robust alainst the radial disorder. These results provide important theoretical instructions on the imperfection during the manufacture possess in the PCWs experiment.

Key words: Photonic Crystal Waveguide (PCW); Disorder; Transmission spectra; Poynting vectors

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0 Introduction

In next generation information networks, light is applied for efficient data transmission between network nodes, but in the time domain the optical signals is difficult to control. Path switching of optical packets and photonic routers to avoid the optical-electronic conversion become very important. A key device is the buffer that temporarily stores and adjusts the timing of optical packets. Solutions with a high data rate, high throughput and low power consumption are required. Slow light provides a promising solution for buffering and time-domain processing of optical signals, such as retiming, multiplexing and performing convolution integrals. It also offers the possibility for spatial compression of optical energy and the enhancement of linear and nonlinear optical effects[1-2].

Photonic-crystal (PC) devices are especially attractive for generating slow light, as they are compatible with on-chip integration and room temperature operation, and can offer wide-bandwidth and dispersion-free propagation. There are several methods of manufacturing photonic crystals, such as interference lithography, photolithographic etching two-dimensional structures, and self-assembling tridimensional structures[3-4]. Although several improvements on the experimental techniques in micro- and sub-micro scale manipulations have been achieved, some disorder may be introduced in the PC structure during the manufacturing processes[5-9]. A line defect in a photonic crystal slab can truly guide modes which do not exhibit out-of-plane radiation loss theoretically if appropriately designed. In line defects, light is confined due to a combination of 2D PBG and 1D TIR[10]. Because 2D simulation method can greatly decrease the complexity of calculation, the influence of the disorder over the transmission of 2D-PCW is simulated for manufacturing practical devices based on these structures.

Recently, disorder in 2D-PC waveguides has been studied taking into account different kinds of imperfections in the fabrication process, for

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First author: ZHANG Xuan (1988—), female, M. S. degree candidate, mainly focuses on photonic crystals and optical communication. Email: zhangxuanbeijing08@163.com

Corresponding author (Contact author): JI Yue-feng (1963—), male, professor, Ph. D. degree, mainly focuses on broadband communication networks and optical communications, with emphasis on key theory, realization of technology and applications.. Email: jyf@bupt.edu.cn

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instance, the positions, radii, and refractive indices of the cylinders that compose the structure[10-13]. Disorder induces inherent, coherent and multiply scattering loss has also been calculated[14-18]. In this paper, we numerically investigated the effect of positional and radial disorder on the transmission spectrum of 2D hexagonal lattice PCW with three different shapes of air-holes for transverse electric TE modes. We mainly concern three interesting regions -- the low and high frequency gap edges and the anti-crossing point. Firstly we calculate that the positional and radial disorder affects the transmission spectrum on the PCW with circular air-holes. Then at the same fill ratio, we compare the robust against the PCWs with circular, square and elliptic air-holes.

1 Model

1.1 The model of perfect PCW without disorder

In this paper, we conduct research on W_1 photonic crystal (PC) waveguide sandwiched between two conventional dielectric waveguides. The waveguides were produced by removing a line of cells from a finite 30×13 air holes in hexagonal lattice as shown in Fig. 1.

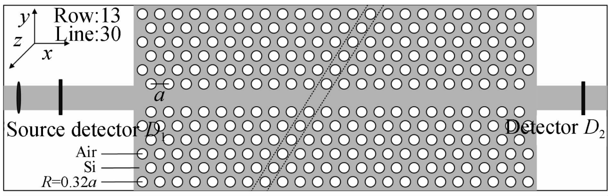


Fig. 1 Schematic diagram of the perfect photonic crystal waveguide (without disorder)

The refractive indices of the background material Si and the air holes are 3.5 and 1. The radii of the circular air holes are $r=0.32a$, where a is the PC lattice constant. A pulse source injects light into the system. The detector line D_1 at the input side of the waveguide is as the reference of the light corresponding to the incident wave. Detector line D_2 is applied to calculate the transmission spectrum through the PC waveguide.

To obtain the transmission spectral properties of the PC devices, one needs to calculate the average power flow, which is computed by spatially integrating the energy flux $S(\omega)$, the Poynting vector. The average power flux is defined by the following formula

$$S_{av}(\omega)=1/2\mathrm{Re}[E(\omega)\times H^*(\omega)] \quad (1)$$

The transmission spectrum is the ratio between the transmitted power and the incident power, while the power flow is computed by integrating Poynting vectors S_x along the x-direction,

$$S_x(\omega)=1/2\mathrm{Re}[E_y(\omega)\times H_z(\omega)^*)] \quad (2)$$

where $E_y(\omega)$ and $H_z(\omega)$ are Fourier transform (FT) of E_y and H_z.

1.2 The model of disorder PCWs with different air-holes

We also compared the transmission spectra of three kinds of PCWs with different air-holes shapes, as circular, square, and elliptic shown in Fig. 2. The radius of circular air-holes shown in Fig. 2 (a) is $0.32a$, where a is the lattice constant. To obtain the same fill ratio, the width of the square air-holes in Fig. 2 (b) is $0.283\,5a$, the semi-major A of elliptic is $0.4a$, the semi-minor B is $0.256a$, as shown in Fig. 2 (c). Air hole positional and radial disorders distributions are independent with that of other air holes. Here we only consider the case $A/B=0.4/0.256$ for the elliptic holes that the light is well confined in the waveguide. So the conclusion might be different for a different value of A/B.

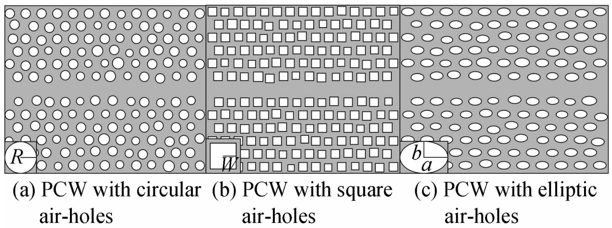


Fig. 2 PCWs with different air-holes shapes with radial and positional disorders

2 Simulation results

2.1 The perfect photonic crystal waveguide without disorder

The band properties of PCW were calculated with the plane-wave expansion (PWE) method. The defect modes inside the band gap are studied with a supercell that is 1 unit in the x-direction and 10 units in the y-direction, as shown in the dashed box diagram of Fig. 1. The common 2D triangle lattice PCW with circular air-holes supports: even and odd in the PBG[19] as shown in Fig. 3 (a). The odd mode is difficult to couple with a mono-mode dielectric waveguide due to the symmetry mismatch. The even gap guided mode concentrate most of its energy in the energy in the unstructured part of the waveguide[20]. The flat band edge of the even mode is theoretically slow light region.

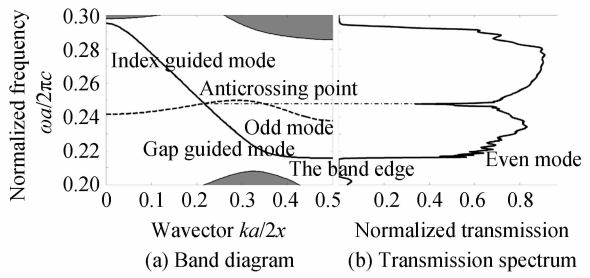


Fig. 3 Band diagram and transmission spectrum of 2D PCW with circular air-holes

The transmission spectra are calculated by employing the Finite-difference time-domain (FDTD). The absorbing boundary conditions were applied to the sides of the structure taking into account perfectly matched layers (PML′s). We performed between the normalized frequencies ranges 0.2～0.3(a/λ) around the PBG. Fig. 3 (b) sketches the transmission spectrum of the PCW with circular air-holes. All the distinctive features of the dispersion behavior of the PCW modes are reflected. We can see that there is a transmission dip at the anti-crossing point for the intrinsic interaction of even mode and odd modes near 0.248 (a/λ). Due to the fast change of group velocity at the anti-crossing point, the group velocity dispersion greatly increases and seriously distorts the signal.

2.2 The photonic crystal waveguide with disorder

In the following we numerically investigated the effect of positional and radial disorders on the transmission of 2D perfect and disorder PC waveguides with circular air-holes for transverse electric TE modes respectively.

2.2.1 The effects of positional disorder on PCW with circular air-holes

The positional disorder is introduced in the structure by randomly changing the position of the air holes from their original position in the ordered waveguide. Here the position is determined by x-axis and y-axis, defined as p_x and p_y. The relationship between its final p_x and p_y and its original position p_{ox} and p_{oy} are given by

$$p_x = p_{ox} + \delta(1-2\gamma_x)/2 \quad (3)$$

$$p_y = p_{oy} + \delta(1-2\gamma_y)/2 \quad (4)$$

Where γ_x and γ_y are the random variable uniformly distributed in the interval[0, 1], and δ quantifies the degree of disorder. The two air-holes were separated by the lattice constant a, the positional disorder lies in the range of[$-0.5a$, $0.5a$] to avoid overlap.

Employing this approach, the effect of positional disorder in the PC waveguide was investigated on the average transmission spectrum. Three different configurations were studied for δ equal to 5%, 10%, and 15%, as shown in Fig. 4. We find the disorder affects the anti-crossing point greatest. Disorder destroys the periodicity of the structure which makes index guided mode difficult to couple to the gap guided mode. The flat region of the transmission becomes narrower as the disorder increases.

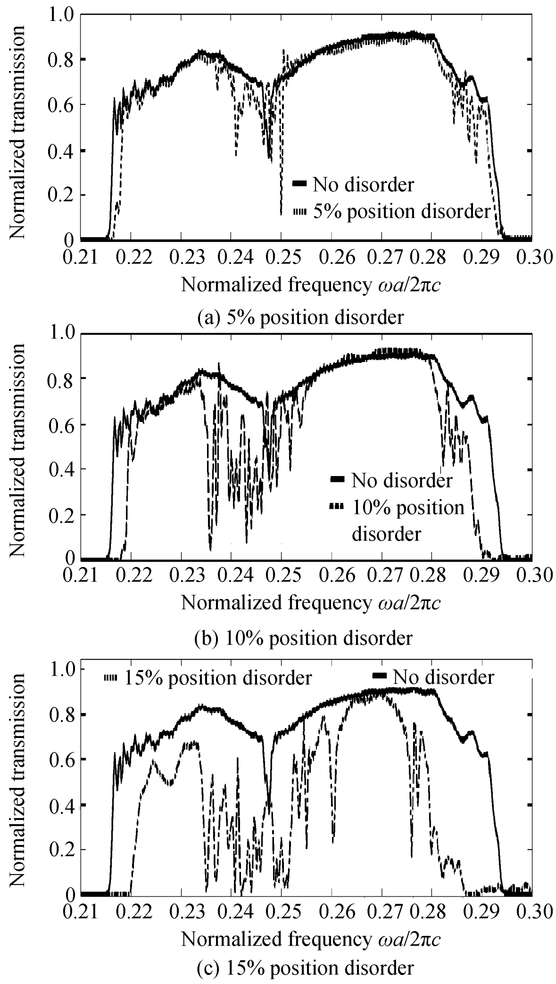


Fig. 4 The normalized transmissions for different degree of the positional disorder PCWs

We then calculated the average transmission and the flat bandwidth in the three regions shown in Table 1. First we can obtain that, in low frequency region, the flat bandwidths of PCW with 5%, 10%, 15% positional disorder are respectively 72%, 49.5%, 42% of the bandwidths of PCW with no disorder. The average transmission here decreases as the disorder degree increases. Correspondingly, we calculated that, in high frequency flat region, the bandwidths of PCW with 5%, 10%, 15% positional disorder are respectively 91%, 80.5%, 54% of those of PCW with no disorder.

Table 1 The average transmission and flat bandwidth in different positional disorder situations

Positional disorders	Bandwidths average	Fow frequency flat region	Anti-crossing point	High frequency flat region
No disorder	Bandwidths	0.216～0.246 5	0.246 5～0.248 5	0.248 5～0.292
	Average	0.713 7	0.578 2	0.813 4
5% disorder	Bandwidths	0.218 5～0.240 5	0.240 5～0.250 5	0.250 5～0.290
	Average	0.705 3	0.576 4	0.795 6
10% disorder	Bandwidths	0.219 8～0.235	0.235～0.252	0.252～0.287
	Average	0.676 5	0.460 7	0.811 6
15% disorder	Bandwidths	0.222～0.234 8	0.234 8～0.256	0.256～0.279 4
	Average	0.550 3	0.309 4	0.734 9

The flat region of PCW with 15% positional disorder becomes significantly narrow. As comparison, we find the average transmission rate in the high frequency is more stable than the low frequency.

2.2.2 The effects of radial disorder on PCW with circular air-holes

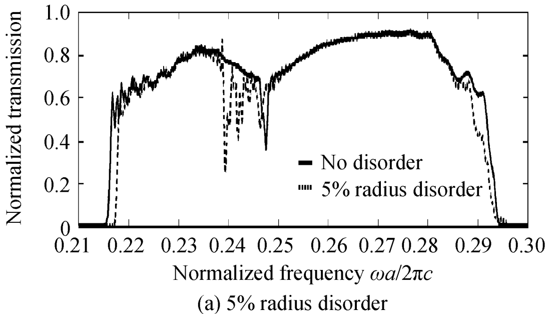
Similarly the radius disorder is also introduced in the structure randomly in the perfect PC waveguide. The relationship between its final R and its original position R_o is given by

$$R=R_o+\delta(1-2\gamma) \tag{5}$$

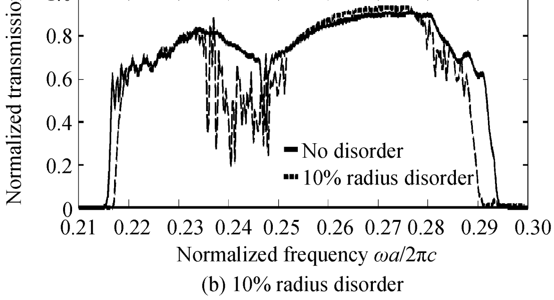
Where γ is the random variable uniformly distributed between the interval [0, 1], and δ quantifies the degree of disorder. The same three different configurations were calculated for δ equal to 5%, 10%, and 15%, as shown in Fig. 5. We find the disorder affect the anti-crossing point greatest. The flat region of the transmission also becomes narrower as the disorder increase.

We also calculated the average transmission and the flat bandwidth in the three regions as shown in Table 2. First we can obtain that, in low frequency region, the flat bandwidths of PCW with 5%, 10%, 15% positional disorder are respectively 68%, 56%, 62% of the bandwidths of PCW with no disorder. Correspondingly we calculated that, in high frequency region, the flat bandwidths of PCW with 5%, 10%, 15% positional disorders are respectively 89%, 82.7%, and 73.5% of PCW with no disorder. We also find the flat bandwidth in the high frequency is more stable than that of the low frequency and the anti-crossing region escalates.

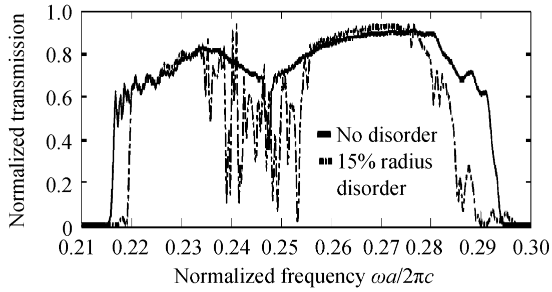
Compared the Fig. 4 with Fig. 5, we find that at the same degree of disorder, the transmission property performs more sensitive on the air-holes positions than the radii because the former breaks periodicity directly while the latter does not. The flat bandwidth in high frequency region with positional disorder performs better than that of low frequency region because the latter is in the slow light region where the guide band is influenced seriously by the periodicity mismatch. We can see that the disorder affect the anti-crossing point greatest. The flat region of the transmission spectrum also becomes narrower as the degree of the disorder increases.



(a) 5% radius disorder



(b) 10% radius disorder



(c) 15% radius disorder

Fig. 5 The normalized transmissions for different degree of the radial disorder PCWs

Table 2 The average transmission and flat bandwidth in different radial disorder situations

Radial disorders	Bandwidths average	Low frequency flat region	Anti-crossing point	High frequency flat region
No	Bandwidths	0.216～00.246 5	0.246 5～0.248 5	0.248 5～0.292
disorder	Average	0.713 7	0.578 2	0.813 4
5%	Bandwidths	0.217 9～0.238 6	0.238 6～0.249 2	0.249 2～0.288
disorder	Average	0.716 7	0.605 4	0.816 7
10%	Bandwidths	0.218 4～0.235 5	0.235 5～0.252	0.252～0.288
disorder	Average	0.708 3	0.522 0	0.839 4
15%	Bandwidths	0.219 6～0.238 5	0.238 5～0.254	0.254～0.286
disorder	Average	0.712 6	0.461 0	0.811 6

2.2.3 The effects of positional and radial disorders on PCWs with three different air-hole shapes

First we focus on the robustness of the PCWs with three different air-holes shapes against the positional disorder in Fig. 6. Fig. 6(a), (d) and (g) describe the 5% positional disorder for three different PCWs. Similarly Fig. 6 (b), (e) and (h) are the 10% positional disorder. Fig. 6(c), (f) and (i) are the 15% positional disorder. We can see that the PCWs with circular and square air-holes are more robust at frequencies 0.215(a/λ) ～0.237(a/λ) near the band edge than that of elliptic air-holes at 10% positional disorder. Continue to increase the degree of disorder, the PCW with square air holes performs better than the other two. But the elliptic is more stable at frequencies 0.25(a/λ) ～ 0.28(a/λ) with 15% positional disorder. We can see that the square air-holes perform more robust against the positional disorder, especially near band edge at the slow light region.

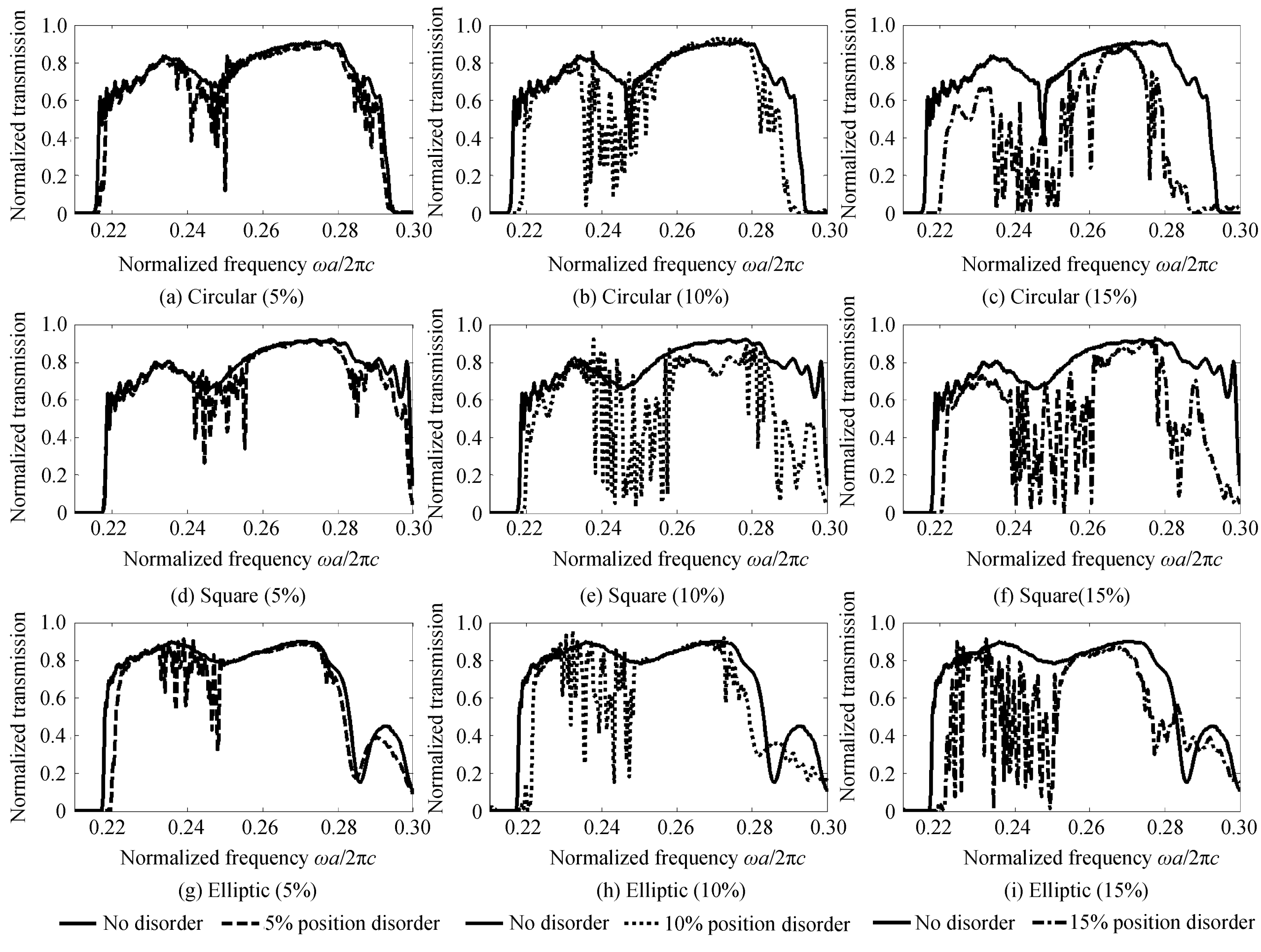


Fig. 6 Normalized transmissions of PCWs with different air-holes shapes with different positional disorder

Then we disorder the radius (R) of circular air-holes, the width (W) of the square air-holes, the semi-major (A) of the elliptic air-holes respectively. To brief our expression, we uniform the three types of disorder to be radial disorder of the air-holes. We research on the robust of the

PCWs with different shapes air-holes against the radial disorder which means disorder R of the circular, W of the square, A of the ellipse simultaneously in Fig. 7. Fig. 7(a), (d) and (g) describe the 5% radial disorders of PCWs with three different shapes air-holes respectively. Fig. 7 (b), (e) and (h) are the 10% radial disorder. Fig. 7(c), (f) and (i) are the 15% radial disorder. We can see that in the PCWs with circular and elliptic air-holes are more robust at frequencies 0.25 (a/λ) ~ 0.28 (a/λ) than that of square air-holes in 15% radial disorder. The PCW with square air holes performs worse than the other two on the radial disorder. The circular air-holes are the most robust against the radial disorder of all shapes. From Fig. 7 we can find that the PCW with the circular air-holes performs more robust against the radial disorder as the disorder increases.

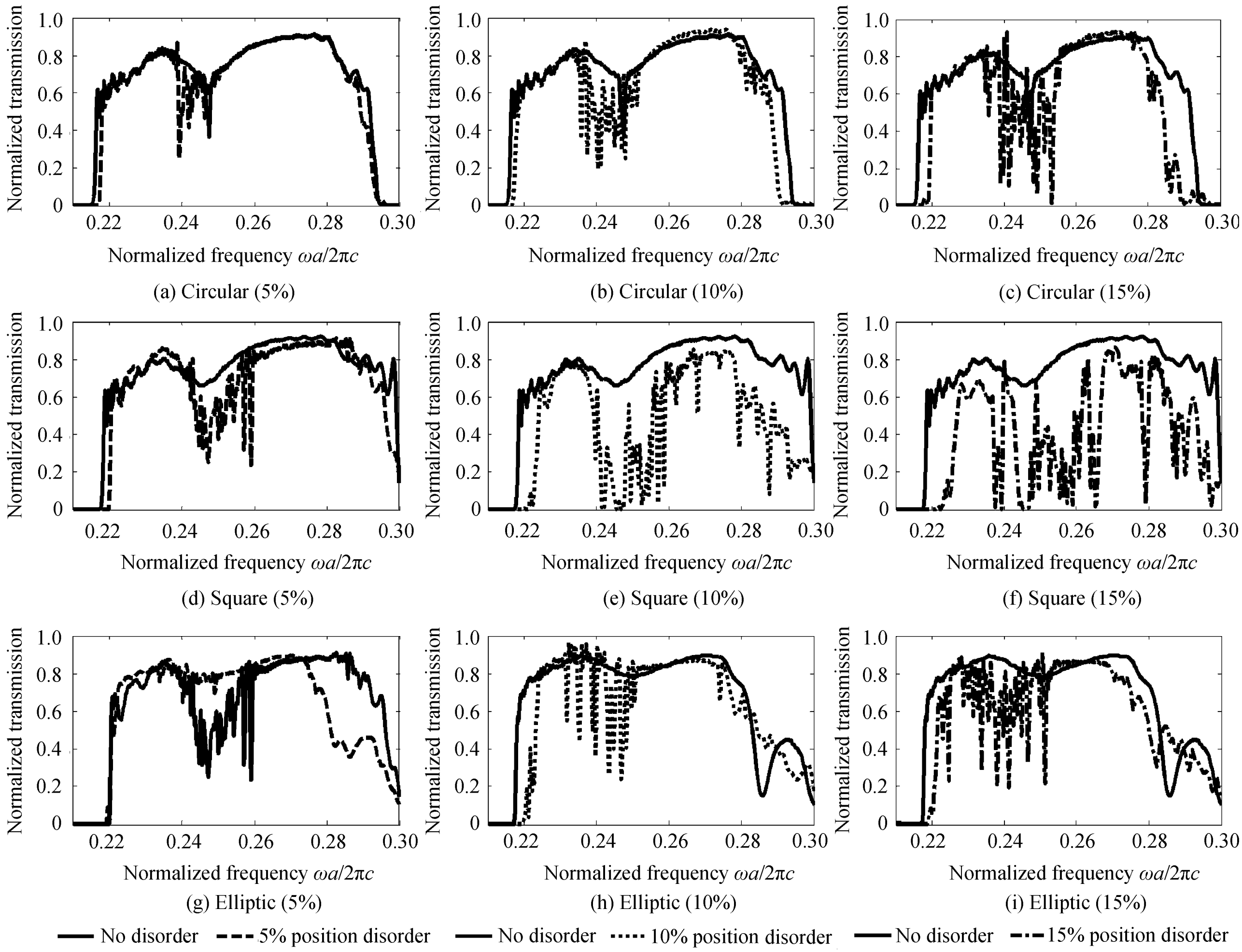


Fig. 7 Normalized transmissions of PCWs with different air-holes shapes with different radial disorder

3 Conclusion

We numerically investigate the effects of positional and radial disorder on the transmission and energy distribution of 2D hexagonal lattice PCWs with different shapes of air-holes, including circular, square and elliptic. We can see that the disorder affect the anti-crossing point most greatly. The flat region of the transmission spectrum also becomes narrower as the degree of the disorder increase. The transmission property performs more sensitive on the air-holes position than the radius, because the former breaks periodicity directly but the latter does not. For different shapes of air-holes, we find that the square air-holes are more robust against the positional disorder, especially in the slow light region and the circular air-holes are better robust again the radial disorder. These researches provide important theoretical instructions for the imperfection during the manufacture possess in the PCWs' experiment and future applications in the optical network.

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半径位置微扰对形状不同的空气孔结构二维光子晶体波导透射的影响

张璇,田慧平,纪越峰

(北京邮电大学 信息与通信工程学院 信息光子学与光通信教育部重点实验室开放互联网实验室,北京 100876)

摘　要:分析了不同空气孔形状的光子晶体受位置和半径微扰对于透射的影响,包括圆形,方形与椭圆形.仿真结果发现及形状相同微扰程度下,通过比较透射平坦带宽部分的分析得到透射率对位置微扰更加敏感.慢光区域正方形空气孔对于位置微扰表现较好,圆形空气孔对于半径微扰更稳健.由于不同形状的空气孔光子晶体波导结构有各自的优势,因此研究微扰对透射率的影响对于光子晶体波导实验和制作过程有重要的理论指导意义.

关键词:光子晶体波导;微扰;透射谱;坡印廷矢量