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Efficient Entanglement Distribution by Separable States

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Abstract: An effcient entanglement distribution protocol was proposed using a qudit (4-dimensional space) as a separable mediating ancilla. Two qubits were used to describe a qudit in order to simplify the description. Firstly, both of the two ancilla qubits and the two distant qubits (a and b) were formed into a four-qubit unlockable bound entangled state, called the Smolin state. Then, a projective measurement in the Bell basis was performed on the two ancilla qubits and the measurement results was delivered to a and b. Based on the measurement results, the qubits a and b can be converted into a standard singlet state locally. At any stage of the protocol, the two ancilla qubits was always separable from the subsystem (a and b). This scheme provides an effective way to distribute the maximally entangled states by separable states, and presents a concise formalism to describe and understand this physical process.

Key words:Entangment; Separable mediating ancilla; Projective measurement

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0 Introduction

The discovery of entanglement, apart from being a fascinating founding feature of quantum theory, which can be employed as a resource for novel or enhanced processing, manipulation, and distribution of information [1], has spurred enormous theoretical and experimental progress.

Entanglement assisted quantum communication requires the communicating parties, Alice and Bob, to establish an entangled state between their qubits a and b. Due to the conservation law of entanglement, e. g. the amount of entanglement can not be increased by local operations and classical communication [2-3], and the generation of entanglement can be done only by using a global quantum operation. There are several ways to establish entanglement between two distant parties.

1 A protoco forentanglement distribution with separated state through the discrete procedure proposed by a qudit as the separable mediating ancill

There are several ways to establish entanglement between two distant parties. The simplest way is that two particles interact together to generate entanglement, then separate and propagate two remote stations. For instance, remote entanglement of photons can now be achieved in a robust manner using the well-developed technology of spontaneous parametric down-conversion, with propagation to remote locations by means of optical fbers. The other way for the case of the two remote particles never meeting together in one place is to send a mediating (or ancilla) particle between them via quantum channel. It is clear that the two remote particles can be entangled by entangling the particle a with the ancilla, sending the ancilla through the channel, and swapping it with the second particle b. This scheme has been demonstrated experimentally[4], in which entangling two remote atomic qubits achieved by generating an entangled state of an atomic qubit and a single photon at site A, transmitting the photon to site B in an adjacent laboratory through an optical fiber, and converting the photon into an atomic qubit. The third way also for the case of the two remote particles never meeting together is that two remote particles are entangled with two ancilla particles respectively, then two ancilla particles meet together to perform

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a joint Bell measurement, so two remote particles convert their state into the entangled state by local operations according the joint measurement results. This scheme corresponds to entanglement swapping[5].

Many such physical protocols have been proposed for entangling two distant atoms, which are essentially based on the first entangling atoms with their emitted photons, and then performing entanglement swapping from the photons onto the atoms by photodetection in a Bell measurement setup[6-8]. This method have been applied to realize entanglement of two distant ions experimentally[9]. In the last two schemes, one would expect naturally that the ancilla necessarily must be entangled with the other two systems. However, the counterintuitive effect of entanglement distribution by separable ancilla was studied[10], in which particle a can be entangled with b by sending the ancilla c that becomes never entangled with the subsystem (*ab*). It turns out that certain eparable states still possess correlations of a quantum nature and indicates that quantum correlations are more general than entanglement[11]. The protocol for entanglement distribution by separable states was also generalized for Gaussian states of infinitely dimensional quantum systems[12]. Another important subject of investigating entanglement is its decoherence and distillation. Entanglement is a very fragile resource, easily destroyed by the decoherence processes to become mixed owing to unwanted coupling with the environment. Therefore, it is important to know which mixed states can be distilled to maximally entangled states from many identical copies by means of local operation and classical communication (LOCC). A surprising discovery in this area is that there exist mixed entangled states from which no pure entanglement can be distilled, and these states are called bound entangled states[13]. This new class of states is between separable and free-entangled states. Much effort has been devoted to the characterization and detection of bound entanglement[14] and various properties and applications of bound entanglement have been found. The distillability of multipartite entangled states, however is much more complicated than that of bipartite entangled states. Usually, a multipartite entangled state is bound entangled if no pure entanglement can be distilled between any two parties by LOCC when all the parties remain spatially separated from each other. However, a multipartite bound entangled state may be unlocked or activated. If all the parties are organized into several groups, and let each group join together and perform collective quantum operations, then pure entanglement may be distilled between some to different groups and this state will be called an unlocked or activable bound entangled state. A famous class of multipartite unlockable bound entangled states is the Smolin state[15], which is a four-qubit state and is generalized recently into an even number of qubits[16-17]. These states have been applied in remote information concentration[18], quantum secret sharing[19], superactivation[20-21]. Especially, the link between multipartite unlockable bound-entangled states and the stabilizer formalism was found[22]. The properties of the multipartite unlockable bound-entangled states can be easily explained from the stabilizer formalism. Recently, the four-qubit unlockable bound-entangled state (Smolin state) was demonstrated experimentally with polarization photons.

In this paper, a protocol for entanglement distribution with separated state through the discrete procedure is proposed by using *a* qudit ($d=4$) as the separable mediating ancilla. Here *a* qudit is described by two qubits in order to present the simple description. In the medium step, the four-qubit unlockable bound-entangled state is generated. Therefore, the last step of this protocol becomes the distillation process of Smolin state: two qubits of ancilla meet to unlock the entanglement. The entanglement that is distilled will be pure and maximal. Compared with the scheme proposed in Ref. [10], in which the ancilla consists of *a* qutrit ($d = 3$) and the mixed (distillable) entangled state is achieved at end, our protocol is more effcient. Due to the introduction of the four-qubit unlockable bound-entangled state, this scheme becomes easy and simple to understand entanglement distribution with separated state.

The protocol is schematically depicted in Fig. 1. The aim of the protocol is to entangle qubit *a* in a station with qubit *b* in the other distant station by a separable mediating ancilla consisted of a qudit, which is described by two qubits *c* and *d*. Here, the discrete procedure is considered, in which the interactions are described by unitary

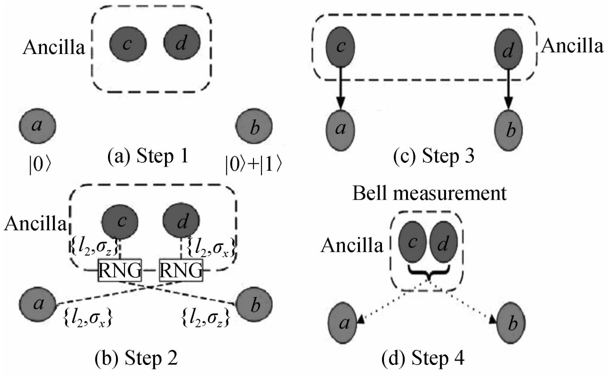

Fig. 1 Schematic of entanglement distribution with separated state through the discrete procedure. The two qubits are used as the separable mediating ancilla

operations. The scheme starts with four-qubit pure state.

Step 1: Preparation of a four-qubit pure state. The qubits a and d are prepared in the eigenstate $|0\rangle f$ Pauli spin operator σz and b, c are prepared in the eigenstate $|+\rangle(|+\rangle\ |0\rangle+|1\rangle)$ of σ_x. Thus the four-qubit pure state ρ_{abcd} is expressed as

$$\rho_{\mathrm{abcd}}=|0++0\rangle\langle 0++0| \tag{1}$$

which is product and fully separable state of the four qubits a, b, c, and d.

Step 2: Preparation of a four-qubit fully separable state by local classically correlated operations. The qubits a and d are transformed randomly and correlatively (the same operations acting on the a and d) the state $|0\rangle$ to $|0\rangle$ or $|1\rangle$. This is done by a random-number generator (RNG) controlling the unitary operations $\{12, \sigma_x\}$ simultaneously. The qubits b and c are also transformed randomly the state $|+\rangle$ to $|+\rangle$ or $|-\rangle$, which is done by a RNG controlling the operations $\{12, \sigma_z\}$ simultaneously. So the four-qubit fully separable state is expressed as

$$\rho^2_{\mathrm{abcd}}=1/4(|0++0\ 0++0|+|1++1\ 1++1|+ \\ |0--0\ 0\ --0|+|1\ --1\ 1\ --1|) \tag{2}$$

This procedure generates the four-qubit mixed state, which possesses classical correlations. The entanglement in this step can not be generated since only local operations and classical communication are used.

Step 3: Generation of the four-qubit unlockable bound-entangled state. A controlled NOT (CNOT) operation is applied on qubits a and c (where c is the control qubit), and the other CNOT gate is applied on qubits b and d (where b is the control qubit). The resulting state is in the form

$$\rho^4_{abcd}=\sum_{i\in\{,1,2,3,4\}}\frac{1}{4}|\Psi^{(i)}_{ac}\langle|\Psi^{(i)}_{ac}|\otimes\|\Psi^{(i)}_{bd}|\rangle\langle|\Psi^{(i)}_{ac} \tag{3}$$

where $|\psi(i)\in\{|\Psi\pm\rangle, |\Phi\pm\rangle\}$, and $\{|\Psi\pm\rangle$, $|\Phi\pm\rangle\}$ are the Bell states given by

$$\Psi^{+-}\rangle=\frac{1}{\sqrt{2}}(|01\rangle\pm|10\rangle)$$

$$\varphi^{+-}\rangle=\frac{1}{\sqrt{2}}(|00\rangle\pm|11\rangle) \tag{4}$$

This state is an equal mixture of all four Bell states, which is four-qubit unlockable bound-entangled state first introduced by Smolin[15]. The Smolin state has been proved that it is the maximally mixed state over the subspace stabilized by g_1 and g_2[22]

$$g_1=\sigma^a_x\sigma^b_x\sigma^c_x\sigma^d_x$$

$$g_1=\sigma^a_z\sigma^b_z\sigma^c_z\sigma^d_z \tag{5}$$

As one know, a unique pure state of n qubits is stabilized by n independent commuting stabilizer generators, which denote a complete set of stabilizer generators. The maximally mixed state over the subspace stabilized only by two stabilizer generators can also expressed as

$$\rho^4_{abcd}=\frac{1}{16}(1+g_1)(1+g_2)= \\ \frac{1}{16}(1+\sum_{i\in\{x,y,z\}}\sigma^a_i\sigma^b_i\sigma^c_i\sigma^d_i) \tag{6}$$

In this form it can be seen easily that the state is invariant when interchanging qubits. An important property of Smolin state is that, if any one of the qubits is taken partial trace, the remaining three qubits are in a maximally mixed state and consequentially there are no correlations left.

The Smolin state is separable with respect to any 2 : 2 partition including $\{\{a, c\}, \{b, d\}\}$, $\{\{a, b\}, \{c, d\}\}$ and $\{\{a, d\}, \{b, c\}\}$, which can be concluded easily since the Smolin state can also be written as

$$\rho^4_{abcd}=\sum_{i\in\{1,2,3,4\}}\frac{1}{4}|\psi^{(i)}_{ab}\rangle\{\psi^{(i)}_{ab}\otimes|\psi^{(i)}_{cd}\rangle\{\psi^{(i)}_{cd}\otimes|= \\ \sum_{i\in\{1,2,3,4\}}\frac{1}{4}|\psi^{(i)}_{ad}\rangle\{\psi^{(i)}_{ad}\otimes|\ \psi^{(i)}_{cb}\rangle\{\psi^{(i)}_{cb}\otimes| \tag{7}$$

Thus the Smolin state satisfies the requirement of this protocol that ancilla (c and d) is separable from (a, b). Compared with entanglement swapping, the entanglement is destroyed by local correlated operations (step 2) that make the auxiliary qubits separable from (a, b). Note that the procedure of generating the four-qubit unlockable bound-entangled state in this scheme is different with that demonstrated experimentally with polarization photons recently[22], in which the product state of two pairs of singlet state is transformed randomly into Smolin state.

Step 4: Entanglement distribution (activation) between a and b. If the ancilla qubits c and d come together and make a projective measurement in Bell basis and send the result of their measurement (classical information) to a and b, who will then know which of the Bell states they share, the qubits a and b.

2 Conclusion

Compared with the schemes above-mentioned, a protocol for entanglement distribution with separated state using a qudit (4-dimensional space) as a separable mediating ancilla is more effective. First, the two qubits of ancilla together with two distant qubits (a and b) are formed into the four-qubit unlockable bound entanglement called the Smolin state. And using a qudit (4-dimensional space) as a separable mediating ancilla, this scheme links the four-qubit unlockable bound-entangled state, the distribution of pure maximal entangled state can be achieved. Two qubits of ancilla perform the projective measurement in the Bell basis and send the result of their measurement to a and b. The qubits a and b can convert their state into the standard singlet state locally. At any stage of the protocol, the ancilla consisted of two qubits is always separable from the subsystem (ab). Because of above the scheme becomes simple and easy to explain entanglement distribution with separated state. It is worth remarking that the discrete procedure was considered in this paper. It is interesting to further investigate whether the protocol can be realized through the continuous procedure. This work provides the new way to perform multipartite entanglement distribution with separable ancilla using different kinds of the multipartite unlockable bound states.

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分离状态下的有效纠缠分配

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摘 要：提出采用两个量子比特（四维空间的 qudit）作为分离的中间媒介实现高效的纠缠分发方案。首先，两个中间媒介量子比特与两个不同站点的量子比特（定义为 a 和 b）形成四个量子比特非锁定束缚纠缠态，也称为 Smolin 态. 然后，对两个附属量子比特在贝尔基矢上进行联合投影测量，测量结果传送给 a 和 b，基于测量结果，量子比特 a 和 b 将可以转换为 EPR 纠缠态. 在整个过程，两个附属量子比特始终与量子比特 a 和 b 保持分离. 该方案提供了一种利用分离态实现最大纠缠态分发的高效方法，并且给出了公式来描述和理解该过程.

关键词：纠缠；中间媒介；联合测量