Article ID:1004-4213(2010)09-1567-5

# Two Components Vector Spatial Solitons in Two-photon Photorefractive Crystals\*

## SU Yan-li, JIANG Qi-chang, JI Xuan-mang

(Department of Physics and Electronic Engineering, Yuncheng University, Yuncheng, Shanxi 044000, China)

**Abstract**: To study the vector spatial solitons in biased two-photon photorefractive crystals, the dynamical evolution equation and the numerical solution of the vector spatial solitons are established. The bright-bright and dark-dark self-coupled vector solitons are predicted by using numerically methods. The dynamical evolutions of the bright-bright self-coupled vector spatial solitons are analyzed numerically. The results indicate that these self-coupled vector solitons can be obtained by using simple numerical integration procedures and their formation are irrespective to the ratio of the intensities of the two optical components. The bright-bright and dark-dark self-coupled vector spatial solitons exist steadily in two-photon photorefractive crystals.

Key words: Nonlinear optics; Vector spatial solitons; Two-photon photorefractive effect CLCN: O437 Document Code: A doi:10.3788/gzxb20103909.1567

## **0** Introduction

Spatial optical solitons have been the focus of extensive research, since waveguides induced by solitons have large potential for applications in generation of reconfigurable optical networks. Thus far, a lot of nonlinear effects have been found to be able to support spatial solitons, for example, (Kerr) nonlinearity<sup>[1]</sup>, cubic-quintic cubic competing nonlinearity<sup>[2]</sup>, photorefractive (PR)</sup> nonlinearity<sup>[3-19]</sup>, and photoisomerization nonlinearity<sup>[20]</sup>, etc. In all kinds of spatial solitons, those forming in PR crystals are particularly interesting, because they can be observed with very elementary experimental apparatus at exceptionally low power levels, which allows the applications in all-optical switching and beam steering, optical interconnection, and novel photonic devices. To date, three difference types of steady-state scalar PR solitons (screening solitons<sup>[3-4]</sup>, photovoltaic solitons<sup>[5-6]</sup> and screening photovoltaic solitons<sup>[7-8]</sup>) have been predicted and experimentally. observed Recently, vector  $solitons^{[9-10]}$ screening and vector screening photovoltaic solitons<sup>[11]</sup> in biased PR crystal

predicted, which involved the two polarization components of an optical beam that are orthogonal to one another. At present, bright-bright and dark-dark, self-coupled or cross-coupled vector screening solitons<sup>[9]</sup>, bright-dark self-coupled vector screening solitons<sup>[10]</sup> and screening photovoltaic solitons<sup>[11]</sup> were predicted.

All of the above-mentioned solitons result from the single-photon process. Very recently, a new model was introduced by Castro-Camus and Magana<sup>[12]</sup>, which involved two-photon PR effect. This model includes a valance band (VB), a conduction band (CB) and an intermediate allowed level (IL). A gating beam is used to maintain a fixed quantity of excited electrons from the VB, which are then excited to the CB by signal beam. The single beam induces a charge distribution identical to its intensity distribution, which in turn gives rise to a nonlinear change of refractive index through space charge field. Based on this model, screening solitons<sup>[13]</sup>, photovoltaic solitons<sup>[14]</sup> and screening photovoltaic solitons<sup>[15]</sup> in two-photon PR crystals were predicted. On the other hand, incoherently coupled bright-bright, dark-dark, bright-dark, and grey-grey soliton pairs whose carrier beams share the same polarization, wavelength, and are mutually incoherent have been predicted for screening solitons or photovoltaic solitons<sup>[16-19]</sup> that result from the two-photon PR effect. The case to the point is that the vector solitons due to the two-photon PR effect are not investigated yet. In this paper, we show that

<sup>\*</sup>Supported by the Science and Technology Development Foundation of Higher Education of Shanxi Province, (200611042) and the Basic Research Foundation of Yuncheng University, China (JC-2009003)

Tel:0359-2090374
 Email:syli1979@163.com

 Received date:2010-02-01
 Revised date:2010-04-22

bright-bright and dark-dark self-coupled vector screening solitons are possible in biased PR crystals with two-photon PR effect.

### **1** Theoretical model

To start, let us consider an optical beam that propagates in a PR crystal with two-photon PR effect along the z axis and is allowed to diffract only along the x direction. Moreover, let us assume that the external bias electric field is also applied along x. For demonstration purposes, let the PR crystal be LiNbO<sub>3</sub>, which is illuminated by a separate gating beam. As previously pointed out, this crystal is a good candidate for the observation of the self-coupled or cross-coupled vector solitons<sup>[9-11]</sup>. More specifically, for the selfcoupled case, the permittivity changes in LiNbO<sub>3</sub> along the extraordinary and ordinary components of the optical beam are equal, i. e.,  $\Delta \varepsilon_{ee} = \Delta \varepsilon_{\infty}$ , provided that the optical c axis of the crystal makes an angle  $\theta \approx 11.9^{\circ}$  with respect to the z axis.  $\Delta \epsilon_{ee}$ and  $\Delta \varepsilon_{\infty}$  represent the diagonal perturbations on the relative permittivity tensor. Moreover, in this case the off-diagonal elements, i. e. ,  $\Delta \varepsilon_{eo}$  and  $\Delta \varepsilon_{oe}$ , are zero. By associating slowly varying envelopes with the extraordinary and ordinary polarizations,  $\varphi_{\rm e}(x,z)$  and  $\varphi_{\rm o}(x,z)$ , then one quickly finds the following set of self-coupled nonlinear evolution equations<sup>[9-11]</sup>

$$2ik_{e}\frac{\partial\varphi_{e}}{\partial z} + \frac{\partial^{2}\varphi_{e}}{\partial x^{2}} + k^{2}\Delta\varepsilon\varphi_{e} = 0$$
(1a)

$$2ik_{o}\frac{\partial\varphi_{o}}{\partial z} + \frac{\partial^{2}\varphi_{o}}{\partial x^{2}} + k^{2}\Delta\varepsilon\varphi_{o} = 0$$
(1b)

where  $k = 2\pi/\lambda$  and  $\lambda$  is the free-space wavelength of the light wave used, and  $\Delta \varepsilon = \Delta \varepsilon_{ee} = \Delta \varepsilon_{oo}$ . The wave numbers  $k_{e}$  and  $k_{o}$  are defined as  $k_{e} = k\hat{n}_{e}$  and  $k_0 = k n_0$ , where  $\hat{n}_e$  and  $n_0$  are the refractive indices of the extraordinary and ordinary components. The relative permittivity changes  $\Delta \varepsilon_{ee}$  and  $\Delta \varepsilon_{oo}$  can be expressed as  $\Delta \epsilon_{\rm ee} = -r_{\rm eff,e} \hat{n}_{\rm e}^4 E_{\rm SC}$  and  $\Delta \epsilon_{\rm oo} = -r_{\rm eff,o}$  $n_0^4 E_{\rm SC}$ , where  $r_{\rm eff,e}$  and  $r_{\rm eff,o}$  are the effective electrooptic coefficients for the extraordinary and ordinary polarizations, respectively. When the optical beam propagates in  $LiNbO_3$  along the z axis at an angle  $\theta = 11.9^{\circ}$  with respect to the *c* axis,  $\Delta \epsilon_{ee} = \Delta \epsilon_{oo} =$ 235. 85 imes 10<sup>-12</sup>  $E_{\rm sc}$  and  $E_{\rm sc}$  represents the spacecharge field, which is measured in units of volts per meter. Moreover, under strong bias conditions and for relatively broad beam configurations, the steady-state space-charge electric field approximately given by

$$E_{sc} = E_{0} \frac{(I_{2\infty} + I_{2d})(I_{2} + I_{2d} + \gamma_{1}N_{A}/s_{2})}{(I_{2\infty} + I_{2d} + \gamma_{1}N_{A}/s_{2})(I_{2} + I_{2d})}$$
(2)

where  $I_2 = I_2(x, z)$  is total power density of the extraordinary and ordinary components, which can be obtained by summing the two Poynting fluxes, i. e. ,  $I_2 = (\hat{n}_{
m e}/2\eta_0) |\varphi_{
m e}|^2 + (n_{
m o}/2\eta_0) |\varphi_{
m o}|^2$ .  $I_{
m 2d}$  is the so-called dark irradiance,  $I_{2\infty}$  represents the total power density the vector pair attains away from the center of the two-photon PR crystal,  $N_{\rm A}$ is the acceptor or trap density,  $\gamma_1$  is the recombination factor of IL to VB transition,  $s_2$  is photoexcitation crossed, and  $E_0$  is the value of the space-charge electric field also at  $x \rightarrow \pm \infty$ . For the sake of convenience, let us adopt the following dimensionless coordinates and variables:  $s = x/x_0$ ,  $\xi = z/(k_0 x_0^2)$ ,  $U = (2\eta_0 I_{2d}/\hat{n}_e)^{-1/2} \varphi_e$  and V = $(2\eta_0 I_{\rm 2d}/n_0)^{-1/2}\varphi_0$ .  $x_0$  is an arbitrary spatial width, and the power densities of the optical beams have been scaled with respect to the dark irradiance  $I_{2d}$ . By employing these latter transformations and by substituting expressing Eq. (2) into Eqs. (1), and after appropriate normalization, we find that the normalized planar envelopes U and V satisfy

$$i\left(\frac{\hat{n}_{e}}{n_{o}}\right)\frac{\partial U}{\partial \xi} + \frac{1}{2}\frac{\partial^{2}U}{\partial s^{2}} - \beta \frac{(1+\rho)(1+\sigma+|U|^{2}+|V|^{2})}{(1+\sigma+\rho)(1+|U|^{2}+|V|^{2})}U = 0 \quad (3a)$$

$$i\frac{\partial V}{\partial \xi} + \frac{1}{2}\frac{\partial^{2}V}{\partial s^{2}} - \beta \frac{(1+\rho)(1+\sigma+|U|^{2}+|V|^{2})}{(1+\sigma+\rho)(1+|U|^{2}+|V|^{2})}V = 0 \quad (3b)$$

$$\arg = \frac{1}{2} - \frac{1}{2} \frac{1}{2$$

where  $ho = I_{2\infty} / I_{2d}$ ,  $\sigma = \gamma_1 N_A / \beta_2$ , and  $eta = -(235.85 imes 10^{-12}/2) (kx_0)^2 E_0$ .

#### 2 Numerical simulation

To find the bright-bright solitary pair solutions of Eqs. (3), let us express the normalized envelopes U and V as follow

 $U = r^{1/2} y(s) \cos \varphi \exp \left[ i (n_o / \hat{n}_e) \mu \xi \right]$  $V = r^{1/2} y(s) \sin \varphi \exp (i \mu \xi)$ 

where r represents the ratio of the intensity at peak to the dark irradiance  $I_{2d}$ ,  $\mu$  is the nonlinear shift of the propagation constant, y(s) is the normalized real function which is bounded as  $0 \leq y(s) \leq 1$ ,  $\varphi$  is an arbitrary angle which describes the relative strength of two components of the composite. Now, substitution expression of U and V into Eqs. (3) yields the following equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}s^2} = 2 \Big[ \mu + \frac{\beta}{1+\sigma} + \frac{\beta\sigma}{1+\sigma(1+ry^2)} \Big] y \tag{4}$$

Eq. (4) can be integrated and yields

$$\left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)^{2} = 2\left(\mu + \frac{\beta}{1+\sigma}\right)\left(y^{2}-1\right) + \frac{2\beta\sigma}{r\left(1+\sigma\right)}\ln\left(\frac{1+ry^{2}}{1+r}\right)$$
(5)

According to the appropriate boundary conditions, i. e., at  $(s \rightarrow \pm \infty)$ ,  $y(\pm \infty) = 0$ , and  $\dot{y}(\infty) = 0$ , we can easily obtain  $\mu$  as

$$\mu = -\frac{\beta}{1+\sigma} - \frac{\beta\sigma}{(1+\sigma)r} \ln(1+r) \tag{6}$$

Inserting Eq. (6) in Eq. (5) we get

$$\left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)^2 = \frac{2\beta\sigma}{(1+\sigma)r} \left[\ln(1+ry^2) - y^2\ln(1+r)\right] \quad (7)$$

We can show that  $\ln(1+ry^2) - y^2 \ln(1+r) > 0$  for  $0 < y^2 < 1$ , and thus we can know that the bright soliton requires  $\beta > 0$ , i. e.,  $E_0 < 0$ . By further integrating Eq. (7), we get

$$\left(\frac{2\beta\sigma}{1+\sigma}\right)^{1/2}s = \pm \int_{y}^{1} \frac{r^{1/2} \operatorname{d} \tilde{y}}{\left[\ln(1+r\,\tilde{y}^{2}) - \tilde{y}^{2}\ln(1+r)\right]^{1/2}}$$
(8)

In order to investigate bright-bright selfcoupled vector solitons, we take LiNbO3 crystal, which is oriented at  $\theta = 11.9^{\circ}$ , biased by  $E_0 =$  $-2.0 \times 10^6$  V/m. If the arbitrary spatial width  $x_0$ is taken to be 40  $\mu$ m and  $\lambda = 0.633 \mu$ m, then  $\beta =$ 37.18. Moreover, we assume that  $\sigma = 6.6 \times 10^6$ ,  $r=10, \varphi = 30^{\circ}$ . Fig. 1 shows the normalized intensity profile of the bright-bright self-coupled vector solitons. It is notable that the ratio of the intensity of two components can be adjusted by changing the value of  $\varphi$ . Thus we can obtain the self-coupled vector solitons by use of the numerical integration procedures in any value of the ratio, which is different from Ref  $\lceil 10 \rceil$ , where the intensities of the two optical components are approximately equal.



Fig. 1 Intensity profiles of bright-bright self-coupled vector solitons when  $r=10, \varphi=30^{\circ}$ 

A dark-dark self-coupled vector solitons can be calculated similarly. We express the normalized envelopes U and V as follow  $U = \rho^{1/2} y(s) \cos \varphi \exp \left[ i (n_o / \hat{n}_e) \nu \xi \right]$  $V = \rho^{1/2} y(s) \sin \varphi \exp \left( i \nu \xi \right)$ 

where  $\rho = I_{2\infty}/I_{2d}$ ,  $\nu$  is the nonlinear shift of the propagation constant. y(s) is a normalized odd function of s and satisfies the following boundary conditions:  $y(0)=0, y(s \rightarrow \pm \infty)=\pm 1$ , and all the derivatives of y(s) vanish at infinity. We take similar methods with bright-bright vector solitons; The following expression can be obtained

$$\left(\frac{-2\beta\sigma}{1+\sigma+\rho}\right)^{1/2} s = \pm \int_{y}^{0} (\mathrm{d}\,\tilde{y}) / \left[(\tilde{y}^{2}-1) - \frac{\rho+1}{\rho} \ln\left(\frac{1+\rho\,\tilde{y}^{2}}{1+\rho}\right)\right]^{1/2}$$
(9)

Taking same parameters with bright-bright vector solitons but  $E_0 = 2$ .  $0 \times 10^6$  V/m. Fig. 2 shows the normalized intensity profile of the dark-dark self-coupled vector solitons.



Fig. 2 Intensity profiles of dark-dark self-coupled vector solitons when  $\rho = 10, \varphi = 30^{\circ}$ 

Moreover, we investigate the evolution of the bright-bright self-coupled vector solitons. The normalized envelopes given by Fig. 1 have been used as the input beam profiles of the two optical components. To solve Eqs. (3), the following Fig. 3 can be obtained. It is shown that the two components of the self-coupled vector solitons can propagate unchanged.





Fig. 3 The dynamical evolution of the U and V components of the self-coupled vector soliton pair when  $r=10, \varphi=30^{\circ}$ 

### **3** Conclusion

It is found that bright-bright and dark-dark self-coupled vector solitons are possible in biased two-photon PR crystal, which involve the two polarization components of an optical beam that are orthogonal to one another. The bright-bright and dark-dark vector solitons exist when the value of  $\beta$ are positive and negative, respectively. Moreover, the evolution of these solitons has been investigated simply.

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## 双光子光折变晶体中两分量矢量空间孤子

苏艳丽,姜其畅,吉选芒 (运城学院物理与电子工程系,山西运城 044000)

摘 要:为了研究外加偏压双光子光折变晶体中的矢量空间孤子,建立了矢量空间孤子的动态演化方程,给 出了矢量空间孤子数值解.采用数值模拟的方法,求解矢量空间孤子的数值表达式,理论预言了稳态条件下 亮-亮、暗-暗自耦合矢量空间孤子的存在;同时,数值求解演化方程,分析了亮-亮自耦合矢量空间孤子的演化 特性.数值结果表明,无论两孤子分量的强度近似相等还是有较大差别,这些自耦合矢量空间孤子都可以由 数值积分程序给出.亮-亮、暗-暗自耦合矢量空间孤子在双光子光折变晶体中稳定存在. 关键词 非优性来学, 在景空间孤子, 双来子来拓峦故立

关键词:非线性光学;矢量空间孤子;双光子光折变效应



**SU Yan-li** was born in 1979, and received her M. S. degree from Shandong Normal University in 2006. Now she works at Yuncheng University and her research interests focus on nonlinear optics and optical spatial solitons.