第 40 卷第 2 期
2011 年 2 月

光 子 学 报
ACTA PHOTONICA SINICA

Vol. 40 No. 2
February 2011

Article ID:1004-4213(2011)02-0307-4

Quantum Information Splitting by Using a Genuinely Entangled Six-qubit State and Bell-state Measurements

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Abstract: Through introducing the new application of a genuinely entangled six-qubit state, quantum information splitting of an arbitrary two-qubit state was investigated. For the quantum information splitting, a genuinely entangled six-qubit state was shared by a sender (Alice), a controller (Charlie) and a receiver (Bob). The sender firstly performed twice Bell-state measurements, and then the controller made a Bell-state measurement. Finally the receiver can reconstruct the arbitrary two-qubit state by performing some appropriate unitary transformations on his qubits after he knew the measured results of both the sender and the controller. This quantum information splitting scheme is deterministic, i. e. the probability of success is 100%. In comparison with the quantum information splitting scheme using the same quantum channel the sender and the controller only need to perform Bell-state measurements, not joint multi-qubit measurement, which makes this scheme simpler and easier, and can be fulfilled based on present experimental technology.

Key words: Quantum information; Quantum information splitting; Six-qubit state; Bell-state measurements

CLCN: O431.2; TN918 **Document Code**: A **doi**: 10.3788/gzxb20114002.0307

0 Introduction

Entanglement lies at the heart of quantum mechanics and plays a crucial role in quantum information processing. Quantum teleportation is an important technique for transfer of information between two or more parties. Since Bennett et al. presented the first protocol of quantum teleportation through an entangled channel of Einstein-Podolsky-Rosen (EPR) pair in 1993[1]. Many teleportation protocols have been devised using multi-partite entangled states[2-3], such as the prototype-GHZ states[4], generalized W states[5] and cluster states[6]. The first scheme for quantum information splitting was proposed using an entangled three-qubit GHZ state[7]. The basic idea of quantum information splitting (QIS) is to share quantum information among a group of participants such that the original information cannot be completely reconstructed by any one of the parties by themselves[8], and it has been under the extensive investigations[9-11]. Recently, attention has turned towards the investigation of the efficacy of a number of multipartite entangled channels for the teleportation of an arbitrary two-qubit state[12-15]. The QIS of an arbitrary two-qubit state was initially realized using four Bell pairs[16], and the same purpose has also been achieved using cluster states[8] and a genuinely entangled five-qubit state[14]. In 2007, Borras et al., introduced a genuinely entangled six-qubit state which is not decomposable into pairs of Bell states[17]. The state is robust against decoherence and its entanglement still prevails after particle loss. Further it has been pointed out that no other pure state of six qubits has been found that evolves to a mixed state with a higher amount of entanglement. Choudhury et al., showed that the Borras et al.'s six-qubit state can be used for teleportation of an arbitrary two-qubit state deterministically[15]. We

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Foundation item: Supported by The National Natural Science Foundation of China(No. 60807014), the Natural Science Foundation of Jiangxi Province, China(No. 2009GZW0005), the Research Foundation of State Key Laboratory of Advanced Optical Communication Systems and Networks, and the Research Foundation of the Education Department of Jiangxi Province, China(No. GJJ09153)

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Received date: 2010-06-07 **Revised date**: 2010-07-30

think the Borras et al.'s six-qubit state will be important resources for quantum processing, which motivate us to study the application of the six-qubit state for the QIS of an arbitrary two-qubit state and thus it is expected to be more interesting.

In this paper, we proposed a new scheme for the QIS of an arbitrary two-qubit state using a genuinely entangled six-qubit state as the quantum channel. In this scheme, a six-qubit state is prepared and shared by a sender, a controller and a receiver. The sender performs Bell-state measurements (BSMs) on her qubit pairs, respectively. Then the controller makes Bell-state measurement (BSM) on her qubit pairs. Finally the receiver applies some appropriate unitary transformations on his qubits according to the measure results of the sender and the controller. Thus the task of QIS of an arbitrary two-qubit state is completed.

1 QIS of an arbitrary two-qubit state with BSMs

In Choudhury et al.'s scheme, the sender Alice first combined the original state with the genuinely entangled six-qubit state and made a four-qubit (or five-qubit) joint measurement on his qubits[15]. In order to recover the original state, the receiver Bob can ask the controller Charlie to perform a BSM (or a single-qubit projective measurement). Having known the outcomes of both their measurements, the receiver Bob can do appropriate unitary transformations to obtain the original state.

In this section, we propose a new protocol for the QIS of an arbitrary two-qubit state using a genuinely entangled six-qubit state as the quantum channel with BSMs. As we know, the joint BSM or joint multi-qubit measurement is the key step in quantum teleportation. In general, the difficulty of implementing joint multi-qubit measurement increases with the number of qubits. In experiment, the joint BSM can be more easy to realize than the multi-qubit measurement. Considering this fact, we think our scheme is more advantageous than Choudhury et al.'s scheme, and we assume that Alice will make the joint BSM instead of the multi-qubit measurement.

This QIS scheme can be described as follows. Suppose the sender Alice has an unknown arbitrary two-qubit state

$$|\psi\rangle_{ab}=\alpha|00\rangle+\mu|10\rangle+\gamma|01\rangle+\beta|11\rangle \tag{1}$$

where $|\alpha|^2+|\mu|^2+|\gamma|^2+|\beta|^2=1$. She prepares a genuinely entangled six-qubit state with six qubits 1,2,3,4,5 and 6[17]. Now, we note that the Borras et al.'s $|\psi_6\rangle$ state also can be given by

$$|\psi\rangle_{123456}=\frac{1}{4}(|00\rangle_{12}|\xi_1\rangle_{3456}+|01\rangle_{12}|\xi_2\rangle_{3456}+|10\rangle_{12}|\xi_3\rangle_{3456}+|11\rangle_{12}|\xi_4\rangle_{3456}) \tag{2}$$

where the $|\xi_i\rangle_{3456}$ $(i=1,2,\cdots,4)$ of qubits 3, 4,5 and 6 are given by

$$|\xi_1\rangle_{3456}=\frac{1}{2}(|\Phi^-\rangle_{34}|00\rangle_{56}+|\Psi^+\rangle_{34}|01\rangle_{56}+|\Psi^-\rangle_{34}|10\rangle_{56}+|\Phi^+\rangle_{34}|11\rangle_{56}) \tag{3}$$

$$|\xi_2\rangle_{3456}=\frac{1}{2}(|\Phi^-\rangle_{34}|10\rangle_{56}-|\Psi^+\rangle_{34}|11\rangle_{56}-|\Psi^-\rangle_{34}|00\rangle_{56}+|\Phi^+\rangle_{34}|01\rangle_{56}) \tag{4}$$

$$|\xi_3\rangle_{3456}=\frac{1}{2}(|\Psi^-\rangle_{34}|11\rangle_{56}-|\Phi^-\rangle_{34}|01\rangle_{56}-|\Psi^+\rangle_{34}|00\rangle_{56}+|\Phi^+\rangle_{34}|10\rangle_{56}) \tag{5}$$

$$|\xi_4\rangle_{3456}=\frac{1}{2}(|\Psi^+\rangle_{34}|10\rangle_{56}-|\Psi^-\rangle_{34}|01\rangle_{56}-|\Phi^-\rangle_{34}|11\rangle_{56}+|\Phi^+\rangle_{34}|00\rangle_{56}) \tag{6}$$

with $|\Phi^\pm\rangle=(|00\rangle\pm|11\rangle)/\sqrt{2}$ and $|\Psi^\pm\rangle=(|01\rangle\pm|10\rangle)/\sqrt{2}$ are four Bell states.

Firstly, Alice sends the qubits 3 and 4 to the controller Charlie, and the qubits 5 and 6 to the receiver Bob, respectively. Then, the combined state of the eight qubits can be expressed as

$$|\Psi\rangle_{ab123456}=|\psi\rangle_{ab}\otimes|\psi\rangle_{123456} \tag{7}$$

To achieve the purpose of QIS, Alice firstly performs BSMs on her qubit pairs $(a,1)$ and $(b,2)$, respectively, and she has 16 kinds of possible measure results with equal probability 1/16. There are also 16 kinds of corresponding collapse states $|\varphi^i\rangle_{3456}$ $(i=1,2,\cdots,16)$ of qubits 3, 4, 5 and 6 after the measurement by Alice. The outcome of the measurement performed by Alice and the corresponding collapse state of Bob-Charlie's system are shown in Table 1.

Neither Bob nor Charlie can reconstruct the original state from the above states by local operations. Then Alice tells the result of her measurement to Bob and Charlie via a classical channel. If the controller Charlie allows Bob to get the initial state that Alice wants to send to Bob, then Charlie carries out a BSM, and tells Bob about his result via a classical channel. According to Alice's and Charlie's measurement results, Bob can apply an appropriate unitary transformation on his qubits to reconstruct the original state $|\psi\rangle_{ab}$. For instance, if Bob-Charlie's system had evolved into the first state given in Table 1. i. e. Bob-Charlie's system collapses to $|\varphi^1\rangle_{3456}$, then the

Table 1　The outcome of the measurement performed by Alice and the corresponding state obtained by Bob and Charlie, where the normalization factors have been omitted for convenience

Alice′s measured result	State obtained by Bob and Charlie
$\lvert\Phi^{+}\rangle_{a1}\lvert\Phi^{+}\rangle_{b2}$	$\lvert\varphi^{1}\rangle_{3456}=\alpha\lvert\xi_1\rangle+\mu\lvert\xi_3\rangle+\gamma\lvert\xi_2\rangle+\beta\lvert\xi_4\rangle$
$\lvert\Phi^{+}\rangle_{a1}\lvert\Phi^{-}\rangle_{b2}$	$\lvert\varphi^{2}\rangle_{3456}=\alpha\lvert\xi_1\rangle+\mu\lvert\xi_3\rangle-\gamma\lvert\xi_2\rangle-\beta\lvert\xi_4\rangle$
$\lvert\Phi^{-}\rangle_{a1}\lvert\Phi^{+}\rangle_{b2}$	$\lvert\varphi^{3}\rangle_{3456}=\alpha\lvert\xi_1\rangle-\mu\lvert\xi_3\rangle+\gamma\lvert\xi_2\rangle-\beta\lvert\xi_4\rangle$
$\lvert\Phi^{-}\rangle_{a1}\lvert\Phi^{-}\rangle_{b2}$	$\lvert\varphi^{4}\rangle_{3456}=\alpha\lvert\xi_1\rangle-\mu\lvert\xi_3\rangle-\gamma\lvert\xi_2\rangle+\beta\lvert\xi_4\rangle$
$\lvert\Phi^{+}\rangle_{a1}\lvert\Psi^{+}\rangle_{b2}$	$\lvert\varphi^{5}\rangle_{3456}=\alpha\lvert\xi_2\rangle+\mu\lvert\xi_4\rangle+\gamma\lvert\xi_1\rangle+\beta\lvert\xi_3\rangle$
$\lvert\Phi^{+}\rangle_{a1}\lvert\Psi^{-}\rangle_{b2}$	$\lvert\varphi^{6}\rangle_{3456}=\alpha\lvert\xi_2\rangle+\mu\lvert\xi_4\rangle-\gamma\lvert\xi_1\rangle-\beta\lvert\xi_3\rangle$
$\lvert\Phi^{-}\rangle_{a1}\lvert\Psi^{+}\rangle_{b2}$	$\lvert\varphi^{7}\rangle_{3456}=\alpha\lvert\xi_2\rangle-\mu\lvert\xi_4\rangle+\gamma\lvert\xi_1\rangle-\beta\lvert\xi_3\rangle$
$\lvert\Phi^{-}\rangle_{a1}\lvert\Psi^{-}\rangle_{b2}$	$\lvert\varphi^{8}\rangle_{3456}=\alpha\lvert\xi_2\rangle-\mu\lvert\xi_4\rangle-\gamma\lvert\xi_1\rangle+\beta\lvert\xi_3\rangle$
$\lvert\Psi^{+}\rangle_{a1}\lvert\Phi^{+}\rangle_{b2}$	$\lvert\varphi^{9}\rangle_{3456}=\alpha\lvert\xi_3\rangle+\mu\lvert\xi_1\rangle+\gamma\lvert\xi_4\rangle+\beta\lvert\xi_2\rangle$
$\lvert\Psi^{+}\rangle_{a1}\lvert\Phi^{-}\rangle_{b2}$	$\lvert\varphi^{10}\rangle_{3456}=\alpha\lvert\xi_3\rangle+\mu\lvert\xi_1\rangle-\gamma\lvert\xi_4\rangle-\beta\lvert\xi_2\rangle$
$\lvert\Psi^{-}\rangle_{a1}\lvert\Phi^{+}\rangle_{b2}$	$\lvert\varphi^{11}\rangle_{3456}=\alpha\lvert\xi_3\rangle-\mu\lvert\xi_1\rangle+\gamma\lvert\xi_4\rangle-\beta\lvert\xi_2\rangle$
$\lvert\Psi^{-}\rangle_{a1}\lvert\Phi^{-}\rangle_{b2}$	$\lvert\varphi^{12}\rangle_{3456}=\alpha\lvert\xi_3\rangle-\mu\lvert\xi_1\rangle-\gamma\lvert\xi_4\rangle+\beta\lvert\xi_2\rangle$
$\lvert\Psi^{+}\rangle_{a1}\lvert\Psi^{+}\rangle_{b2}$	$\lvert\varphi^{13}\rangle_{3456}=\alpha\lvert\xi_4\rangle+\mu\lvert\xi_2\rangle+\gamma\lvert\xi_3\rangle+\beta\lvert\xi_1\rangle$
$\lvert\Psi^{+}\rangle_{a1}\lvert\Psi^{-}\rangle_{b2}$	$\lvert\varphi^{14}\rangle_{3456}=\alpha\lvert\xi_4\rangle+\mu\lvert\xi_2\rangle-\gamma\lvert\xi_3\rangle-\beta\lvert\xi_1\rangle$
$\lvert\Psi^{-}\rangle_{a1}\lvert\Psi^{+}\rangle_{b2}$	$\lvert\varphi^{15}\rangle_{3456}=\alpha\lvert\xi_4\rangle-\mu\lvert\xi_2\rangle+\gamma\lvert\xi_3\rangle-\beta\lvert\xi_1\rangle$
$\lvert\Psi^{-}\rangle_{a1}\lvert\Psi^{-}\rangle_{b2}$	$\lvert\varphi^{16}\rangle_{3456}=\alpha\lvert\xi_4\rangle-\mu\lvert\xi_2\rangle-\gamma\lvert\xi_3\rangle+\beta\lvert\xi_1\rangle$

outcome of the measurement performed by Charlie will be one of the Bell-states. the outcome of the measurement performed by Charlie, the corresponding state obtained by Bob and Bob′s operation are shown in the Table 2.

Table 2　Charlie′s measure result, the corresponding state obtained by Bob and Bob′s operation, where the normalization factors have been omitted for convenience

Charlie′s result	States obtained by Bob	Bob′s operation
$\lvert\Phi^{+}\rangle_{34}$	$\lvert\varphi^{1}\rangle_{56}=\alpha\lvert 11\rangle+\mu\lvert 10\rangle+\gamma\lvert 01\rangle+\beta\lvert 00\rangle$	$\begin{pmatrix}0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0\end{pmatrix}$
$\lvert\Phi^{-}\rangle_{34}$	$\lvert\varphi^{2}\rangle_{56}=\alpha\lvert 00\rangle-\mu\lvert 01\rangle+\gamma\lvert 10\rangle-\beta\lvert 11\rangle$	$\begin{pmatrix}1&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&-1\end{pmatrix}$
$\lvert\Psi^{+}\rangle_{34}$	$\lvert\varphi^{3}\rangle_{56}=\alpha\lvert 01\rangle-\mu\lvert 00\rangle-\gamma\lvert 11\rangle+\beta\lvert 10\rangle$	$\begin{pmatrix}0&0&1&0\\-1&0&0&0\\0&0&0&-1\\0&1&0&0\end{pmatrix}$
$\lvert\Psi^{-}\rangle_{34}$	$\lvert\varphi^{4}\rangle_{56}=\alpha\lvert 10\rangle+\mu\lvert 11\rangle-\gamma\lvert 00\rangle-\beta\lvert 01\rangle$	$\begin{pmatrix}0&1&0&0\\0&0&0&1\\-1&0&0&0\\0&0&-1&0\end{pmatrix}$

It is evident that there are 64 kinds of possible collapse states $\lvert\varphi^{i}\rangle_{56}$ $(i=1,2,\cdots,64)$ of qubits 5 and 6 after the measurement by Charlie. Finally, Bob can apply an appropriate unitary transformation on his qubits to reconstruct the original state $\lvert\psi\rangle_{ab}$.

Another possible scenario is that, Alice can send the result of her measurement to Bob by four classical bits of information. Then Bob and Charlie can co-operate and apply four-qubit joint unitary transformations on their qubits and convert their all possible states $\lvert\varphi^{i}\rangle_{3456}$ $(i=1,2,\cdots,16)$ into $\lvert\varphi^{1}\rangle_{3456}=\alpha\lvert\xi_1\rangle+\mu\lvert\xi_3\rangle+\gamma\lvert\xi_2\rangle+\beta\lvert\xi_4\rangle$. After performing the unitary transformation, Bob and Charlie can be spatially separated. Charlie can perform a BSM on his qubits and Bob can obtain the original state by applying an appropriate unitary operator on own qubits. These measure result performed by Charlie, the corresponding state obtained by Bob and Bob′s operation are same as in Table 2.

2　Conclusions

In this paper, we proposed a new scheme for QIS of an arbitrary two-qubit state using a genuinely entangled six-qubit state. In this scheme, a genuinely entangled six-qubit state is prepared and shared by a sender, a controller and a receiver. The sender only needs to perform BSMs on her qubits, respectively. Then the controller makes a BSM on his qubit pairs. Finally the receiver applies some appropriate unitary transformation on his qubits according to the measure results from the sender and the controller. Thus the task of QIS of an arbitrary two-qubit state is completed, and the probability of successful QIS is 100%. Without the cooperation of controller, the receiver cannot reconstruct the initial arbitrary two-qubit state by himself. Compare with Choudhury et al.′s scheme, ours is more advantageous than that, because in our scheme only need to make joint Bell-state measurements, and not to make joint multi-qubit measurements. In general, the difficulty of implementing joint multi-qubit measurement increases with the number of qubits. In experiment, the joint Bell-state measurement can be more easily to be realized than the multi-qubit measurement. Nowadays, it has been shown that the genuinely entangled six-qubit state are important resources for teleportation of an arbitrary two-qubit state, so our QIS scheme may be helpful to realize the Brown state′s potential characteristic and the protocol may be realized in the realm of current experimental technology.

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基于六粒子纠缠态和Bell态测量的量子信息分离

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摘　要：通过介绍六粒子纠缠态的新应用研究，提出了一个二粒子任意态的信息分离方案.在这个方案中，发送者Alice、控制者Charlie和接受者Bob共享一个六粒子纠缠态，发送者先执行两次Bell基测量；然后控制者执行一次Bell基测量；最后接受者根据发送者和控制者的测量结果，对自己拥有的粒子做适当的幺正变换，从而能够重建要发送的二粒子任意态.这个信息分离方案是决定性的，即成功概率为100%.与使用相同的量子信道进行二粒子任意态的信息分离方案相比，本文提出的方案只需要进行Bell基测量而不需要执行多粒子的联合测量，从而使得这个方案更简单、更容易，并且在目前的实验室技术条件下是能够实现的.

关键词：量子信息；量子信息分离；六粒子态；Bell基测量