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Arbitrarily-orientated Fan Filter and Its Application in Direction Detection of Images

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Abstract: Fan filter palys an important role in direction detection of images. In this paper, a simple but efficient method is proposed for the design of two-dimensional (2D) fan filters with arbitrary orientations. The proposed method is based on the polar Fourier transform (PFT) and a wedge-shaped filter. Firstly, the wedge-shaped filter is transformed to the polar Fourier domain. Then, by taking the rotation advantage of PFT, this wedge-shaped filter is rotated via shifting its PFT along the horizontal axis. By doing so, a series of arbitrarily-orientated fan filters are obtained and these obtained fan filters can detect the image information in arbitrary directions. Furthermore, since the design process involves no 2D optimization, the advantage of easy design is also achieved. To varify their directional sensetivity, the designed fan filters are applied to detect the texture direction of images. Experiment results show that the proposed fan filters has great potential in texture detection.

Key words: Fan filter; Polar Fourier transform; Arbitrary direction; Direction detection

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0 Introduction

Extracting the directional information of images plays an important role in image processing applications[1-3]. Two-dimensional (2D) fan filters, due to their directional sensitivity, have been widely used to this purpose.

For the design of 2D fan filters, many works have been done over the years. In the early time, fan filters were obtained by directly optimizing 2D filters[4-5], while the optimization is of high complexity. Therefore, much attention was later paid to the transformation-based design method[6-9], in which a 2D fan filter can be obtained by mapping a one-dimensional (1D) filter via the McClellan transformation. Although this method avoids the direct optimization encountered in Refs. [4-5], the coefficients of the McClellan transformation have to be computed with a complicated optimization. More seriously, the main directions of the resulting filters concentrate only in the vertical and horizontal. That is to say, the resulting fan filters can only detect the image information along the directions of 0° and 90°. To increase more orientation selections, tree-structure is recently introduced to design 2D fan filters[10,11]. However, due to the tree-structure architecture, the number of fan filters is fixed to 2^n. Since natural images typically have abundant textures and details, the fixed directions are insufficient to detect them. As a result, there is a need for an efficient method to design the fan filters with arbitrary orientations.

In this paper, a simple but efficient method is proposed for designing 2D arbitrarily-orientated fan filters. It is based on the polar Fourier transform (PFT)[12-13], of which the vertical and horizontal axes respectively denote the radial and angular directions. By utilizing this characteristic of PFT, a 2D wedge-shaped filter is rotated by shifting its PFT along the horizontal axis, and then a series of fan filters are obtained. In practice, the shift operation can be performed via multiplying the PFT by permutation matrices. Due to the fact that the permutation matrix can be computed with respect to the desired shift, the resulting fan filters can be arbitrarily-orientated. In addition, since the design process involves no 2D optimization, the

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proposed method has low design complexity. At the end of this paper, the fan filters with 12 directions are designed and then applied to the direction detection of two images, Finger and Barbara. The simulation results show that the proposed fan filter can detect the directions of texture images exactly.

In what follows, we use boldface lowercase letters to denote vectors and the boldface uppercase letters to denote matrices.

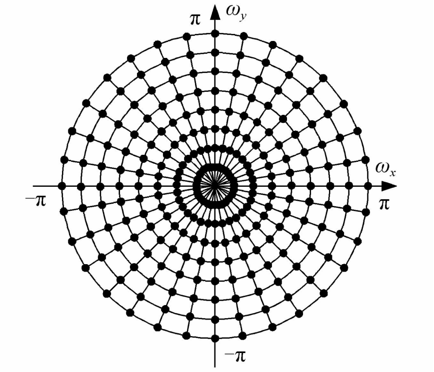
1 Polar Fourier transform

In this section, we briefly review the basic principle of PFT. The PFT proposed in Refs. [12-13] evaluates the Fourier transform on the polar grid. Such grid is composed of equispaced points along rays, where different rays are equispaced in angle. The sampled frequency points of the polar grid are given by

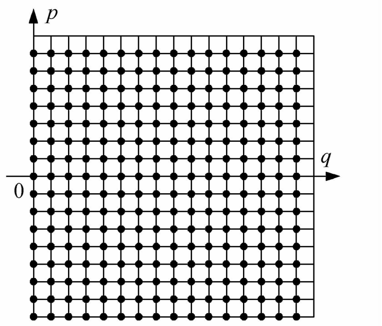
$$\begin{cases}\xi_x[p,q]=\dfrac{\pi p}{N}\cos(\pi q/2N)\\ \xi_y[p,q]=\dfrac{\pi p}{N}\sin(\pi q/2N)\end{cases}\qquad(1)$$

$$-N\leqslant p\leqslant N-1,\ 0\leqslant q\leqslant 2N-1$$

where N is the number of equispaced rays, p is the radial variable, and q is the angular variable. Fig. 1 shows an example of the polar grid with $N=8$ in the Cartesian coordinates. In the polar Fourier domain, this grid is rearranged as shown in Fig. 1(b), where the vertical axis denotes the radial direction and the horizontal denotes the angular direction.



(a) Polar grid in the Cartesian Fourier domain



(b) Polar grid in the polar Fourier domain

Fig. 1 Example of polar grid with $N=8$

2 Fan filters with arbitrary direction

From Fig. 1 it is clear that the operation of shift along the horizontal axis in polar Fourier domain is equivalent to that of rotation in Cartesian Fourier domain. Therefore, if we shift the PFT of a pre-designed wedge-shaped filter along the horizontal axis, we can obtain the fan filters with desired directions. Practically, the shift operation can be performed via multiplying the PFT by permutation matrices. Inspirited by this idea, we propose a simple but efficient method for designing arbitrarily-orientated fan filters. It involves the following procedures: 1) design a 2D wedge-shaped filter by using the existing methods; 2) compute the PFT of this filter and multiply its PFT by permutation matrices, achieving the PFT of rotated fan filters; 3) with the inverse PFT, obtain the rotated fan filters in spatial domain.

2.1 Wedge-shaped filter

With the above design procedures, a 2D wedge-shaped filter should be designed in advance. Fig. 2(a) shows the passband support of an ideal wedge-shaped filter. It can be obtained by firstly designing a parallelepiped-shaped filter as depicted in Fig. 2(b) with a simple technique of downsampling[14] and then modulating the parallelepiped-shaped filter by π along ω_y axis. The detailed steps are given below

1)Design a 1D lowpass linear-phase filter $h(n)$ with the cutoff frequency $\pi/J(\mathbf{M})$, where $\mathbf{M}=[K,-K;1,1]$, and $J(\mathbf{M})=|\det(\mathbf{M})|=2K$.

2)Construct a separable 2D filter $h(\mathbf{n})$ from $h(n)$ as

$$h(\mathbf{n})=h(n_1)h(n_2)\qquad(2)$$

3) Downsample $h(\mathbf{n})$ by $\hat{\mathbf{M}}$ to obtain a 2D parallelepiped-shaped filter $h_p(\mathbf{n})$

$$h_p(\mathbf{n})=c_0h(\hat{\mathbf{M}}\mathbf{n})\qquad(3)$$

where $c_0=2N$ and $\hat{\mathbf{M}}=[1,K;-1,K]$

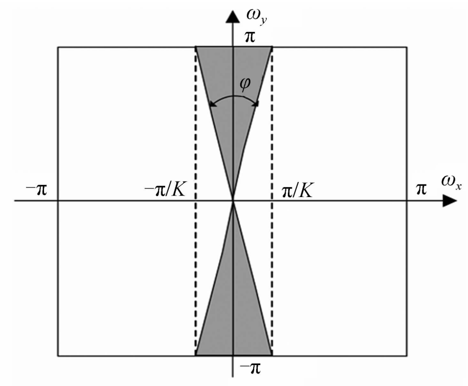
4) Obtain the wedge-shaped filter $p(\mathbf{n})$ by modulating $h_p(\mathbf{n})$ by π along ω_y axis, where

$$p(\mathbf{n})=e^{j\pi n_2}h_p(n_1,n_2)\qquad(4)$$

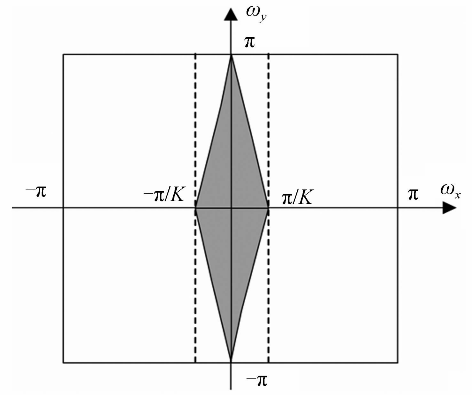
Thus, the wedge-shaped filter is obtained (see Fig. 2(a)), and its angle φ between two passband edges can be expressed as

$$\varphi=2\arctan(1/K)\qquad(5)$$

By adjusting K, the wedge-shaped filter with arbitrary bandwidth can also be achieved.



(a) Wedge-shaped filter



(b) Parallelepiped-shaped filter

Fig. 2　Passband supports of wedge-shaped and parallelepiped-shaped filters

2.2　Fan filters based on rotation

Observing Fig. 2(a)， we see that， the fan filters with different orientations can be obtained by rotating the wedge-shaped filter by different angles. Based on the above analysis in the beginning of Section 2， it is further realized that the rotation can be performed via multiplying the PFT by permutation matrices.

Here we assume the wedge-shaped filter $p(n)$ has the size $N \times N$， and denote its PFT by P. From the polar grid in Eq. (1) and Fig. 1， it is clear that the size of P is $2N \times 2N$， and its horizontal axis denotes the angular variable q. With Eq. (1)， we further know that a value of q equal to i corresponds to the angle of $\pi i/2N$， $0 \leqslant i \leqslant 2N-1$. As a result， the PFT P_i of a fan filter orientated at $\pi i/2N$ can be obtained， where

$$P_i = P \cdot \Phi_i \tag{6}$$

Φ_i is the permutation matrix and has the form below

$$\Phi_i = \begin{bmatrix} 0 & I_{i \times i} \\ I_{(2N-i)\times(2N-i)} & 0 \end{bmatrix} \tag{7}$$

where $I_{i \times i}$ and $I_{(2N-i)\times(2N-i)}$ are both identity matrices and the parameter i is determined by the number of shifted columns. With different Φ_i， we can obtain fan filters with different orientations.

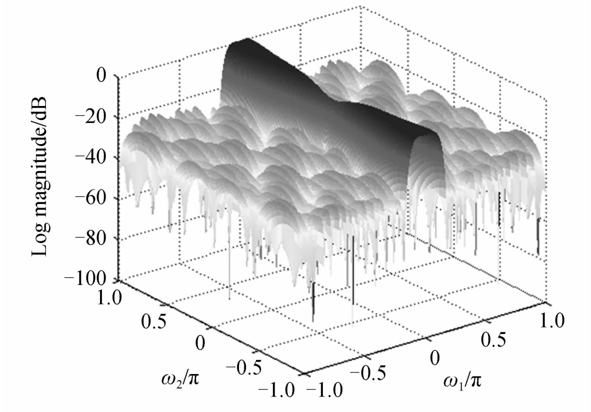
Furthermore， it should be noticed that during the above design， only a 1D low-pass filter needs to be designed， leading to low design complexity.

3　Design example and its application on direction detection of images

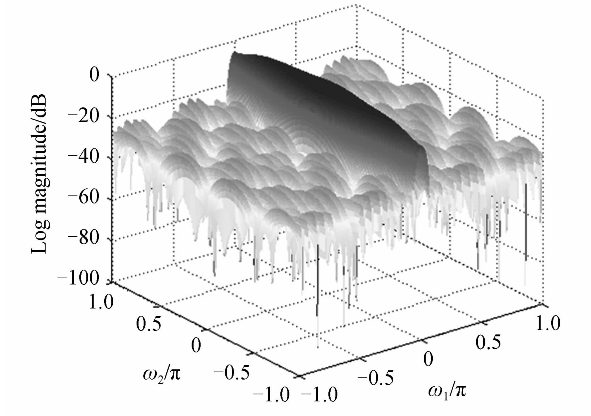
In this section， we first give a design example of a series of fan filters with 12 different orientations， and then apply these fan filters to detect the directional information of images for illustrating their directional sensitivity.

3.1　Example of fan filters with different orientations

With the design procedures given in Section 2， a wedge-shaped filter $p(n)$ which has the size of 100×100 is firstly designed. Its parameter K is chosen as 8， thus the angle φ between two passband edges of $p(n)$ is about $\pi/12$. The magnitude responses of the designed wedge-shaped filter and parallelepiped-shaped filter are shown in Fig. 3(a) and (b)， respectively.



(a) Wedge-shaped filter



(b) Parallelepiped-shaped filter

Fig. 3　Magnitude responses of wedge-shaped filter and parallelepiped-shaped filter

Then we transform this wedge-shaped filter $p(n)$ to the polar Fourier domain. Since $p(n)$ has the angle φ about $\pi/12$， the 12 fan filters orientated at $k\pi/12$， $k=0,1,\cdots,11$， are chosen to be designed. They can tile the whole frequency plane. For each fan filter, the corresponding permutation matrix is computed according to its orientation, and finally the fan filters are obtained via Eq. (6). Here we choose six fan filters for illustration, which have the orientations of $k\pi/12$, $k=0,1,\cdots,5$. Their magnitude responses in the PFT domain are shown in Fig. 4 (a) ~ (f), respectively.

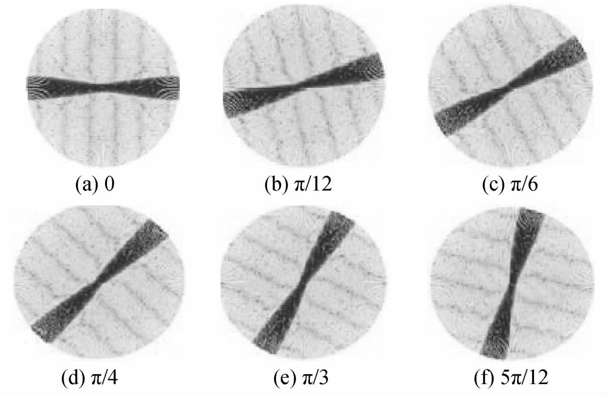

(a) 0 (b) π/12 (c) π/6
(d) π/4 (e) π/3 (f) 5π/12

Fig. 4 Magnitude responses of fan filters with the orientations of $k\pi/12$, $k=0,1,\cdots,5$

From the analysis in Section 2 we observe that, the design complexity of the 12 fan filters is equal to that of a 1D low-pass filter. This demonstrates that the proposed method has much lower design complexity than the existing methods[6-11] in which the fan filters are designed individually and the design of each fan filter has to involve complexity optimization.

3.2 Direction detection of images

To validate the directional sensitivity, we apply the 12 fan filters designed above to detect the directions of images with abundant textures and details. The detailed procedures are as follows. Firstly, we pass the input image through a highpass filter to avoid the influence of DC energy. Then, we partition the pre-processed image into non-overlapped uniform subblocks. Next, we filter each subblock by those 12 fan filters in PFT domain and compute the spectral energy of each filtered subblock. For each subblock, its spectral energy in the k-th direction denoted as E_k, can be computed by

$$E_k=\sum_{(p,q)\in\text{polar grid}}\left|P_k(p,q)\right|^2 \tag{8}$$

where $P_k(p,q)$ is the PFT coefficient of the subblock filtered by the fan filter in the direction $k\pi/12$, $k=0,1,\cdots,11$. Finally, the main direction of this subblock is determined to be the one corresponding to $E_{\max}=\max\{E_k\}$.

Further, we notice that for the smooth regions of an image, they have little directional information, and E_k have similar values. In order to distinguish these smooth regions, we set a threshold T. If $\frac{E_{\max}}{E_{\text{avg}}}\geqslant T$ where E_{avg} denotes the average of E_k, the subblock is considered to be a texture region and its main direction is determined by $E_{\max}$; otherwise it is a smooth region. The threshold T is determined by the spectral energy distribution of an image and its value varies with different images. Here we take the commonly used test images Finger (256×256) and Barbara (512×512) as examples. For the both images, the separable 9/7 wavelet filter is employed to remove their DC energy and the size of each subblock is set to be 16×16. Then the above 12 fan filters are applied to detect the main direction of each subblock. The threshold T is set to be 2.55 for Finger and 2.85 for Barbara. Their estimated orientation maps are shown in Fig. 5(a) and (b), respectively. It is obvious that the 12 fan filters can detect the directions of images exactly.


(a) Direction detection by the proposed 12 fan filter (Finger)


(b) Direction detection by the proposed 12 fan filter (Barbara)


(c) Direction detection by the 8 fan filters (Barbara)

Fig. 5 Estimated orientation maps of Finger and Barbara

For a comparison, we also use the tree-structure-based fan filters[10-11] to detect the directions of image Barbara. Due to the fact that the number of these fan filters is fixed to 2^n, some directions cannot be detected. Fig. 5(c) displays the estimated orientation map of Barbara by using 8 fan filters. Since the tree-structure cannot provide the fan filters orientated at 0° and 90°, the directions of table legs cannot be detected. Comparing Fig. 5(b) and (c), we see that the proposed arbitrarily-orientated fan filters have greater potential in direction detection of images than the traditional fan filters.

4 Conclusion

This paper proposes a simple but effective method for designing 2D fan filters. The proposed method is based on the PFT and a wedge-shaped filter. It cannot only design the fan filters with arbitrary orientation, but also has the advantage of easy design. Applying the designed fan filters to the application of direction detection of images, the results show that the proposed fan filters with arbitrary orientation can detect the directions of textures and details exactly.

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具有任意方向的扇形滤波器及其在图像方向检测中的应用

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摘　要:扇形滤波器在图像的方向检测中是非常重要的.本文基于极傅里叶变换和一个楔形滤波器,设计出具有任意方向的二维扇形滤波器.首先,楔形滤波器被变换到极傅里叶域,再利用极傅里叶变换的旋转特性,楔形滤波器可以通过将其极傅里叶变换沿着水平方向进行移动来实现旋转,得到一系列具有任意方向导向的扇形滤波器,它们能够检测图像中所包含的任意方向信息.由于整个设计过程不涉及二维优化,因此所提出的设计方法具有设计简单的优点.为了验证扇形滤波器的方向敏感性,将所设计的扇形滤波器应用于图像的纹理方向检测,结果表明,具有任意方向的扇形滤波器在图像纹理方向检测中具有很大的潜能.

关键词:扇形滤波器;极傅立叶变换;任意方向;方向检测