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Oscillating Guided Modes in a Symmetric Five-layer Slab Waveguide with Left-handed Material*

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Abstract: A symmetric five-layer slab waveguide with a core of left-handed material and surrounded by four normal dielectrics is investigated. Under the consideration of material dispersion, the dispersion equations for the TE oscillating guided modes are obtained and mode dispersion curves are plotted. It is found that zero-order TE oscillating guided mode is exist. With the increase of mode number, mode dispersion curves move to left, and their cutoff frequencies decrease. Besides, some modes have three dispersion properties: normal dispersion, abnormal dispersion and mode double-degeneracy.

Key words: Slab waveguide; Left-handed material; Dispersion equation; Dispersion curves

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0 Introduction

Since Smith et al[1] made firstly the left—handed material, the issue for negative index has been investigated a few years. Now, many scholars[2-6] have explored slab waveguide containing left-handed material(LHM). L. F. Shen[5] and Y. He[6] studied a five-layer slab waveguide, however, they neglected material dispersion, and the latter applied a graphical method to investigate guided optical modes. We know that the material dispersion is one of inherent properties of LHM and the graphical method can only determine the existence of modes. Therefore, we should investigate guided modes in slab waveguide under considering material dispersion further.

In this paper, a five-layer slab waveguide with LHM in the core and right-handed materials (RHMs) in other layers is examined. Through Maxwell's equations, by a transfer matrix method, dispersion equations for TE oscillating guided modes are obtained. From these equations, we plot some corresponding dispersion curves, and discuss dispersion properties for TE oscillating guided modes as frequency from 4 GHz to 6 GHz.

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1 Dispersion equations for TE oscillating guided modes

A five-layer slab waveguide including LHM is shown in Fig. 1. The core layer is the LHM, i. e. its dielectric permittivity (ε_1), magnetic permeability (μ_1) and refractive index (n_1) are negative. However, the inner claddings and the outer claddings are different conventional materials, i. e. their dielectric permittivity (ε_2 and ε_3), magnetic permeability (μ_1 and μ_2) and refractive index (n_2 and n_3) are positive. The thicknesses of the core and inner claddings are $2h_1$ and h_2, respectively. Besides, we assume that the outer layers extend to infinity. The time-and z- factor $\exp[\mathrm{i}(\omega t-\beta z)]$ that multiplies all the field components is neglected from all equations. Where ω and β denote the

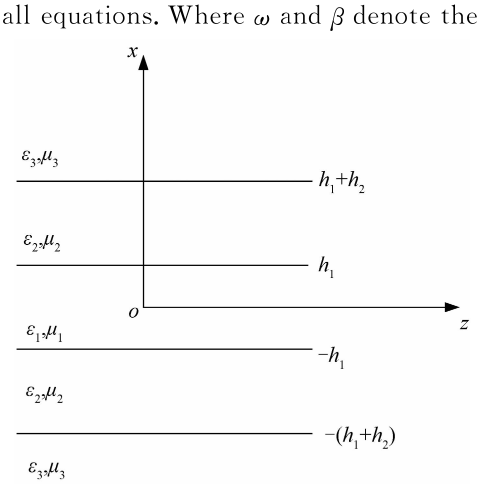

Fig. 1 Schematic geometry of symmetric five-layer LHM slab waveguide

angular frequency and the longitudinal propagation constant. In general, a slab waveguide can support TE and TM modes.

In this paper, we discuss TE oscillating guided modes. For TM modes, they will be investigated in other papers. Through Maxwell's equations, electric fields for TE oscillating guided modes satisfy the following equation

$$\frac{\partial E_y}{\partial x^2}+(k_0^2\varepsilon_i\mu_i-\beta^2)E_y=0 \tag{1}$$

Where $k_0=\frac{2\pi}{\lambda}$, λ is the wavelength in vacuum, $i=1,2$ and 3. For the oscillating guided modes, there exist two cases:

Case 1 $k_0n_3<\beta<\min(k_0n_2, k_0|n_1|)$

Under this case, guided modes decay in the outer layers, but oscillate in both the core layer and inner claddings. We define these modes as the first oscillating guided modes and noted by TE_m^1. Where m stands for mode number. From Eq. (1), their electric fields in the five layer slab waveguide are written as

$$E_{y_1}(x)=A\cos\varphi_{23}\exp[-q_3(x-h_1-h_2)],\quad x\geqslant h_1+h_2 \tag{2a}$$

$$E_{y_2}(x)=A\cos[k_2x-k_2(h_1+h_2)+\varphi_{23}],\quad h_1\leqslant x\leqslant h_1+h_2 \tag{2b}$$

$$E_{y_3}(x)=AB\cos(k_1x-k_1h_1+\varphi_{12}),\quad 0\leqslant x\leqslant h_1 \tag{2c}$$

$$E_{y_4}(x)=AB\cos(k_1x+k_1h_1-\varphi_{12}),\quad -h_1\leqslant x\leqslant 0 \tag{2d}$$

$$E_{y_5}(x)=A\cos[k_2x+k_2(h_1+h_2)-\varphi_{23}],\quad -(h_1+h_2)\leqslant x\leqslant -h_1 \tag{2e}$$

$$E_{y_6}(x)=A\cos\varphi_{23}\exp[q_3(x+h_1+h_2)],\quad x\leqslant -(h_1+h_2) \tag{2f}$$

Where $\kappa_1=\sqrt{\varepsilon_1\mu_1k_0^2-\beta^2}$, $\kappa_2=\sqrt{\varepsilon_2\mu_2k_0^2-\beta^2}$, $q_3=\sqrt{\beta^2-\varepsilon_3\mu_3k_0^2}$, $\varphi_{23}=\arctan\left(\frac{\mu_2q_3}{\mu_3k_2}\right)$, $\tan\varphi_{12}=-\left(\frac{\mu_1k_2}{\mu_2k_1}\right)\tan(k_2h_2-\varphi_{23})$, $B=\frac{\cos(k_2h_2-\varphi_{23})}{\cos\varphi_{12}}$

A is an undetermined constant. With continuous conditions for the transverse electromagnetic fields by employing a transfer matrix method, a dispersion equation for the first oscillating guided modes is expressed as

$$\begin{bmatrix}-\frac{q_3}{\mu_3} & 1\end{bmatrix}M_2M_1M_2\begin{bmatrix}1\\ -\frac{q_3}{\mu_3}\end{bmatrix}=0 \tag{3}$$

Where

$$M_1=\begin{bmatrix}\cos(2k_1h_1) & \frac{\mu_1}{k_1}\sin(2k_1h_1)\\ -\frac{k_1}{\mu_1}\sin(2k_1h_1) & \cos(2k_1h_1)\end{bmatrix},$$

$$M_2=\begin{bmatrix}\cos(k_2h_2) & \frac{\mu_2}{k_2}\sin(k_2h_2)\\ -\frac{k_2}{\mu_2}\sin(k_2h_2) & \cos(k_2h_2)\end{bmatrix}.$$

After some algebraic manipulation, Eq. (3) can be rewritten as

$$2k_1h_1=m\pi+2\arctan\left(\frac{\mu_1q_2}{\mu_2k_1}\right) \tag{4}$$

Where

$m=0,1,2,3$, $q_2=k_2\tan\left[\arctan\left(\frac{\mu_2q_3}{\mu_3k_2}\right)-k_2h_2\right]$.

Case 2 $\max(k_0n_3, k_0n_2)<\beta<k_0|n_1|$

In this case, the electric fields decay both inner and outer claddings, and oscillate in the core layer. We call these modes the second oscillating guided modes and denote them as TE_m^2. Let $\kappa_2=i\alpha$, $\alpha=\sqrt{\beta^2-k_0^2\varepsilon_2\mu_2}$, after some algebraic manipulation like case 1, a dispersion equation for the second TE oscillating guided modes can be written as

$$2kh_1=m\pi+2\arctan\left(\frac{\mu_1q_2}{\mu_2k_1}\right) \tag{5}$$

Where $q_2=\alpha_2\tan h\left[\arctan h\left(\frac{\mu_2p_3}{\mu_3\alpha_2}\right)+\alpha_2h_2\right]$

Although the form of the dispersion equations (4) and (5) are the same, the physical solutions are different.

2 Dispersion properties of oscillating guided modes

Material dispersion should be considered since this is one of inherent properties for LHM[7]. We employ an experimental model[6] with dielectric permittivity and magnetic permeability written as

$$\varepsilon_1(\omega)=1-\frac{\omega_P^2}{\omega^2},\ \mu_1(\omega)=1-\frac{F\omega^2}{\omega^2-\omega_0^2}.$$

Where $F=0.56$, $\frac{\omega_0}{2\pi}=4$ GHz, $\frac{\omega_p}{2\pi}=10$ GHz. When frequency changes from 4 GHz to 6 GHz, dielectric permittivity and magnetic permeability are negative simultaneously. Usually, the dispersive properties of the oscillating guided modes depend on the layer thicknesses of the waveguide and there exist three cases: $h_1<h_2$, $h_1=h_2$ and $h_1>h_2$. To compare clearly, we denote these modes by TE_m^{1n} and TE_m^{2n} again. That $n=1,2,3$ corresponds to $h_1<h_2$, $h_1=h_2$ and $h_1>h_2$, and discussed in the case 1, 2 and 3, respectively. Besides, as $h_1=h_2$, we find eleven TE oscillating guided modes as frequency from 4 GHz to 6 GHz and examine them in subsection 2.2.2.

2.1 The dispersive properties for lower-order TE oscillating guided modes

In general, we are interested in lower-order modes. From Eqs. (4),(5),some dispersion curves for TE oscillating guided modes ($m=0$, 1) are shown in Fig. 2.

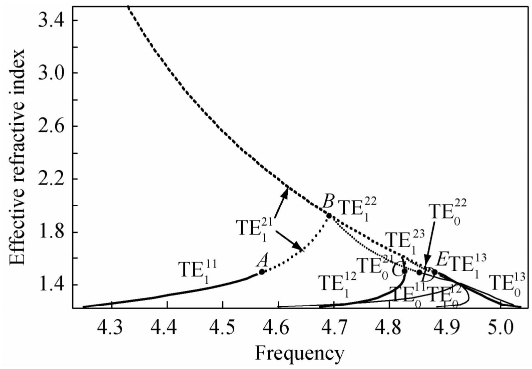

Fig. 2 Dispersion curves for the lower-order TE oscillating guided modes ($m=0$,1), the effective-refractive-index is a function with frequency from 4 GHz to 6 GHz

As h_2 fixed(0. 02 m),and h_1 has three cases, such as 0. 008 m, 0. 02 m and 0. 08 m. The core layer is dispersive LHM. The inner and outer claddings are different right—handed material with dielectric permittivity ($\varepsilon_2=2.25$ and $\varepsilon_3=1.5$), magnetic permeability($\mu_2=\mu_3=1.0$).

As h_1 increases, TE_m^{1n} mode dispersion curves shift to right,and their cutoff frequencies increases too; TE_1^{2n} mode dispersion curves go down and frequency range gets wider. It is interesting that they superpose each other in some frequency ranges. Detailed dispersive properties for lower-order TE oscillating guided modes are as follows:

Case 1 $h_1<h_2$

For TE_0^{11} mode, $h_1<h_2$, the effective refractive index increases as frequency changes from 4. 6 GHz to 4. 85GHz. Its group velocity($v_g=\frac{d\omega}{d\beta}$) is positive and energy flux travels along z-axis. As frequency between 4. 85 GHz and 4. 925 GHz, mode double-degeneracy appears. Therefore, its group velocity may be positive or negative in this frequency region. For TE_0^{21} mode, it connects with the former at point D. Its effective refractive index decreases as frequency changes from 4. 695 GHz to 4. 85 GHz. Its group velocity is negative and energy flux moves along anti-z-axis. As mode number increases and $m=1$, for TE_1^{11} mode, its effective refractive index increases with frequency changing from 4. 25 GHz to 4. 57 GHz. So, it has a positive group velocity. But, for TE_1^{21} mode, dispersion curve is continuous with that of TE_0^{21} mode at point B. Its effective refractive index decreases as frequency from 4. 33 GHz to 4. 57 GHz,and it has a negative group velocity. As frequency between 4. 57 GHz and 4. 695 GHz, mode double-degeneracy appears and its group velocity may be positive or negative in this frequency region,just like TE_0^{11} mode.

Case 2 $h_1=h_2$

Provided $h_1=h_2$, for TE_0^{12} mode, mode double-degeneracy appears as frequency between 4. 885 GHz and 4. 943 GHz. It has positive or negative group velocity. For TE_0^{22} mode, its effective refractive index decreases monotonously as frequency from 4. 833 GHz to 4. 87 GHz. So,it has negative group velocity and energy flux travels along anti-z axis. With the increase of mode number when $m=1$, two modes are observed in Fig. 2. For TE_1^{12} mode, its effective refractive index increases as frequency from 4. 68 GHz to 4. 829 GHz. It has positive group velocity and energy flux moves along z-axis. For TE_1^{22} mode, dispersion curve connects with that of the former at point C. Its effective refractive index decreases with frequency between 4. 33 GHz and 4. 829 GHz,and it has a negative group velocity in that frequency region.

Case 3 $h_1>h_2$

In this case, for TE_0^{13} mode, its effective refractive index decreases with frequency from 4. 93 GHz to 5. 024 GHz. It has negative group velocity in that frequency region. However, we don't find TE_0^{23} mode at any frequency regions. As mode number increases when $m=1$, two oscillating guided modes are shown in Fig. 2. For TE_1^{13} mode, its effective refractive index decreases as frequency from 4. 888 GHz to 5. 037 GHz,and it has negative group velocity. For TE_1^{23} mode, its dispersion curve connects with the former at point E. Its effective refractive index decreases with frequency from 4. 333 GHz to 4. 888 GHz, and it has negative group velocity.

2.2 The dispersive properties for higher-order TE oscillating guided modes

2.2.1 The dispersion properties depend on waveguide thickness

1) $2\leqslant m\leqslant 6$

As mode number from 2 to 6, TE oscillating guided modes have similar properties. Like above lower-modes, we select $m=2$ and 6, fix and change h_2, and plot their dispersion curves in Fig. 3(a), respectively. As $h_1\leqslant h_2$, for TE_m^{1n} modes, their frequency ranges get bigger as increases. As $h_1>h_2$, no mode can be found with frequency between 4 GHz and 6 GHz. For TE_m^{2n} modes, their frequency regions get bigger as h_1 increases. Mode double-degeneracy appears and most obviously seen at $h_1 = h_2$. For $h_1 > h_2$, their effective refractive index decreases monotonously as frequency. So, they have negative group velocity and energy flux travels along anti-z axis.

2) $6 < m \leqslant 10$

pAs $m > 6$, dispersion curves corresponding to m = 7, 8, 9, 10 are plotted in Figs. 3(b), (c), respectively. For TE_m^{1n} and TE_m^{2n} modes, their frequency regions get wider as h_1 increases. As $h_1 > h_2$, the former is different from the latter. For TE_m^{1n} modes, mode double-degeneracy appears. While for TE_m^{2n} modes, effective-refractive index decreases monotonously with frequency. So, the former has either negative or positive group velocity and the latter always has negative group velocity in the corresponding frequency regions.

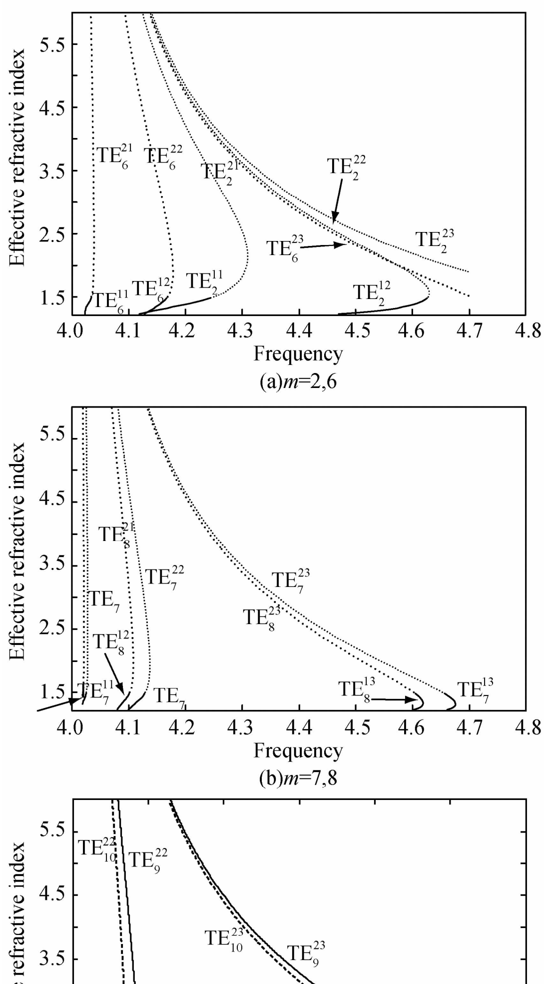

Fig. 3 Dispersion curves for higher-order TE oscillating guided modes (m=2,6,7,8,9,10), the effective-refractive-index is a function with frequency. The corresponding parameters employed are the same as that of above lower-order modes

2.2.2 The dispersion properties depend on mode number

Set $h_1 = h_2$, for $m \geqslant 2$, employed Eqs. (4) and (5), mode dispersion curves are shown in Fig. 4. For TE_m^{12} modes, effective refractive indexes increase monotonously as frequency. As m increases, the group velocities get bigger, but, their bandwidth becomes narrower. As $m = 10$, its bandwidth is only 0.014 GHz and its cutoff frequency is 4.053 GHz. For TE_m^{22} modes, their frequency gets less and bandwidth becomes narrower as m increases. Mode double-degeneracy appears in some frequency regions. Moreover, this property is more obvious as $m = 3, 4, 5$. However, for TE_m^{12} and TE_m^{22} modes, they have a common property that their dispersion curves shift to left as m increases. It is different from that of RHM slab waveguides[8] and LHM slab waveguides neglecting material dispersion[4]. The difference is because for a RHM slab waveguide or a LHM slab waveguide without material dispersion when the frequency increases, the field is more condensed or trapped inside the waveguide. However, for a LHM slab waveguide, when the frequency increases, the absolute value of refractive index decreases rapidly, so the field is less trapped inside the waveguide and causes the dispersion curves shift to left as m increases.

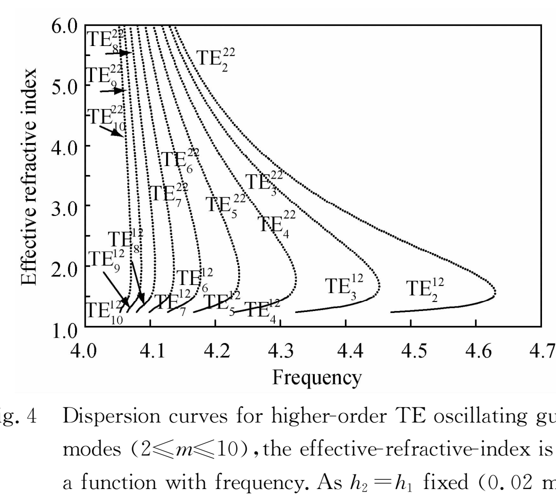

Fig. 4 Dispersion curves for higher-order TE oscillating guided modes ($2 \leqslant m \leqslant 10$), the effective-refractive-index is a function with frequency. As $h_2 = h_1$ fixed (0.02 m), and the other corresponding parameters employed are the same as that of above lower-order modes

Synthesizing above discuss, we find three kinds dispersion properties of the TE oscillating guided modes:

1) Normal dispersion (The effective-refractive-index increases as frequency);

2) Abnormal dispersion (The effective-refractive-index decreases as frequency);

3) Mode double-degeneracy (The effective-

refractive-index has two different values as the same frequency).

The different behavior of TE oscillating guided modes are listed in Table 1.

Table 1 The different behavior of TE oscillating guided modes

Normal dispersion	TE_1^{11}, TE_1^{12}, TE_2^{11}, TE_2^{12}, TE_3^{11}, TE_3^{12}, TE_4^{11}, TE_4^{12}, TE_5^{11}, TE_5^{12}, TE_6^{11}, TE_6^{12}, TE_7^{11}, TE_7^{21}, TE_8^{11}, TE_9^{12}, TE_{10}^{12}
Abnormal dispersion	TE_0^{21}, TE_0^{22}, TE_0^{13}, TE_1^{22}, TE_1^{13}, TE_1^{23}, TE_2^{23}, TE_3^{23}, TE_4^{23}, TE_5^{23}, TE_6^{23}, TE_6^{23}, TE_7^{23}, TE_8^{23}, TE_9^{23}, TE_9^{12}, TE_{10}^{23}
Mode double-degeneracy	TE_0^{11}, TE_0^{12}, TE_1^{21}, TE_2^{21}, TE_2^{22}, TE_3^{21}, TE_4^{21}, TE_4^{22}, TE_5^{21}, TE_5^{22}, TE_6^{21}, TE_6^{22}, TE_7^{21}, TE_7^{21}, TE_7^{22}, TE_8^{13}, TE_8^{22} TE_9^{13}, TE_9^{22}, TE_{10}^{13}, TE_{10}^{22}

3 Conclusions

A five-layer slab waveguide with LHM in the core layer and RHMs in other layers is investigated. The dispersion equations for the TE oscillating guided modes are obtained and corresponding dispersion curves are plotted. We find six TE_m^{1n} modes and eleven TE_m^{2n} modes as frequency from 4 GHz to 6 GHz. Considering material dispersion, we find zero-order TE oscillating guided modes are exist. With the increase of mode number, mode dispersion curves move to left, their cutoff frequencies get less. Besides, we find some modes have following properties. 1) The effective refractive index increases as frequency; 2) The effective refractive index decreases as frequency; 3) The effective refractive index has two different values as the same frequency. These properties may help us construct new devices.

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含左手材料对称五层平面波导模场特性

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摘 要:对芯层为左手材料而内外包层都是普通材料的对称五层平面波导进行了探讨.得到了两类 TE 振荡模的色散方程,且在考虑材料色散的条件下,画出了相关模式的色散曲线.随着模阶数的增加,模色散曲线左移,它们的截止频率变小.其次,发现零阶 TE 振荡模的存在.除此以外,TE 振荡模有三类色散特性:正常色散;反常色散;双模简并.

关键词:平面波导;左手材料;色散方程;色散曲线

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