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Surface Measurement for Long Focal Length Mirror with Phase Retrieval*

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Abstract:In order to solve the difficulties in testing surface of long focal length mirror using interferometer, a novel optical metrology using phase retrieval is developed. In this method, tested mirrors is illuminated by a coherent point light source, a series of diffraction intensity patterns near focus are collected, then phase retrieval algorithm reconstructs the surface error of tested mirror. Compared with interferometer, it simplifies the metrology setup and requires no special environment. Mathematic model of phase retrieval measurement is found according to diffraction theory, and is solved by iterative algorithm which derives from Gerchberg-Saxton algorithm. Validity of this method is tested by simulations and an experiment. A spherical mirror with curvature radius of 8 700 mm and diameter of 145 mm is tested using phase retrieval. The position of intensity patterns are corrected and appropriate patterns are chosen to be computed. The reconstructed result is correlated well with that tested by simultaneous phase-measurement interferometer.

Key words:Long focal length mirror; Surface metrology; Phase retrieval; Diffraction

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0 Introduction

Long focal length optical components have wide application for large telescopes and space detailed survey cameras. With the development of space technology, space resolution and image quality of optical imaging system have become increasingly demanding. The corresponding figure errors of the optical components have become increasingly stringent accuracy requirements that provide the manufacture of optical components challenges, and give birth to a new generation of precision optical processing technology delegated by magneto-rheological finishing.

The achievement of deterministic processing requires a support from quantitative measurement. Main devices for quantitative measurement at present include three-coordinate measuring machines, Hartmann wavefront sensing and interferometer, etc. Because of high precision and high spatial resolution, interferometer becomes the main measuring equipment in modern optical processing. However, conventional interferometer is sensitive to vibration and air disturbance which requires professional chamber. In particular, in order to avoid vibration, interferometer and the tested mirror need to be placed on the same isolation mounting. That result in a problem: when measuring long focus mirror, it's difficult to find a ten meters or even hundreds of meters isolation platform. Although there is simultaneous phase-measurement technology now, it is not mature yet, and the simultaneous phase-measurement interferometer are high cost.

Phase retrieval measurement is a method employing intensity information to restore the phase of optical wave field. It's widely known that the iterative algorithm was first proposed by Gerchberg in 1972 and laid the application foundation for this technology [1]. Since then, the phase retrieval techniques were promoted to multiple scopes. It has been successfully applied in the crystal measurement, binary optics, as well as the assembly of large space telescope measurements, etc, typically including aberration detection and correction of Hubble Space Telescope and the processing and assembly of James Web Space Telescope[2-3]. Researchers at the University of Rochester have done some basic research about phase retrieval in optical metrology, but did not relate to specific about mirror measurement[4-6].

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Interferometer can produce straightforward interference fringes with additional optical components, while phase retrieval measurement can get the diffraction intensity pattern with very simple devices, but further translation is required.

This paper puts forward a method based on phase retrieval techniques to meet the needs of measurement of long focal length mirror. The measuring device is simple and practical, and ability of anti-vibration is good, with acceptable accuracy. The paper verifies the feasibility of the method by measuring an 8 700 mm- curvature-radius spherical mirror in the last part.

1 Primary principle of phase retrieval testing and simulation

Phase retrieval can be applied in mirror testing with significant simplicity apparatus. Fig. 1 shows the schematic diagram of the testing. A point coherent light source is placed at the curvature centre of tested mirror to emit a spherical wave. The spherical wave is reflected back carrying the aberration of the surface in the wave front. The returning light is collected by a CCD camera through a beam splitter.

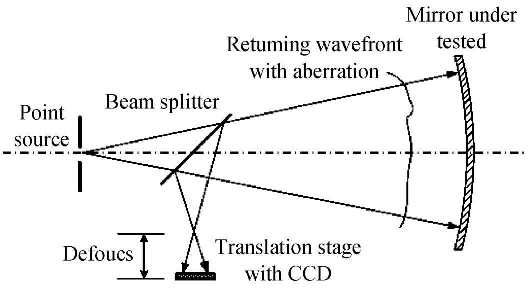


Fig. 1 Phase retrieval schematic diagram of ball testing

1.1 The mathematic model of wave front propagation

It′s a kind of convergent wavefront propagation from the surface of tested mirror to the plane of CCD. The mathematic expression of convergent wavefront includes a spherical-wave factor $(1/z)\exp[-j\pi(x^2+y^2)/(\lambda z)]$. The factor increases the spread of the transform in wave-vector space for λ is very small generally. This may require sample space small enough. To avoid too many sample spots to numerical calculation, the coordinate transformation according to Sziklas and Siegman is operated[7]. The transformation severs the spherical factor from wavefront expression, so the problem of convergent spherical wavefront propagation is transformed into plane wave propagation.

Furthermore, according to the angular spectrum theory of plane waves[8], the complex scalar field $U(x,y,0)$ propagates from $(x,y,0)$ to (x,y,z), the wavefront function $U(x,y,z)$ can be given by

$$U(x,y,z)=F^{-1}\left\{F\{U(x,y,0)\}\cdot \exp\left(\mathrm{j}\frac{2\pi}{\lambda}\sqrt{1-\alpha^2-\beta^2}\,z\times \operatorname{circ}\left(\sqrt{\alpha^2+\beta^2}\right)\right)\right\} \tag{1}$$

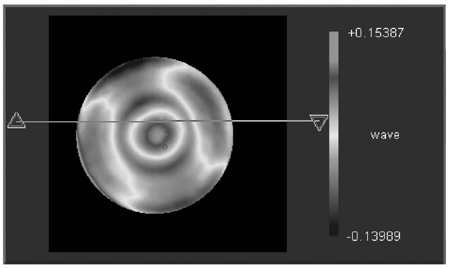
Where

$$\alpha=\lambda f_X,\beta=\lambda f_Y,\operatorname{circ}\sqrt{\alpha^2+\beta^2}=\begin{Bmatrix}1,\sqrt{\alpha^2+\beta^2}\leqslant 1\\0,\sqrt{\alpha^2+\beta^2}>1\end{Bmatrix}$$

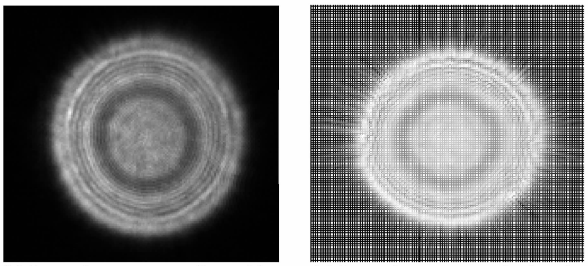
$F\{\ \}$ represents Fourier transform, $F^{-1}\{\ \}$ represents inverse Fourier transform. For convenience, Eq. (1) can be expressed as $u(z)=T\{u(0),z\}$, where $T\{\ \}$ is a linear propagation operator.

1.2 A comparison for validating the model

A comparison has been performed in order to validate the model above-mentioned. We put a spherical mirror of diameter 200 mm and curvature radius 1 000 mm at the situation as Fig. 1, whose surface error distribution is shown as Fig. 2 (a). Then collect the intensity pattern in the plane 4 mm after the focus. The pattern is shown in Fig. 2(b).



(a) Surface error distribution of the mirror



(b) Intensity pattern collected by CCD in position of 4mm after focus

(c) Intensity pattern calculated according as the mathematical model in the same position

Fig. 2 Comparison between intensity patterns collected by CCD and calculated according as the model

The intensity pattern is calculated according to the model correspondingly. The surface errors of mirror are regarded as phases of the complex scalar field at input plane, and the amplitudes are all supposed as 1. The square of amplitudes at output plane is shown in Fig. 2(c). Comparing Fig. 2 (b) with Fig. 2(c), the patterns are almost the same, which certifies that the mathematic model of wavefront propagation is correct.

1.3 The algorithm of testing mirrors

As mentioned above, phase retrieval testing can be described as

$U_1 = A_1 e^{j\varphi_1}, U_2 = A_2 e^{j\varphi_2}, U_2 = T\{U_1, z\}$, A_2 is known , φ_1 is to be solved. Where, U_1 represents the function of optical wave field at tested mirror (the input plane), U_2 represents the field at observation plane (the output plane), and A_1、A_2, φ_1、φ_2 represents the amplitude and phase of the field respectively.

An algorithm based on Gerchberg-Saxton, (GS) algorithm is proposed to reconstruct mirror surface error. GS algorithm is a classic algorithm in solving phase retrieval problem[9]. The algorithm computes iteratively between input and output plane. In our algorithm, every iteration includes a computing $T\{\ \}$ and an inverse computing $T^{-1}\{\}$. When the i-th direct iterative is operated, phases at input plane φ_1^i are limited by the amplitudes A_1, and processed by $T\{\ \}$ operator, then produce optical wave filed U_2^i at output plane:

$$U_2^i = T\{A_1 e^{j\varphi_1^i}, z\} \tag{2}$$

In the i-th inverse iterative, keep the phase φ_2^i of U_2^i, make square root of the intensity pattern which are collected by CCD as A_2^i, form a new optical wave field function at output plane $A_2^i e^{j\varphi_2^i}$, then compute inversely , obtain a estimate of the optical wave field function at input plane

$$U_1^{i+1} = T^{-1}\{A_2^i e^{j\varphi_2^i}, z\} \tag{3}$$

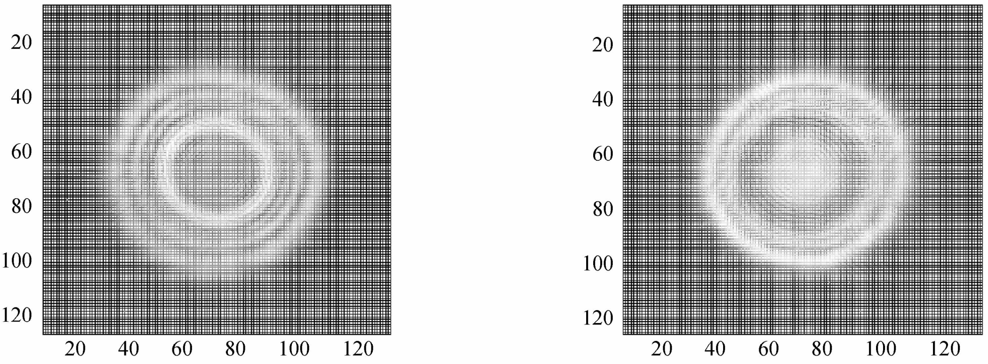
Then turn into next iteration. Suppose E is the difference of intensity pattern between calculated and collected at output plane.

$$E = \sum |I(x,y) - |A_2^i(x,y)|^2| \tag{4}$$

The difference reduces after every iterative, when E is smaller than a given value, algorithm was thought to be convergent. The phases of wavefront function φ_1^i can be thought as surface errors of tested mirror at now.

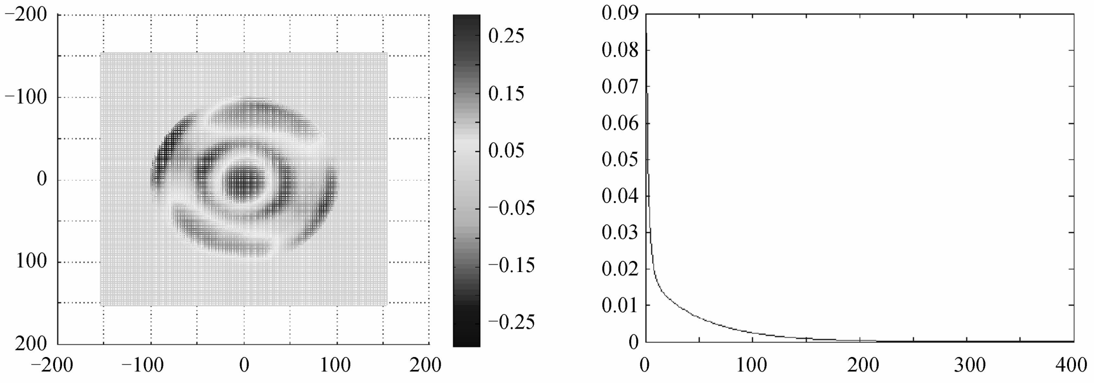
1.4 Simulation for validating the algorithm

A numerical simulation of phase retrieval testing algorithm is performed. The surface errors are known quantity as Fig. 2(a) shown. First, intensity patterns in position of 2 mm behind and before focus are calculated as segment 2.1. They are imported to the phase retrieval algorithm, then surface errors are reconstructed (Fig. 3(c)).



(a) Intensity pattern in position of 2 mm before focus

(b) Intensity pattern in position of 2 mm behind focus



(c) Surface errors reconstructed by phase retrieval

(d) Error curve

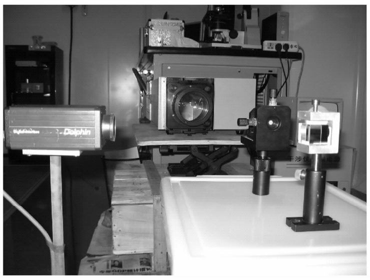
Fig. 3 Numerical simulation of phase retrieval testing algorithm

Fig. 3(c) agrees with Fig. 2 (a) accurately, and the error curve (Fig. 3 (d)) shows that the difference between phase retrieval result and initial surface error is small than 10^{-3}. This proves that the testing algorithm is effective.

2 Experimental setup and results

We use the method above to test a spherical mirror of diameter 145 mm and curvature radius 8 700 mm.

The experiment setup is shown in Fig. 4. The interferometer is used to produce the coherent point source and a pin hole is placed at the point to purify the light. CCD is riveted on a device which can move back and forth. The whole measurement system is adjusted by dint of the interferometer′s laying device. Full view of the setup through the tested mirror is shown in Fig. 4 (b). And the part of the setup shown in Fig. 4 (a) becomes too small to identify in Fig. 4 (b) for the long light path.

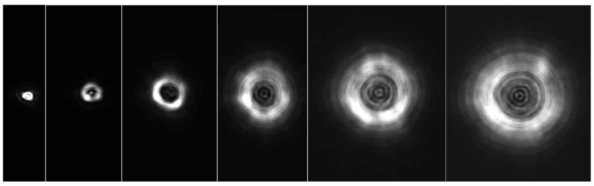


(a) A part of the experimental setup

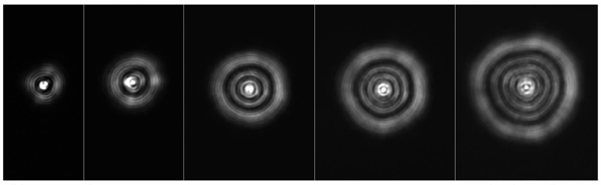
(b) Full view through the tested mirror

Fig. 4 Experimental setup of phase retrieval testing long focal length mirror

The measured intensity patterns are shown in Fig. 5. In theory, the patterns relative to focus are symmetrical. But the patterns in Fig. 5 are unsymmetrical, the size of patterns before focus is bigger than that behind in the same position. That′s because the focus presents as a spot since the aberration of mirror, and we can′t record the pattern′s position exactly (Fig. 6). Pretreatment is required before phase retrieval to compensate the position error.



(a) Intensity patterns 10mm,20mm,30mm,40mm,50mm,60mm behind focus



(b) Intensity patterns 10mm,20mm,30mm,40mm,50mm before focus

Fig. 5 Intensity patterns recorded by CCD

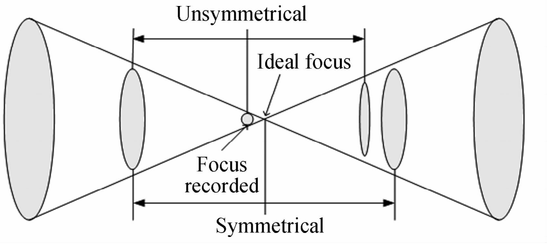


Fig. 6 Position error

In geometric optics, size of pattern is determined by its position and the f/# of light beam. Here the size of pattern can be measured, the distance of every two patterns and f/# is known exactly. Therefore we can calculate the position, compensate result is listed in Table 1. About 8 mm should be subtracted from the recorded data. One must note this method is an approximative one since faintness of the patterns and only suit for the patterns far away from the focus, as the spot near focus is not faithful.

Table 1 Result compensated for the position error/mm

Position recorded	−50	−40	−30	30	40	50
Position compensated	−58.085	−48.085	−37.964	22.71	32.051	42.051

The patterns above are input to phase retrieval algorithm. One pattern is enough for retrieving surface error. But multiple intensity patterns is generally helpful to ensure the convergent of the algorithm. Because the patterns near focus are disturbed by airflow more seriously than the farther ones, patterns 60 mm and 50 mm away from focus both before and after are chosen. The result of reconstruction is shown in Fig. 7 (a), PV 0.379 waves, RMS 0.035 waves. On the other hand, the mirror is tested by 4D interferometer and Fig. 7 (b) displays the result, PV 0.348 waves, RMS 0.057 waves.

The two error distributions are similar, and the 4D interferometer performs better in specifics. But the phase retrieval is accepted because of the use of simple equipment. And it is thought that a more accurate result can be obtained with a more advanced CCD.

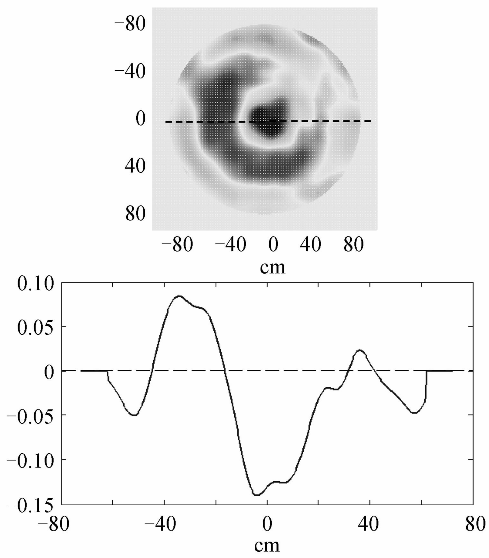
3 Error analysis

The excellent performance in numerical

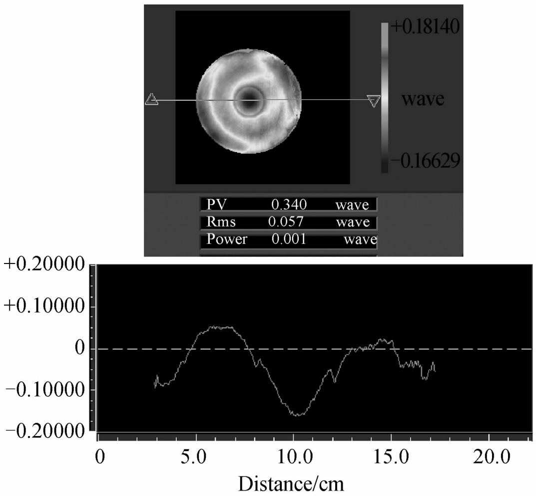
simulation presents nicer foreground of phase retrieval in testing long focal length mirror. But the experimental result is not as good as the simulation. Although the setup of phase retrieval measurement is much simpler than interferometer, it's still a complex process which is affected by a lot of error sources. There are two kinds of errors basically. One works on the measure precision by changing the wavefront reflected by tested mirror. The other influence the precision of recording intensity patterns.

The previous one includes mis-alignment error between point source and tested mirror, the wavefront error brought by point source itself, the beam splitter error, the position error when taking pictures and the stress error when installing the mirror. The mis-alignment between point source and tested mirror may cause small defocus and tilt which can be easily removed in final result. The spherical and coma is small, hence can be ignored. The wavefront error brought by point source is about 1/30 waves, and stress error can be controlled under 1/20 waves. These errors are the same as interferometer and their numerical value are estimated from experience. The wavefront error brought by beam splitter is also calibrated. We have tested a spherical mirror with interferometer at first. Then we put the beam splitter on its work position and measure the mirror again. The former subtracted from the latter gives PV 0. 16 waves, RMS 0. 085 waves. This error was compensated in the algorithm. The position error was also compensated as section 2 described.

The second kind of errors includes CCD errors and environmental errors. CCD is an important element in phase retrieval test where the accuracy is typically limited by spatial resolution of CCD. The CCD used in our experiment is industrial level and the maximal array of intensity patterns is 200×200, so testing spatial resolution is about 1. 37 pixel/mm. the interferometer detection is science level and virtual pixels are 614×602 in this test. That's one reason why Fig. 7 (a) can't present more detail like Fig. 7(b). For high spatial resolution, magnification device is suggested[6]. Furthermore, there are conversion error between photic signal and electrical signal, A/D noise and linearity error. For example, the light intensity should not exceed the dynamic range of CCD; otherwise conversion error will result in bad calculation. In our experiment, all intensity patterns are strictly controlled under the photic signal limit. The SNR is higher than 256:1 for most industrial CCD and this error is below 0. 01 waves.



(a) Surface error reconstructed by phase retrieval, PV 0.379 waves, RMS 0.035 waves



(b) Measurement result using 4D interferometer, waves, RMS 0.057 wavesPV 0.348

Fig. 7 Measurement results using two methods

Environmental errors include the vibration of the support stage, the light disturber from surroundings and the air turbulence. When the vibration's swing is 5 micron, the test result's error is no higher than 0. 005 waves[10]. If there are no thundering vibrations, this error is negligible in in-situ test. Light from surroundings will increase the noise entering CCD, so we turned off all lights when taking pictures. It's suggested covering the point source and CCD with a black box like most optical instruments in the future. So this error is able to be controlled. But air turbulence is out of control. For we tested the mirror in a normal plant, air flowed continually in long light path. That result in irregular light and shade in Fig. 5 (one may see clearly compared with the simulation picture Fig. 3(a)), which vitiates the quality of intensity pattern badly. For phase retrieval is a wave-front sensor technology based on intensity patterns, it's sensitive for intensity errors. This phenomenon is too prominent in the ones near focus to distil real message from them. That's the reason why we chose the further ones as algorithm's input. But the further ones are also disturbed. For example, the protuberance in right up of Fig. 7 (a) derives from the untruthful light spot in the pattern which is 60 mm away from focus (see Fig. 5 (a)). A useful method to reduce this error is disturbing the airflow artificially with fan as simultaneous phase-measurement interferometer, and then solving the algorithm with multiply pictures.

Even though there is limitation, phase retrieval can still expand to lens and other optical systems with problem in measure environment. It requires much looser condition than interferometer after all. By reason of that, it is a promising tool for in-situ surface metrology.

4 Conclusions

We applied phase retrieval to long focal length mirror testing and presented the testing device and algorithm which are validated by simulations and an experiment of testing a diameter 145 mm and radius of curvature 8 700 mm mirror. The experimental result is compared with 4D interferometer test, and the error distributions are mostly agreement. Besides the simplicity for experimental apparatus, this method is much more tolerant of vibration and disturbance of surroundings than interferometer. It can be a good shop tool of surface metrology or provide referenced result for some high accuracy measurement task to cover the shortage relying on one test method. Because of the limitations of our experimental equipment, further study is required to control the error sources and improve the accuracy.

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相位恢复测量长焦距镜面

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摘　要：针对长焦距光学镜面检测中测量光路长，振动干扰较大，不容易用干涉仪进行面形检测的难题，提出了一种基于相位恢复技术的测量方法. 该方法用相干点光源照射被测镜，采集一系列焦点附近的衍射光强图像，然后运用相位恢复算法得到镜面面形误差分布. 利用衍射光学理论建立了测量模型，并用基于Gerchberg-Saxton 算法的迭代算法求解模型. 然后仿真验证了光场传播模型的可靠性和测量算法的有效性，并用该方法测量了一块曲率半径 8 700 mm，口径 145 mm 的球面镜. 通过对光强图像位置进行优化，并选择适当离焦位置的图像，最终恢复出了镜面面形. 相位恢复测量的结果与动态干涉仪测量结果基本一致，并且测量装置简单，对环境要求低.

关键词：长焦距镜面；面形测量；相位恢复；衍射

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