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Oscillating Guided Modes in a Slab Waveguide with Anisotropy Left-handed Material*

SHEN Lu-fa[1,3], WANG Zi-hua[2]

(1 *Centre for Optical and Electromagnetic Research*, *Zhejiang University*, *Hangzhou* 310058, *China*)

(2 *Key Laboratory of Specialty Fiber Optics and Optical Access Networks*, *Shanghai University*, *Shanghai* 200072, *China*)

(3 *Department of Sciences*, *Huzhou Teachers College*, *Huzhou*, *Zhejiang* 313000, *China*.)

Abstract: An asymmetric three-layer slab waveguide with a core of the left-handed material and sandwiched by two conventional materials is investigated. Considering the dispersive and an isotropic properties of the left-handed material, the dispersion equation and power fluxes for TE oscillating guided modes are obtained and mode dispersion curves are plotted, respectively, through Maxuell′s equations. Eight TE oscillating guided modes are found, including the fundamental mode. With the increase of mode number, mode dispersion curves move to right and the power flux curves move to right bottom. However, with the increase of waveguide thickness, mode dispersion curves move to left and mode power flux curves move up. Besides, all modes have abnormal dispersion properties and employ negative group velocities which reveals the special properties of the left-handed materials.

Key words: Slab waveguide; Left-handed material; Dispersion properties; Normalized power flux

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0 Introduction

Since Smith et al firstly made the left-handed material (LHM) in microwave band[1], the issue for negative permittivity and permeability has attracted much attention due to their unusual electromagnetic properties. The typical features include negative refraction, reversed Doppler shift, reversed Cherenkov radiation[2], and reversed Goose-Hanchen shift[3-4], etc. The study of LHMs waveguide is an interesting topic for many scholars. Symmetric and asymmetric slab waveguides with LHM have been discussed[5-13]. They have some typical properties, such as: the absence or presence of the fundamental mode is dependent on neglecting or considering material dispersion; existence of surface wave propagation like metal waveguide; double-mode degeneracy appearance, etc. But, they all considered isotropic LHM in the slab waveguide. However, LHM is usually constructed by periodic arrays of metallic wire and split-ring resonator (SRR). The anisotropic material feature should be considered. Some scholars have studied slab waveguides with anisotropic LHM. Liu Z.[14] found the enhancement of photon tunneling by a slab of anisotropic negative refractive index material; Pan T.[15] obtained the dispersion properties and energy flux properties in a slab waveguide; Hamidreza Salehi et al.[16] investigated single negative LHM slab waveguide and discussed their device applications. However, they all considered the symmetric slab waveguides and solved dispersion equations by using a graphical solution method. But, this method can decide the solutions to exist or not. Thus, in this paper, we will consider anisotropic left-handed material and calculate numerically dispersion equations of oscillating guided modes in the asymmetric slab waveguide.

A three-layer slab waveguide with LHM in the core and right-handed materials (RHMs) in other layers is examined in this paper. Through Maxwell′s equations, considering anisotropic LHM, a dispersion equation for TE oscillating guided modes is obtained. From the equation, we plot some corresponding dispersion curves, and find some dispersion properties as frequency from 4 GHz to 6 GHz. Besides, the power flux of the fields in the slab waveguide is studied and found some new power flux properties for TE oscillating guided modes.

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Tel:0572-2321163 Email:shenlufa@hutc.zj.cn
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1 Dispersion equation and power flow on anisotropic slab waveguide

1.1 Dispersion equation for the TE oscillating guided modes

A three-layer slab waveguide including LHM is shown in Fig. 1. The core layer of the thickness h is the anisotropic LHM with permittivity tensor $\varepsilon_2(\varepsilon_x, \varepsilon_y, \varepsilon_z)$ and permeability tensor $\mu_2(\mu_x, \mu_y, \mu_z)$. The claddings are different RHMs with permittivity (ε_1 and ε_3), permeability (μ_1 and μ_3). To simplify, we assume that the claddings extend to infinity, and the time-and z-factor $\exp[i(\omega t-\beta z)]$ that multiplies all the field components is neglected from all equations. Where ω and β denote the angular frequency and the longitudinal propagation constant. For the oscillating guided modes, mode fields decay in the claddings, but oscillate in the core layer. A slab waveguide can support TE modes and TM modes. In this paper, we mainly discuss TE modes. For TE modes, they have E_y, H_x and H_z components. From Maxwell's equations, the electromagnetic fields in the core satisfy following equations

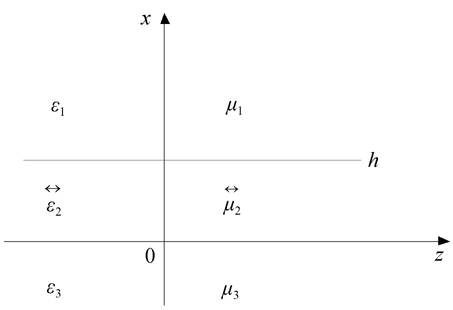

Fig. 1 Schematic geometry of an asymmetric three-layer LHM slab waveguide

$$-i\beta H_x-\frac{\partial H_x}{\partial x}=i\omega\varepsilon_y E_y \tag{1a}$$

$$H_x=-\frac{\beta}{\omega\mu_x}E_y \tag{1b}$$

$$\frac{\partial E_y}{\partial x}=-i\omega\mu_z H_z \tag{1c}$$

In the claddings, the electromagnetic fields equations can be obtained by replace of the corresponding permittivity and permeability in Eq. (1a-1c). From all equations, after some algebraic manipulations, the electric fields in the slab waveguide are written as

$$E_{y_1}=E_1\exp[-\alpha_1(x-h)],\ x>h \tag{2a}$$

$$E_{y_2}=E_2\cos(k_2x-\varphi_s),\ 0<x<h \tag{2b}$$

$$E_{y_3}=E_3\exp(\alpha_3 x),\ x<0 \tag{2c}$$

Where E_1, E_2 and E_3 are the amplitudes for the electric fields in each layer. α_1 and α_3 are evanescent coefficients, which can be written as $\alpha_1=\sqrt{\beta^2-k_0^2\varepsilon_1\mu_1}$ and $\alpha_3=\sqrt{\beta^2-k_0^2\varepsilon_3\mu_3}$, k_2 is the transverse wave number, which can be expressed as $k_2=\sqrt{\frac{\mu_z}{\mu_x}(k_0^2\varepsilon_y\mu_x-\beta^2)}$, k_0 and φ_s represent the vacuum wave number and the phase constant, respectively.

From continuous boundary conditions, after some algebraic manipulations, a dispersion relation for the TE oscillating guided modes is expressed as

$$k_2h-\tan^{-1}\left(\frac{\mu_z\alpha_1}{\mu_1k_2}\right)-\tan^{-1}\left(\frac{\mu_z\alpha_3}{\mu_3k_2}\right)=m\pi \tag{3}$$

For TM oscillating guided modes, their dispersion equations can be written that the permeability is replaced by the corresponding permittivity in Eq. (3).

1.2 Power flow for the TE oscillating guided modes

Power fluxes each layer in the waveguide are calculated by an integral of Poynting vector. For the TE oscillating guided modes, their power fluxes (P_i) are calculated as follows

$$P_i=\frac{\beta}{2\omega}\int\frac{1}{\mu_i}\ |E_{yi}|^2\mathrm{d}x(i=1,2,3) \tag{4}$$

Substituted Eqs. (2a) ~ (2c) into Eq. (4), after some algebraic manipulation, their power fluxes in the waveguide are expressed as follows.

$$P_1=\frac{\beta E_2^2}{4\omega\mu_1\alpha_1}\cos^2(k_2h-\varphi_s) \tag{5a}$$

$$P_2=\frac{\beta E_2^2}{4\omega\mu_xk_2}[k_2h+\sin(k_2h)\cos(k_2h-2\varphi_s)] \tag{5b}$$

$$P_3=\frac{\beta E_2^2}{4\omega\mu_3\alpha_3}\cos^2\varphi_s \tag{5c}$$

Where P_1, P_2 and P_3 denote the power fluxes in the cover, the core and the substrate, respectively.

To describe the energy transmission of the fields in the waveguide, the normalized power flux for the LHM waveguide is defined as[9]

$$P=\frac{P_1+P_2+P_3}{|P_1|+|P_2|+|P_3|} \tag{6}$$

We have known that power fluxes propagate forward in the conventional media and they are all positive, i. e. P_1 and $P_3>0$. However, the direction of the wave vector and Ponyting vector in the LHM medium is opposite, so, the power flux is negative, namely, $P_2<0$. In terms of Eq. (6), there exist three cases: (1) $P>0$, it means $P_1+P_3>|P_2|$. It is a case for the forward wave; (2) $P<0$, it implies $P_1+P_3<|P_2|$. It is a case for the backward wave; (3) $P=0$, it means $P_1+P_3=|P_2|$. Under this case, electromagnetic waves are stopped and all energy is stored in the waveguide.

2 Numerical results

For the practical left-handed materials, both material dispersion and anisotropic optical property should be considered. For material dispersion, we employ an experimental model[9] with dielectric permittivity and magnetic permeability written as

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega^2},\ \mu(\omega)=1-\frac{F\omega^2}{\omega^2-\omega_0^2} \tag{7}$$

Where $F=0.56$, $\omega_0/2\pi=4$ GHz, $\omega_p/2\pi=10$ GHz. When frequency increases from 4 GHz to 6 GHz, the dielectric permittivity and magnetic permeability are negative simultaneously. Considering anisotropic LHM, the permittivity tensor $\varepsilon_2(\varepsilon_x,\varepsilon_y,\varepsilon_z)$ and permeability tensor $\mu_2(\mu_x,\mu_y,\mu_z)$ are assumed as: $\varepsilon_x=\varepsilon_y=\varepsilon(\omega)<0$, $\varepsilon_z=1>0$; $\mu_x=\mu_y=\mu(\omega)<0$, $\mu_z=1>0$. Under this case, for $k_2=\sqrt{\mu_z/\mu_x(k_0^2\varepsilon_y\mu_x-\beta^2)}$, as $\beta>k_0\sqrt{\varepsilon_y\mu_x}$, we define the modes as TE oscillating guided modes, but, as $\beta<k_0\sqrt{\varepsilon_y\mu_x}$, we call the modes as TE surface modes and discussed in another papers.

2.1 The dispersion properties for the TE oscillating guided modes

In order to explore the influence of the left-handed materials for the slab waveguide, we assume that the cladding permittivity and permeability are fixed and equals to $\varepsilon_1=2.25$, $\mu_1=1$, $\varepsilon_3=2.0$, $\mu_3=1$. The dispersion properties of two cases are discussed as follows.

2.1.1 *The dispersion properties depend on mode number*

For $h=2$ cm, from Eq. (3), some dispersion curves (the effective refractive index verse frequency) for TE oscillating guided modes are plotted in Fig. 2. As frequency increases from 4 GHz to 6 GHz, we find eight TE_m modes, and m stands for mode number. For $m=0$, the fundamental mode is exist. As m increases from 1 to 7, the dispersion curves shift to right and their cutoff frequencies becomes bigger. It is different with an isotropic LHM waveguide considering material dispersion[17]. The difference is because for an isotropic LHM slab waveguide considering material dispersion, when the frequency increases, the absolute value of refractive index decreases rapidly, so the field is less trapped inside the waveguide and causes the dispersion curves shift to left as m increases. However, for an anisotropic LHM with material dispersion slab waveguide, the transverse permittivity and permeability are negative and change with frequency while the longitudinal permittivity and permeability are positive ($\varepsilon_z=\mu_z=1$) and without material dispersion. When the field propagates along z-axis, it is more condensed or trapped inside the waveguide and causes the dispersion curves shift to right as m increases. Besides, for a TE_m mode, as frequency increases, the effective refractive index decreases. That shows abnormal dispersion property and their group velocities ($v_g=\frac{d\omega}{d\beta}$) are negative. Negative group velocity implies energy propagates backward and reveals special property in the LHM slab waveguide.

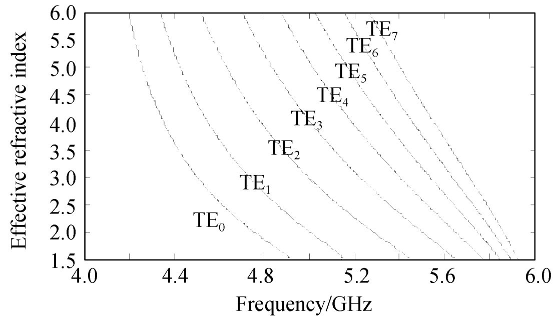

Fig. 2 Dispersion curves for the TE oscillating guided modes depend on mode number and the effective-refractive-index is a function with frequency from 4 GHz to 6 GHz. As fixed (2 cm), and m increases from 0 to 7. The core layer is the anisotropic-dispersive LHM. The cover and substrate are different right-handed material with dielectric permittivity ($\varepsilon_1=2.25$ and $\varepsilon_2=2.0$), magnetic permeability ($\mu_1=\mu_3=1.0$)

2.1.2 *The dispersion properties depend on waveguide thickness*

As mode number increases from 0 to 7, all TE oscillating guided modes have similar dispersion properties. Usually, we are interested in the fundamental mode. As $m=0$, from Eq. (3), the dispersion curves with different waveguide thickness h are plotted in Fig. 3. As h increases, the curves move to left. It is different from that of a conventional slab waveguide[18]. The difference is because when h increases, for the conventional slab waveguide, the effective thickness increases, too. However, for the LHM slab waveguide, the

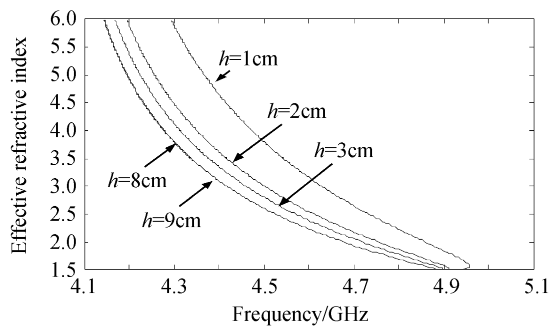

Fig. 3 Dispersion curves for TE oscillating guided modes depend on waveguide thickness. The effective-refractive-index verses the frequency from 4 GHz to 6 GHz. As $m=0$, and h equals to 1 cm, 2 cm, 3 cm, 8 cm and 9 cm. The other corresponding parameters employed are the same as Fig. 2

effective thickness decreases since negative permeability and causes curves move to left. Besides, near cutoff, for one frequency, the effective refractive index has two different values and double-mode degeneracy appears. But, as h increases, the double-mode degeneracy becomes not obvious and the interval of two curves gets small. As $h>8$ cm, the dispersion curves almost overlap together.

2.2 The power flux properties for the TE oscillating guided modes

By using equations (3)～(7), we choose the same parameters as section 2.1 and plot some curves for normalized power flux verse the frequency. The power fluxes of two cases are discussed in the following details.

2.2.1 *The power flux properties depend on mode number*

For $h=2$ cm, as mode number m increases from 0 to 7, the normalized power flux (NPF) curves are plotted in Fig. 4, respectively. They all have similar power flux properties. As frequency increases, first, their NPF increases slowly. As frequency is enough big, their NPF increases abruptly. That property may help us to design a sensor device with high sensitivity. For $m=0$, 1and 2, their NPF increases from negative, zero, to positive as frequency. As NPF equals to zero, it implies that the electromagnetic waves are stopped and energy is stored in the slab waveguide completely. For $m=3$, 4, 5, 6 and 7, their NPF keeps negative. It means that the energy in the core is bigger than that of sum of the cover and the substrate. As mode number increases, the curves move to right bottom. Both the absolute value of NPF and operating frequency get bigger. For TE_7 mode, its curve is at the bottom of Fig. 4 and the absolute value of NPF is the biggest. It is because

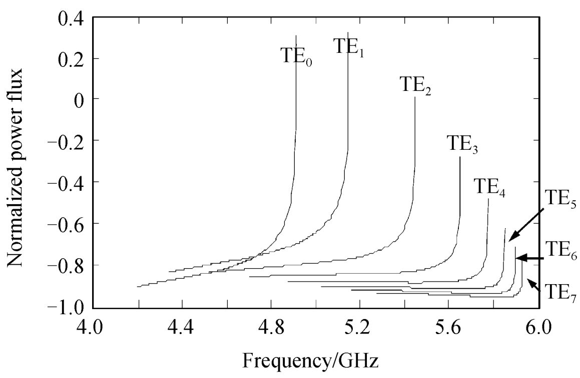

Fig. 4 The power flux curves for TE oscillating guided modes depend on mode number. The normalized power flux verses frequency from 4 GHz to 6 GHz. The corresponding parameters employed are the same as Fig. 2

the energy increases in the core as mode number increases. That property has been discussed in subsection 2.1.1. However, for TE_0 mode, the absolute value of NPF is bigger than that of TE_1 mode in low-frequency regions and appears some abnormal properties. That property will be studied in the future.

2.2.2 *The power flux properties depend on waveguide thickness*

From above discussions, the NPF of each mode has similar property. As $m=0$, we study the NPF with dependence of waveguide thickness, and the corresponding curves are plotted in Fig. 5, respectively. As h increases, the curves move up. It means that energy in the core decreases as h increases. It is because for LHM slab waveguide, its effective thickness including negative permeability gets small as h increases. Besides, we find that the NPF has an abnormal property as $h=1$ cm, which the absolute value of its NPF is smaller than that of $h=2$ cm and 3 cm on some low frequency regions. It may cause of the double-mode degeneracy in the LHM waveguide.

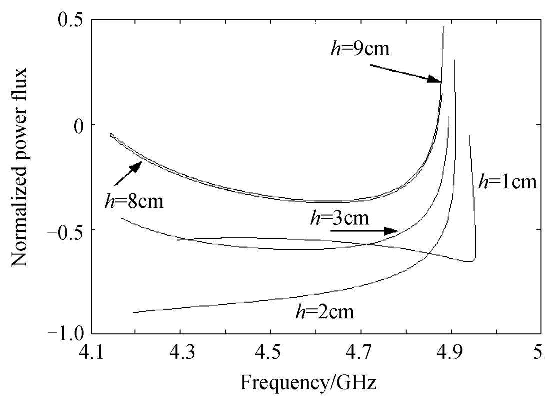

Fig. 5 The power flux curves for TE oscillating guided modes depend on waveguide thickness. The normalized power flux verses frequency from 4 GHz to 6 GHz. The corresponding parameters employed are the same as Fig. 3

3 Conclusions

A three-layer slab waveguide with anisotropic LHM in the core and RHMs in the claddings is investigated. A dispersion equation for the TE oscillating guided modes is obtained and corresponding dispersion curves are plotted. We find eight TE oscillating guided modes, including zero-order mode. Besides, the power fluxes of TE oscillating guided modes are calculated and found some new properties. The TE oscillating guided modes have following properties:

1) The effective refractive index decreases as frequency increases and reveals the abnormal dispersion property;

2) With the increase of mode number, mode dispersion curves move to right and their cutoff frequencies get bigger; the normalized power flux curves move to right bottom and the energy in the core increases;

3) With the increase of waveguide thickness, mode dispersion curves move to left and their cutoff frequencies get smaller; the normalized power flux curves move up and the energy in the core decreases.

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各向异性色散左手材料平面波导的导模特性

沈陆发[1,3]，王子华[2]

(1 浙江大学 光及电磁波研究中心，杭州 310058)

(2 上海大学 特种光纤与光接入网省部共建重点实验室，上海 200072)

(3 湖州师范学院 理学院，浙江 湖州 313000)

摘　要：对芯层为左手材料而内外包层都是普通材料的非对称三层平面波导TE振荡模进行了分析.在考虑左手材料色散和各向异性的情况下，从Maxwell方程组出发，得到了TE振荡模的色散方程和功率流分布，并且画出了相应的色散曲线.我们找到了8个TE振荡模，而且包括基模.随着模阶数的增加，模色散曲线右移，功率流曲线下移.但是，随着波导厚度的增加，色散曲线左移，功率流曲线上移.此外，TE振荡模有反常色散特性和负的群速，这正揭示了左手材料的本质特性.

关键词：平面波导；左手材料；色散方程；归一化功率流

SHEN Lu-fa was born in 1963. He received his Ph. D. degree from Shanghai University in 2008. Now, he is a Post Doctor at Zhejiang University, and his research interests focus on waveguide optics and optical fiber.