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## Properties of One-dimensional Doped Photonic Crystals Surrounded by an $\epsilon$ -negative Layer and a $\mu$ -negative Layer\*

DONG Li-juan<sup>1,2</sup>, JIANG Hai-tao<sup>2,†</sup>, LI Yun-hui<sup>2</sup>, DU Gui-qiang<sup>2,3</sup>,  
SHI Yun-long<sup>1,2</sup>, YANG Cheng-quan<sup>1</sup>

(1 Institute of Solid State Physics, Shanxi Datong University, Datong, Shanxi 037009, China)

(2 Pohl Institute of Solid State Physics, Tongji University, Shanghai 200092, China)

(3 School of Space Science and Physics, Shandong University at Weihai, Weihai, Shandong 264209, China)

**Abstract:** The transmission properties of defect modes for a doped one-dimensional photonic crystal (1DPC) surrounded by an  $\mu$ -negative layer and a  $\mu$ -negative layer are studied. The reflection phase spectra of  $\mu$ -negative and a  $\mu$ -negative media and the transmission phase spectra of the doped 1DPC are calculated by means of the transfer matrix method. It is found that the sum of the reflection phases of  $\mu$ -negative and a  $\mu$ -negative layers and the double transmission phases of the doped 1DPC under special conditions are zero or integer multiple of  $2\pi$ . The results show that only one defect mode appears in the photonic gap when the doped 1DPC surrounded by an  $\mu$ -negative layer and a  $\mu$ -negative layer, regardless of the thickness of the defect. Moreover, the quality factor of the single defect mode is enhanced greatly due to the confinement effect of the  $\epsilon$ - and  $\mu$ -negative layers.

**Key words:** Photonic crystal; Defect mode;  $\epsilon$ -negative material;  $\mu$ -negative material

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## 0 Introduction

Metamaterials made of subwavelength unit cells have attracted widespread study due to their special electromagnetic (EM) properties<sup>[1-5]</sup>. They contain double-negative materials whose permittivity ( $\epsilon$ ) and permeability ( $\mu$ ) are simultaneously negative<sup>[1]</sup> and single-negative materials including  $\epsilon$ -negative materials ( $\epsilon < 0, \mu > 0$ ) and  $\mu$ -negative materials ( $\mu < 0, \epsilon > 0$ )<sup>[2-3]</sup>. It is known that the wave vector in single-negative materials is imaginary, so the single-negative materials are opaque to EM waves. However, the paired structure made of  $\epsilon$ -negative and  $\mu$ -negative media can be transparent under impedance and phase matching conditions<sup>[3-5]</sup>.

On the other hand, photonic crystals made of periodic dielectric have received considerable development for potential applications in optical devices<sup>[6-10]</sup> since the work of Yablonovitch<sup>[11]</sup> and

John<sup>[12]</sup> in 1987. The period length of photonic crystal is in the scale of incident wavelength. Photonic crystals have photonic gaps<sup>[13-14]</sup> within which the propagations of EM waves are prohibited due to Bragg scattering. If the photonic crystal is doped with a defect, the translation symmetry will be broken and defect modes will appear in the photonic gaps<sup>[11]</sup>.

Recently, some new physical phenomena and characteristics are found in some complicated structures containing single-negative materials and photonic crystals<sup>[15-17]</sup>. For example, the localized gap-edge fields were found in a one-dimensional photonic crystal (1DPC with dielectrics) doped with an  $\epsilon$ -negative layer and a  $\mu$ -negative layer (so called effective zero-index defect) in the middle<sup>[17]</sup>. For an all-dielectric 1DPC with a dielectric defect in the middle, usually the number of the localized defect modes in the photonic gap increases as the thickness of the defect enlarges. In this paper, we study the transmission properties of a doped dielectric 1DPC surrounded by an  $\epsilon$ -negative layer and a  $\mu$ -negative layer. We find only one defect mode appears, regardless of the thickness of the defect. Moreover, the quality factor of the single defect mode is enhanced noticeably because of the confinement effect of the  $\epsilon$ - and  $\mu$ -negative layers. Based on the resonance condition, the physical origin

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† Tel: 021-65988661 Email: jiang-haitao@tongji.edu.cn

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of the single-mode property is given in Sec. 1. The loss of the single-negative materials is considered in Sec. 2. Finally, we conclude in Sec. 3.

### 1 Single-mode transmission with high quality factor

A dielectric 1DPC surrounded by an  $\epsilon$ -negative layer and a  $\mu$ -negative layer is schematically shown

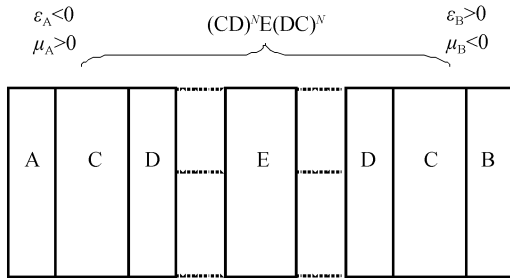


Fig. 1 Schematic of a doped 1DPC surrounded by an  $\epsilon$ -negative layer and a  $\mu$ -negative layer in the air in Fig. 1. The structure is denoted as A (CD)<sup>N</sup>E (DC)<sup>N</sup>B. Here, A and B indicate the  $\epsilon$ - and  $\mu$ -negative layers, respectively. C, D and E represent three types of dielectric materials, respectively.  $N$  is the periodic number. We describe the single-negative materials with the Drude model,

$$\epsilon_A = 1 - \frac{\omega_{ep}^2}{\omega^2}, \mu_A = a \tag{1}$$

in  $\epsilon$ -negative medium and

$$\epsilon_B = b, \mu_B = 1 - \frac{\omega_{mp}^2}{\omega^2} \tag{2}$$

in  $\mu$ -negative medium, respectively. In Eqs. (1) and (2),  $\omega$  is the angular frequency measured in gigahertz (GHz), and  $\omega_{ep}$  ( $\omega_{mp}$ ) denotes electronic (magnetic) plasma frequency. The refractive-index of C, D and E layers are supposed to be  $n_C$ ,  $n_D$  and  $n_E$ , respectively. The thickness of A, B, C, D and E layers are assumed to be  $d_A$ ,  $d_B$ ,  $d_C$ ,  $d_D$  and  $d_E$ , respectively. In the following calculation, we choose  $\omega_{ep}^2 = \omega_{mp}^2 = 133.24$  GHz<sup>2</sup>,  $a = b = 5$ ,  $n_C = 1.2$ ,  $n_D = 3.5$ ,  $n_E = 1.4$ ,  $n_C d_C = n_D d_D = 0.1$  m and  $N = 3$ .

We only consider normal incidence in this paper. The transmission properties of the structure are studied by means of the transfer matrix method<sup>[15]</sup>. Fig. 2 shows the transmittances of the (CD)<sup>3</sup>E (DC)<sup>3</sup> and A (CD)<sup>3</sup>E (DC)<sup>3</sup>B structures, respectively. We suppose  $n_E d_E = 0.8$  m and  $d_A = d_B = 0.02$  m in Fig. 2. It is seen from Fig. 2 (a) that three defect modes emerge in the photonic gap of the (CD)<sup>3</sup>E (DC)<sup>3</sup> structure with a thick defect in the middle. In contrast, only one defect mode appears and the other defect modes are avoided in the photonic gap of the A (CD)<sup>3</sup>E

(DC)<sup>3</sup>B structure, as shown in Fig. 2 (b). The physical origin is discussed below.

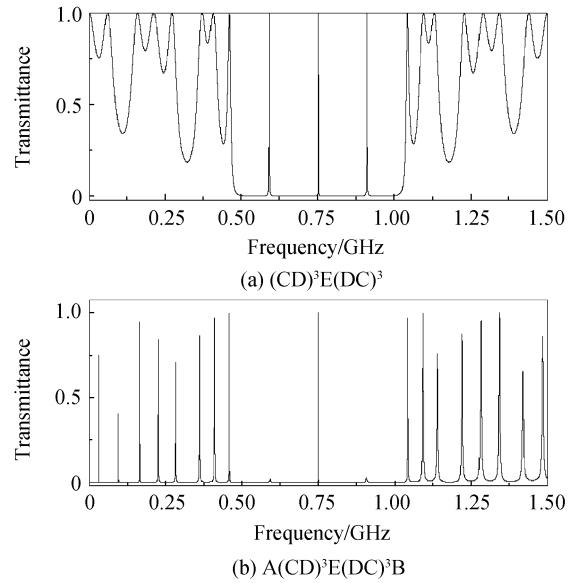


Fig. 2 Transmittances of (CD)<sup>3</sup>E (DC)<sup>3</sup> structure and A (CD)<sup>3</sup>E (DC)<sup>3</sup>B structure respectively

In our structure, A and B layers can be considered as reflectors because EM waves decay in the single-negative materials. It should be pointed out that the reflection phase of an  $\epsilon$ -negative medium is positive while that of a  $\mu$ -negative layer is negative. Therefore, the A (CD)<sup>3</sup>E (DC)<sup>3</sup>B structure can be considered as a doped 1DPC surrounded by two kinds of reflectors (one is the electrical wall and the other is the magnetic wall), as shown in Fig. 3. Resonance modes will appear if

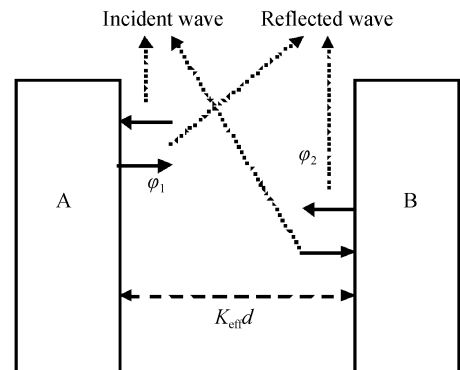


Fig. 3 Schematic of resonance cavity

$$\varphi_1 + \varphi_2 + 2k_{\text{eff}}d = m2\pi, (m = 0, 1, 2, \dots) \tag{3}$$

where  $\varphi_1$  ( $\varphi_2$ ) is reflection phase of the  $\epsilon$ -negative ( $\mu$ -negative) reflector, and  $2k_{\text{eff}}d$  denotes the phase accumulation in the round trip of the dielectric 1DPC (CD)<sup>3</sup>E (DC)<sup>3</sup>. When the thickness of the (CD)<sup>3</sup>E (DC)<sup>3</sup> is zero, the A (CD)<sup>3</sup>E (DC)<sup>3</sup>B structure reduce to AB pairing structure. Fig. 4(a) shows the variance of the reflection phase for A and B layers, respectively. One can see the reflection phases  $\varphi_1$  and  $\varphi_2$  are 79.57 and  $-79.57^\circ$

at the frequency of 0.75 GHz shown by the solid and the dashed lines in Fig. 4(a), respectively. Therefore, the sum of  $\varphi_1 + \varphi_2$  is zero at 0.75 GHz for AB structure, as indicated by the second black solid circle in the dotted line. So zero-order resonance mode will appear at 0.75 GHz. Now, if we put  $(CD)^3E(DC)^3$  into the middle of the AB

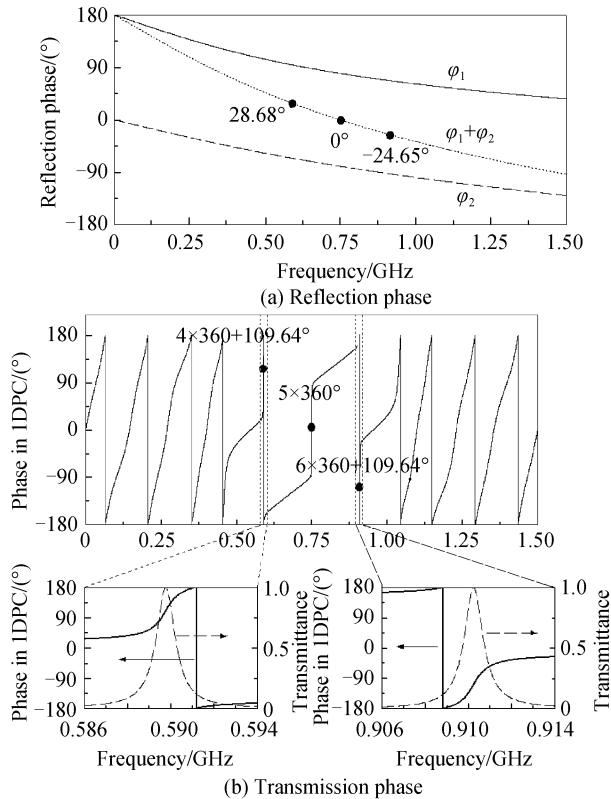


Fig. 4 Reflection phase of the  $\epsilon^-$  and  $\mu^-$ -negative layers and Transmission phase in  $(CD)^3E(DC)^3$  structure, resonance modes will emerge when Eq. (3) is satisfied. We select the same parameters of  $(CD)^3E(DC)^3$  used in Fig. 2(a). The transmission phase of  $(CD)^3E(DC)^3$  structure is given in Fig. 4(b). The frequencies of three defect modes in Fig. 2(a) are 0.5898 GHz, 0.75 GHz and 0.9102 GHz, respectively. For clearness, the transmission phases of  $(CD)^3E(DC)^3$  structure at the region around 0.5898 GHz and the one around 0.9102 GHz indicated by vertical dashed lines in Fig. 4(b) are enlarged. Meanwhile, the corresponding transmittance spectra are shown in the two enlarged figures. At the frequency of 0.5898 GHz, the sum of reflection phase of AB pair is 28.68° indicated by the first black solid circle in the dotted line in Fig. 4(a), and the transmission phase of  $(CD)^3E(DC)^3$  is  $4 \times 360^\circ + 109.64^\circ$  indicated by the first black solid circle (the left enlarged figure) in Fig. 4(b). Then the sum of phases on the left side of Eq. (3) (double of the transmission phase of  $(CD)^3E(DC)^3$  adding the

reflection phase of AB) is  $8 \times 360^\circ + 247.96^\circ$  at 0.5898 GHz. So no resonance mode appears at 0.5898 GHz, as seen in Fig. 2(b). However, at the frequency of 0.75 GHz, the transmission phase of  $(CD)^3E(DC)^3$  is  $5 \times 360^\circ$  indicated by the second black solid circle in Fig. 4(b). Then the sum of phases at 0.75 GHz is  $10 \times 360^\circ$  because the sum of reflection phase in AB pair is zero at this frequency. So a resonance mode emerges at 0.75 GHz, as shown in Fig. 2(b). Similarly, the total sum of phases at the frequency of 0.9102 GHz is  $12 \times 360^\circ - 243.93^\circ$  from the two values of the third black solid circles in Fig. 4(a) and (b) (the right enlarged figure), respectively. So no resonance mode emerges for  $A(CD)^3E(DC)^3B$  structure at 0.9102 GHz, as shown in Fig. 2(b). It is the resonance condition that allows only one defect mode exists in the photonic gap of  $(CD)^3E(DC)^3$ . Later we will find this single-mode property is maintained in spite of the thickness of the defect layer E.

Here we select  $n_E d_E = 1.6 \text{ m}$  for defect layer E in the  $(CD)^3E(DC)^3$  structure. Fig. 5 shows the transmittances of the  $(CD)^3E(DC)^3$  and  $A(CD)^3E(DC)^3B$  structures, respectively. There are seven defect modes in the photonic gap of the  $(CD)^3E(DC)^3$  structure, as shown in Fig. 5(a). However, only one defect mode exists in the photonic gap of the  $A(CD)^3E(DC)^3B$  structure, as shown in Fig. 5(b). Therefore, single defect mode can be realized in  $A(CD)^3E(DC)^3B$  structure, regardless of the thickness of the defect.

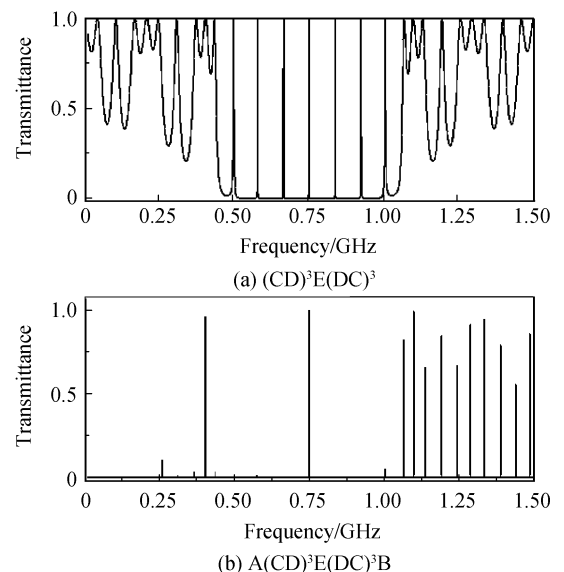


Fig. 5 Transmittances of  $(CD)^3E(DC)^3$  structure and  $A(CD)^3E(DC)^3B$  structure, respectively

Now, the quality factor of the single defect mode is studied in the  $(CD)^3E(DC)^3$  and  $A(CD)^3E$

(DC)<sup>3</sup>B structures. The quality factor of the defect mode at the frequency of 0.75 GHz is 1 630 in the (CD)<sup>3</sup>E(DC)<sup>3</sup> structure in Fig. 2(a). However, the quality factor of the single defect mode reaches 18 750 in the A(CD)<sup>3</sup>E(DC)<sup>3</sup>B structure in Fig. 2(b). The rapid increase of the quality factor of the defect mode is due to the confinement effect of  $\epsilon$ - and  $\mu$ -negative reflectors<sup>[3-5]</sup>.

## 2 Influence of losses in single-negative materials

In the above calculations, we considered the lossless single-negative materials. In practice, the losses in the single-negative materials are inevitable. When the losses are involved, the permittivity of the  $\epsilon$ -negative layer in Eq. (1)

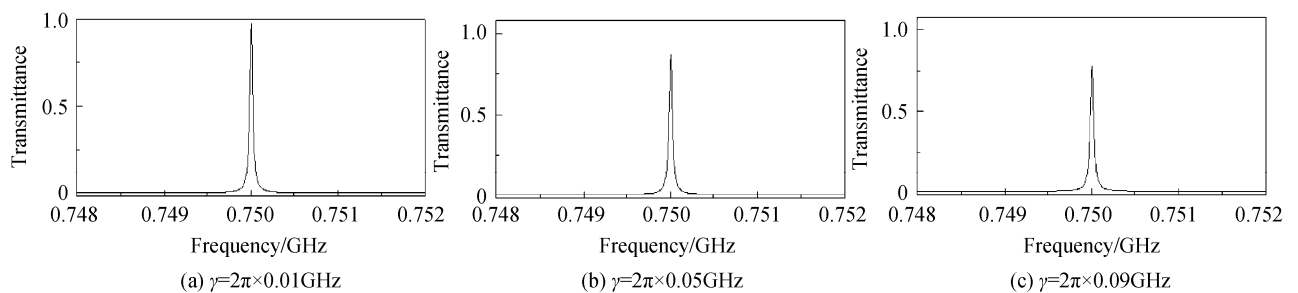


Fig. 6 Transmittances of A(CD)<sup>3</sup>E(DC)<sup>3</sup>B structure with different dissipation coefficient  $\gamma$

## 3 Conclusion

In conclusion, we studied the transmission properties of a doped 1DPC with a thick defect surrounded by an  $\epsilon$ -negative layer and a  $\mu$ -negative layer. In this structure, the single defect mode with the high quality factor can be realized in the photonic band gap, regardless of the thickness of the defect, which is distinct from the case of the all-dielectric doped 1DPC. This property may be used in the applications requiring single mode with the high quality factor.

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should be modified as  $\epsilon_A = 1 - \frac{\omega_{ep}^2}{\omega^2 + i\omega\gamma_e}$ , and the permeability of the  $\mu$ -negative layer in Eq. (2) should be modified as  $\mu_B = 1 - \frac{\omega_{mp}^2}{\omega^2 + i\omega\gamma_m}$ , where  $\gamma_e$  ( $\gamma_m$ ) denotes the dissipation coefficient of  $\epsilon$ -negative ( $\mu$ -negative) material. We assume  $\gamma_e = \gamma_m = \gamma$ . The variances of the transmittance of the A(CD)<sup>3</sup>E(DC)<sup>3</sup>B structure with  $\gamma$  are shown in Fig. 6. The other parameters of A(CD)<sup>3</sup>E(DC)<sup>3</sup>B are the same with those of A(CD)<sup>3</sup>E(DC)<sup>3</sup>B in Fig. 2. One can see from Fig. 6 that the transmittance decreases when  $\gamma$  increases. Meanwhile, the quality factor are somewhat decreased. But the single-mode property is still maintained even when losses are involved.

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## 一维掺杂光子晶体嵌入负介电常数材料和负磁导率材料的性质

董丽娟<sup>1,2</sup>, 江海涛<sup>2,\*</sup>, 李云辉<sup>2</sup>, 杜桂强<sup>2,3</sup>, 石云龙<sup>1,2</sup>, 杨成全<sup>1</sup>

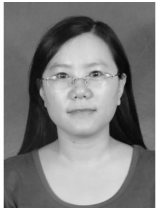
(1 山西大同大学 固体物理研究所, 山西 大同 037009)

(2 同济大学 波耳固体物理研究所, 上海 200092)

(3 山东大学 威海分校 空间科学与物理学院, 山东 威海, 264209)

**摘要:** 对一维掺杂光子晶体嵌入负介电常数材料和负磁导率材料中缺陷模的透射性质进行了研究. 利用转移矩阵方法, 分别计算了负介电常数材料和负磁导率材料的反射相位谱和一维掺杂光子晶体的透射相位谱. 研究发现, 在特定条件下, 负介电常数材料和负磁导率材料的反射相位以及一维掺杂光子晶体的往返透射相位之和是  $0$  或者  $2\pi$  的整数倍. 这样的研究结果表明, 在满足一定的条件下, 一维掺杂的光子晶体嵌入负介电常数材料和负磁导率材料中后, 无论杂质的厚度多大, 在光子带隙中仅出现一个缺陷模. 而且, 由于负介电常数材料和负磁导率材料性质的限制, 单个缺陷模的品质因子会大大提高.

**关键词:** 光子晶体; 缺陷模; 负介电常数材料; 负磁导率材料



**DONG Li-juan** was born in 1976. She received the M. S. degree in theory physics from Hebei University of Technology in 2003. Now she is working for the Ph. D. degree at Tongji University, and her main research interests focus on photonic crystals and metamaterials.