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Evaluate the Dispersion Parameters for Ultrashort Pulses Propagating in Photonic Crystals

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Abstract: Using the least-squares spline approximation to analyze the photonic band structure, the dispersion parameters of a two-dimensional photonic crystal are quantitatively calculated. Considering the large bandwidth of an ultrashort pulse, average group velocity and average group velocity dispersion are achieved by introducing a weighting factor, which reveals the share of different frequency components. So the pulse evolution can be predicted theoretically. Propagation of ultrashort pulses through the photonic crystals is simulated by using the finite-difference time-domain method. The results of numerical simulations are well consistent with the theoretical predictions. This process for achieving dispersion parameters is valuable for designing photonic-crystal dispersion devices such as dispersion compensator, pulse compressor and pulse stretcher.

Key words: Photonic crystal; Group velocity; Group velocity dispersion; Ultrashort pulse propagation

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0 Introduction

Due to its periodic modulation of the refractive index, photonic crystal (PhC) exhibits large controllable dispersion[1-5]. Designing the spatial dispersion properly, negative refraction and self-collimating for continuous waves propagating in PhCs have been demonstrated theoretically and experimentally[5-9]. Since the temporal dispersion can be tailored by tuning the structural and material parameters of PhC, its effects on ultrashort pulses propagation have attracted a great deal of attention in recent years[2-5, 10-15].

On one hand, nearly-zero dispersion is necessary for slow-light PhC structures because the group velocity dispersion (GVD) and higher-order dispersion distort optical pulses severely. In 2005, Mori et al. have designed a low dispersion PhC coupled waveguide, which allows robust slow light with 40 GHz bandwidth[10]. An asymmetric PhC waveguide to obtain flat band modes with low group velocity and low dispersion has been proposed in 2008[11]. In the same year, light bullet propagation has been demonstrated by combining zero GVD and self-collimation in a two-dimensional (2D) PhC[12].

On the other hand, large GVD is prerequisite for realizing the so-called dispersion control of ultrashort optical pulses, such as dispersion compensation and pulse compression. At first, Notomi et al. have discovered the extremely large group velocity dispersion in PhC waveguides[3]. Using these large dispersion PhC waveguides, a new dispersion compensator of a few tens of millimeters in size has been proposed[13]. In 2004, Karle et al. have compressed pulse width from 1.91 ps to 1.17 ps using planar PhC coupled cavity waveguides, whose equivalent dispersion value is two orders of magnitude larger than that of the conventional single-mode fibers[5]. In recent years, even larger dispersion in PhC waveguides has been realized one after another[14-15].

Although we know that the PhC has controllable temporal dispersion, it is still quite difficult to obtain its accurate dispersion parameters, especially for ultrashort pulses propagation. Furthermore, these parameters are prerequisite for designing dispersion devices based on PhC such as dispersion compensators and pulse

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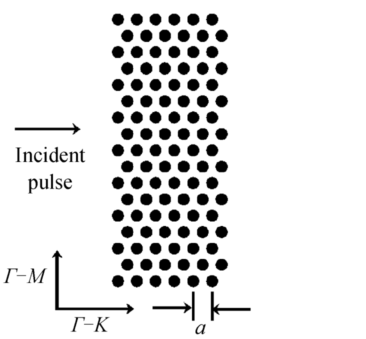
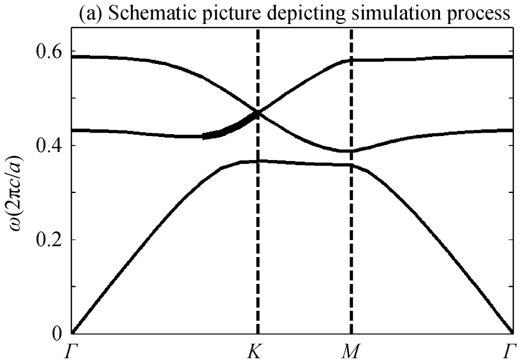
compressors. For this purpose, in this paper we systematically present a process to evaluate dispersion parameters for a PhC, and verify its validity by using the finite-difference time-domain (FDTD) method.

1 Dispersion analysis

PhC waveguides, whose dispersion properties have been studied intensively before, are not applicable for high-power ultrashort pulses due to its small effective mode area. Considering this problem, in this paper we take a perfect 2D PhC as a kind of dispersion device. This PhC structure is made of dielectric rods arranged in triangle lattice and the ultrashort pulses are incident along the Γ-K direction as shown in Fig. 1(a). The rods have dielectric constant $\varepsilon=11.4$ and radius $r=0.3a$, where a is the lattice constant. We only consider the TE polarization that the magnetic field H_z is parallel to these rods. Using the plane wave expansion method, the photonic band structure of this PhC is calculated and plotted in Fig. 1(b).



(a) Schematic picture depicting simulation process



(b) Photonic band structure for the TE polarization

Fig. 1 Schematic diagram of the photonic crystal and photonic band structure

Dispersion parameters of a PhC can be investigated by analyzing its photonic band structure. Petrov et al. have used a simple parabolic approximation to calculate the PhC′s dispersion parameters[16]. However, Andreev has pointed out that for the femtosecond pulse propagation this method is not efficient any more[17]. Alternatively, in this paper we use the least-squares spline approximation[18] to analyze the photonic band structure. Take a part of the second band as an example, which is denoted by the bold curve as shown in Fig. 1(b). The calculated group velocity $v_g=1/(\mathrm{d}\omega/\mathrm{d}k)$ and GVD parameter $\beta_2=\mathrm{d}^2k/\mathrm{d}\omega^2$ are plotted in Fig. 2. The dashed and solid line are the group velocity and GVD parameter of the PhC respectively. The inset of Fig. 2 is a magnified plot of the GVD parameter in the frequency range $\omega_0=0.42\sim0.44(2\pi c/a)$. For simplicity, we neglect the higher order dispersions.

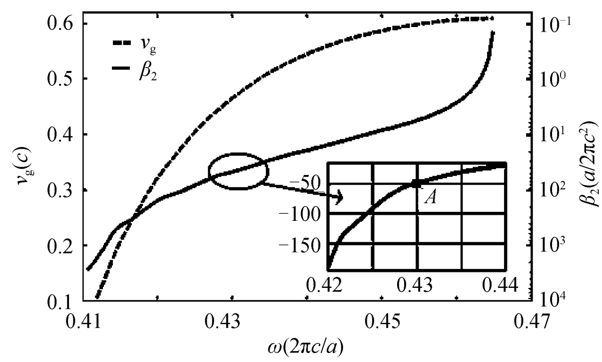


Fig. 2 Group velocity and GVD parameter of the PhC vs. the normalized frequency

Due to neglecting the higher order dispersions of the PhC, the incident Gaussian pulse maintains its original shape on propagation. Using the average GVD parameter, the dispersion-induced broadening factor is given by[19]

$$\frac{T_{\mathrm{out}}(L)}{T_{\mathrm{inc}}}=\left[\left(1+\frac{C\beta_2 L}{T_{\mathrm{inc}}^2}\right)^2+\left(\frac{\beta_2 L}{T_{\mathrm{inc}}^2}\right)^2\right]^{1/2} \tag{1}$$

where C is a chirp parameter and L is the propagation distance in PhC, T_{inc} and T_{out} are the temporal width of input and output Gaussian pulse, respectively.

As we all know, the quasi-monochromatic approximation is questionable for studying ultrashort pulses propagation due to its large bandwidth. So we cannot predict dispersive broadening for ultrashort pulses directly using the GVD parameter β_2 which we have obtained from the band structure. Considering the contribution of different frequency components, as a modification we introduce an average GVD parameter $\tilde{\beta}_2$ defined as[11]

$$\tilde{\beta}_2=\frac{1}{\Delta\omega}\int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} f(\omega)\beta_2(\omega)\,\mathrm{d}\omega \tag{2}$$

where $f(\omega)$ is a weighting factor derived from the Fourier decomposition of the incident time signal. For a Gaussian pulse, $f(\omega)$ often takes the form

$$f(\omega)=1-\frac{(\omega-\omega_0)^2}{2\Delta\omega^2} \tag{3}$$

where ω_0 and $\Delta\omega$ are the central frequency and spectral half width (at $1/e$-intensity point) of the incident Gaussian pulse, respectively.

For a pulse with central frequency $\omega_0=0.43\times(2\pi c/a)$ (namely point A in the inset of Fig. 2) and spectral half width $\Delta\omega=0.01\times(2\pi c/a)$, substituting Eq. (3) in Eq. (2), we figure out the average GVD dispersion parameter of the PhC $\tilde{\beta}_2=-64.76\times[a/(2\pi c^2)]$, while the GVD parameter β_2 is about $-46.73\times[a/(2\pi c^2)]$ as shown in Fig. 2. Substituting β_2 and $\tilde{\beta}_2$ in Eq. (1), we calculate the broadening factor $T_{out}(L)/T_{inc}$ for an initially unchirped pulse (i. e. chirp parameter $C=0$) propagating in the PhC. The results are denoted by the dash and solid lines in Fig. 3.

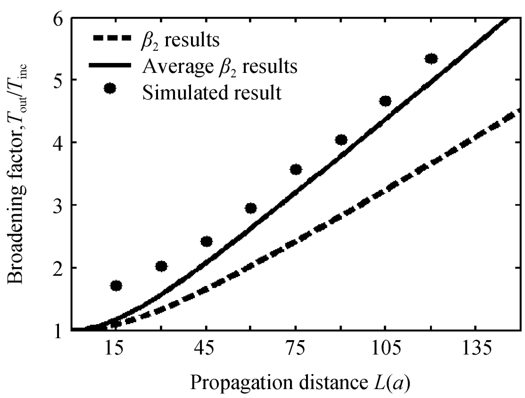


Fig. 3 Variation of broadening factor vs. the propagation distance

2 Numerical simulations

To validate the dispersion parameters obtained in Section 2, we simulate ultrashort pulses propagating in the PhC by using the FDTD method. Firstly, we simulate an initially unchirped Gaussian pulse ($C=0$) with central frequency $\omega_0=0.43\times(2\pi c/a)$ and spectral halfwidth $\Delta\omega=0.01\times(2\pi c/a)$ incident to the PhC. For different thickness such as $L=30a$, $60a$, $90a$, $120a$, the pulse evolutions are shown in Fig. 4. It should be mentioned that oscillations near the trailing edge of the pulse in Fig. 4(d) and Fig. 4(e) are resulted from high order dispersions which we have neglected in the theoretical analysis. However, the oscillations relative positions can be predicted by the positive slope at point A in Fig. 2 indicating positive third order dispersion. By measuring temporal widths of the input and output Gaussian pulses, broadening factors are obtained and plotted with dotted line in Fig. 3. It is clear that the theoretical prediction of dispersion-induced broadening by using average $\tilde{\beta}_2$ is more consistent with the numerical simulations than using β_2.

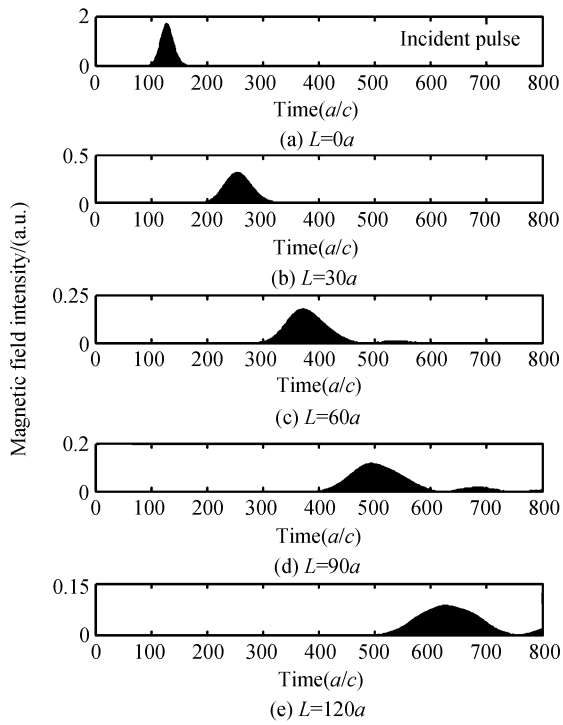
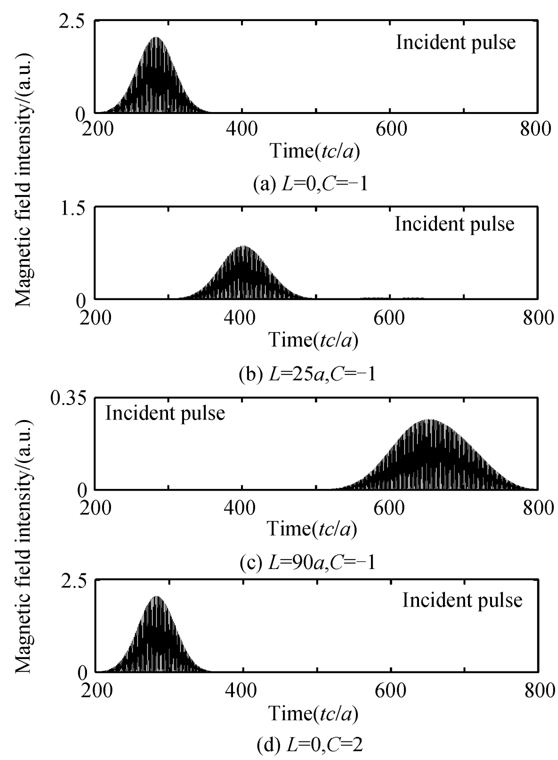


Fig. 4 The time evolution of a Gaussian pulse propagating in the PhC with different thickness

Secondly, we simulate an initially chirped Gaussian pulse with $C=-1$ falling on the PhC. For $\tilde{\beta}_2 C>0$, the incident pulse broadens monotonically as shown in Fig. 5(a)～(c). Compared Fig. 4 with Fig. 5(a)～(c), it is



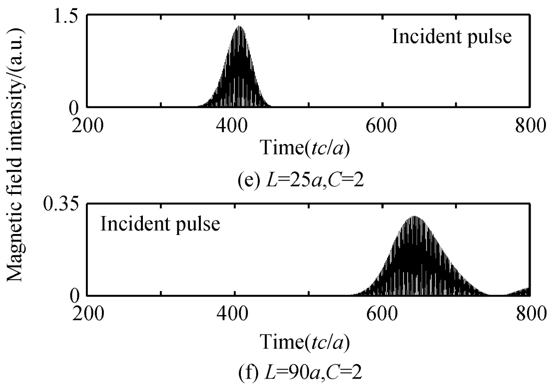


Fig. 5 The time evolution of two chirped Gaussian pulse propagating in the PhC with different thickness

obviously that the broadening rate of the chirped pulse is faster than that of the unchirped one.

Finally, we simulate an initially chirped Gaussian pulse with $C=2$ propagating in the PhC. Since $\tilde{\beta}_2 C<0$, the incident pulse goes through an initial narrowing stage as seen from Fig. 5(d) ~ (e). By using Eq. (1), theoretical calculation indicates that the pulse temporal width becomes minimum at a distance $L_{\min}=24.58a$, which is very consistent with the numerical simulation result $L_{\min}=25a$. After the first stage of compression, the pulse begins to broaden with a further increase in the propagation distance.

3 Conclusions

In summary, we have systematically presented a process to analyze the propagation of ultrashort pulses through PhCs. By using the least-squares spline approximation to analyze the photonic band structure, we calculate the dispersion parameters. Considering the large bandwidth of an ultrashort pulses, the average GVD parameter has been calculated by introducing a weighting factor. With this parameter, we have theoretically predicted the evolution of an ultrashort pulse propagating in the PhC, and the theoretical predictions agree well with the FDTD simulations results. Although, we have only discussed a specific ultrashort pulse propagating in a 2D triangular lattice PhC in this paper, the method can be extended to other situations such as ultrashort pulses with different parameters propagating in other kinds of PhCs. Therefore, it is very useful for designing photonic-crystal dispersion devices such as dispersion compensator, pulse compressor and pulse stretcher. At last, we should mention that for a real photonic crystal with unavoidable defects, the dispersion parameters would be less than what we have calculated.

References

[1] YANG S Y, CHANG C T. Chromatic dispersion compensators via highly dispersive photonic crystals[J]. *Appl Phys*, 2005, **98**(2): 023108.

[2] HUANG Yong-qing, LIU Nian-hua. Propagation of a pulse through one-dimensional photonic crystals containing negative materials[J]. *Acta Photonica Sinica*, 2007,**36**(7):1211-214.

[3] NOTOMI M, YAMADA K, SHINYA K, *et al*. Extremely large group-velocity dispersion of line-defect waveguides in photonic crystal slabs[J]. *Phys Rev Lett*, 2001, **87**(25): 253902.

[4] LU Shu-yuan, ZHAO Jian-lin, ZHANG Dong. Slow light transmission in two-dimension square photonic crystal waveguide[J]. *Acta Photonica Sinica*, 2009, **36**(7): 2224-2228.

[5] KARLE T J, CHAI Y J, MORGAN C N, *et al*. Observation of pulse compression in photonic crystal coupled cavity waveguides[J]. *J Lightwave Technol*, 2004, **22**(2): 514-519.

[6] KOSAKA H, KAWASHIMA T, TOMITA A, *et al*. Superprism phenomena in photonic crystals[J]. *Phys Rev B*, 1998, **58**(16): R10096.

[7] KOSAKA H, KAWASHIMA H, TOMITA A, *et al*. Self-collimating phenomena in photonic crystals[J]. *Appl Phys Lett*, 1999, **74**(9): 1212-1214.

[8] GRALAK B, ENOCH S, TAYEB G. Anomalous refractive properties of photonic crystals[J]. *JOSA A*, 2000, **17**(6): 1012-1020.

[9] NOTOMI M. Theory of light propagation in strongly modulated photonic crystals: Refractionlike behavior in the vicinity of the photonic band gap[J]. *Phys Rev B*, 2000, **20**(16): 10696.

[10] MORI D, BABA T. Wideband and low dispersion slow light by chirped photonic crystal coupled waveguide[J]. *Opt Express*, 2005, **13**(23): 9398-9408.

[11] JING M, CHUN J. Flatband slow light in asymmetric line-defect photonic crystal waveguide featuring low group velocity and dispersion[J]. *IEEE J Quant Electron*, 2008, **44**(8): 763-769.

[12] ZHOU C, GONG Q, YAO P, *et al*. Bulletlike light pulses in photonic crystals[J]. *Appl Phys Lett*, 2008, **93**(6): 061103-3.

[13] HOSOMI K, KATSUYAMA T. A dispersion compensator using coupled defects in a photonic crystal[J]. *IEEE J Quant Electron*, 2002, **38**(7): 825-829.

[14] ASANO T, KIYOTA K, KUMAMOTO D, *et al*. Time-domain measurement of picosecond light-pulse propagation in a two- dimensional photonic crystal-slab waveguide[J]. *Appl Phys Lett*, 2004, **84**(26): 4690-4692.

[15] CHEN Q, ALLSOPP DWE. Group velocity delay in coupled-cavity waveguides based on ultrahigh-Q cavities with Bragg reflectors[J]. *J Opt A: Pure Appl Opt*, 2009, **11**(5): 054010.

[16] PETROV A Y, EICH M. Dispersion compensation with photonic crystal line-defect waveguides[J]. *IEEE J Sel Areas Commun*, 2005, **23**(7): 1396-1401.

[17] ANDREEV A V, BALAKIN A V, OZHEREDOV I A, *et al*. Compression of femtosecond laser pulses in thin one-dimensional photonic crystals[J]. *Phys Rev E*, 2000, **63**(1): 016602.

[18] BOOR C D. A Practical guide to splines springer[M]. 2nd ed. New York: Heidelberg, 2001.

[19] AGRAWAL G P, Nonlinear fiber optics[M]. 3rd ed. UK: Academic Press, 2001: 63-86.

超短脉冲在光子晶体中传播时的色散参量

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摘　要：利用最小二乘拟合法分析了光子晶体的能带结构，定量分析了二维光子晶体的色散参量．为了考虑脉冲各频率的作用，通过引入与频率分量相关的权重因子，从而解析了群速度和平均群速度色散，进而解析出脉冲波络渐变的理论结果．通过 FDTD 数值模拟后，发现数值模拟结果与理论解析结果完全相吻合．本文处理光子晶体的色散参量的方法，为制作光子晶体色散器件如光子晶体色散补偿器、脉冲展宽和压缩器有一定的借鉴作用．

关键词：光子晶体；群速度；群速度色散；超短脉冲

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