光　子　学　报
ACTA PHOTONICA SINICA

Article ID:1004-4213(2010)04-0577-7

Comparison of Few-cycle Laser Pulse Propagation Behaviors in Dilute and Dense Ladder-type Atomic Mediums*

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Abstract: Using the numerical solution of the full Maxwell-Bloch equations without the slowly varying envelope approximation and the rotating-wave approximation, propagation behavior of few-cycle laser pulse in a dense ladder-type three-level atomic medium is investigated and it is compared with the case in the corresponding dilute medium. It is found that, in the propagation process, time evolution of the pulse in the dense medium is obviously different from that in the dilute medium, and the difference will become more evident with increasing of the initial pulse area. When the initial pulse area is smaller, in the propagating process, the pulse shape in the dilute medium just have a small change; the pulse shape in the dense medium has considerable variation but the pulse splitting doesn't occur. When the initial pulse area is larger enough, for the dense medium case, the pulse splits into sub-pulses with different numbers and shapes at different propagation distances; while for the dilute medium case, variation of the pulse shape is still smaller in propagation process. The cause producing above difference is from two effects in the dense medium: the effect of the local-field correction (LFC) from the near dipole-dipole interaction and effect of stronger polarization field than that in the dilute medium. In which the effect of stronger polarization field plays a main role, but the role of LFC also cannot be neglected, and the stronger polarization field is due to the atomic density increasing .

Key words: Few-cycle laser pulse; Near dipole-dipole interaction; Dense medium; Propagation behavior

CLCN: O437　　**Document Code**: A　　**doi**: 10.3788/gzxb20103904.0577

0 Introduction

Recently, the study of the interaction of few-cycle laser pulse with matter has attracted many attentions for example, see review papers[1-3] and the propagation of a few-cycle laser pulse in atom or molecular media is an important part of the study. Usually, investigations for the interaction of the laser field with mater employ the semiclassical theory, i. e., electromagnetic field is described by Maxwell equations, while the atomic or molecular medium is described by the density matrix. For long laser pulses, we can use the Maxwell-Bloch equations with the slowly varying envelope approximation (SVEA) and the rotating-wave approximation (RWA) in order to simplify problem. However, for few-cycle laser pulse, these approximations fail clearly[4-5]. In addition, people often adopt a simplified level model of atomic, molecular system in the theory study due to the complexity of the actual substance system. The propagating behaviors of few-cycle laser pulse in two-level atomic or molecular mediums have been extensively studied. When number of main levels participating in the interaction of the laser field with mater is more than two, the two-level model is no longer valid. In order to describe exactly the interaction of ultrashort pulses and a medium, a multilevel model must be adopted. Some papers[6-12] have studied few-cycle pulses propagating in dilute three-level atomic mediums. For the case of a dilute atomic medium, we only need to consider the interaction between the atoms and the ultrashort pulse. However, in a dense atomic medium, where the atomic density is so high that there are, in the sense of average, many atoms within a cubic resonance wavelength such that the near-dipole-dipole (NDD) interaction must be considered, and NDD interaction will lead to Lorentz local-field correction (LFC)[13]. At

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* Supported by the National Basic Research Program of China (2006CB806000), the Natural Science Foundation of China (10875072) and the Open fund of the State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics

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Received date:2009-07-13　　Revised date:2010-01-12

present, there are only few studies about few-cycle pulse propagating in a dense atomic medium. Recently, Song, et al.[14] investigated the propagation of attosecond pulse in a dense two-level medium, they found that significantly higher spectral components can be obtained in all small area pulses propagating due to strong carrier reshaping. Xia, et al.[15] studied the propagation of femtosecond pulse in a dense two-level atomic medium; they found that difference of the carrier-envelope phase between the two cases with and without LFC increases at some location in the pulse propagating process. We explored the effect of the ratio, γ, of transition dipole moments on few-cycle pulse propagation in a dense V-type three-level atomic medium. In this paper, we will investigate propagating behavior of femtosecond few cycle pulse in a dense three-level ladder-type atomic medium, and compare it with the case in the corresponding dilute medium.

1 Theoretical model

Let us consider propagation of a linearly polarized few-cycle laser pulse in the dense medium filled with ladder-type three-level atoms, for example, Rb atom[17] as illustrated in Fig. 1. In addition, the values of the atomic parameters given bellow also are corresponding to Rb atom. We assume that the electric field and magnetic field are polarized along x- and y-axis directions, respectively. So we can represent the electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ by the scalars E_x and H_y, respectively. The Maxwell equations take the form

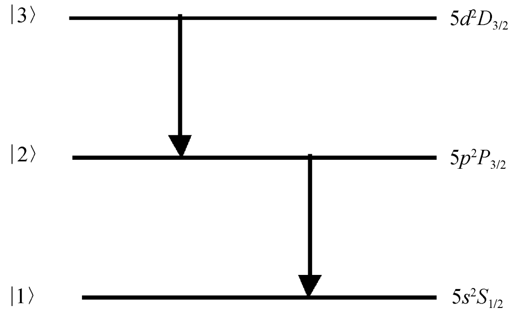


Fig. 1 The ladder-type three-level configuration

$$\partial_t H_y = -\frac{1}{\mu_0}\partial_z E_x \tag{1a}$$

$$\partial_t E_x = -\frac{1}{\varepsilon_0}\partial_z H_y - \frac{1}{\varepsilon_0}\partial_t P_x \tag{1b}$$

Where ε_0 is the electric permittivity in the vacuum and μ_0 is the magnetic permeability in the vacuum. The Bloch equations can be written as follows

$$\dot{\rho}_{12} = -\mathrm{i}\{-\omega_1\rho_{12} + \Omega(\rho_{22} - \rho_{11}) - \gamma\Omega\rho_{13}\} \tag{2a}$$

$$\dot{\rho}_{13} = -\mathrm{i}\{-(\omega_1 + \omega_2)\rho_{13} + \Omega\rho_{23} - \gamma\Omega\rho_{12}\} \tag{2b}$$

$$\dot{\rho}_{23} = \text{-}\mathrm{i}\{-\omega_2\rho_{23} + \Omega\rho_{13} + \gamma\Omega(\rho_{33} - \rho_{22})\} \tag{2c}$$

$$\dot{\rho}_{11} = -\mathrm{i}\{\Omega(\rho_{21} - \rho_{12})\} \tag{2d}$$

$$\dot{\rho}_{22} = -\mathrm{i}\{\Omega(\rho_{12} - \rho_{21}) + \gamma\Omega(\rho_{32} - \rho_{23})\} \tag{2e}$$

$$\dot{\rho}_{33} = -\mathrm{i}\{\gamma\Omega(\rho_{23} - \rho_{32})\} \tag{2f}$$

Where ω_1 and ω_2 represent the transition frequencies form level |1⟩ to level |2⟩ and from level |2⟩ to level |3⟩, respectively; $\Omega = \mu_{12}E_x/\hbar$ is the Rabi frequency of the pulse laser field; $\gamma = \mu_{23}/\mu_{12}$, μ_{ij} are the dipole moments between level i and j.

We introduce the real three-vector representation of the density matrix through the relations

$u_1 = \rho_{12} + \rho_{21}$, $u_2 = \rho_{23} + \rho_{32}$, $u_3 = \rho_{13} + \rho_{31}$, $u_4 = -\mathrm{i}(\rho_{12} - \rho_{21})$, $u_5 = -\mathrm{i}(\rho_{23} - \rho_{32})$, $u_6 = -\mathrm{i}(\rho_{13} - \rho_{31})$, $u_7 = \rho_{22} - \rho_{11}$, $u_8 = \rho_{33} - \rho_{11}$

and adopt the initial condition $\rho_{11} = 1$, $\rho_{22} = \rho_{33} = 0$, $\rho_{ij} = 0\ (i, j = 1, 2, 3, i \neq j)$, which means that all atoms are in level |1⟩. The Bloch equations can be easily derived as follows

$$\dot{u}_1 = -\omega_1 u_4 - \gamma\Omega u_6 - \frac{1}{T_1}u_1 \tag{3a}$$

$$\dot{u}_2 = -\omega_2 u_5 + \Omega u_6 - \frac{1}{T_2}u_2 \tag{3b}$$

$$\dot{u}_3 = -(\omega_1 + \omega_2)u_6 + \Omega u_5 - \gamma\Omega u_4 - \frac{1}{T_3}u_3 \tag{3c}$$

$$\dot{u}_4 = \omega_1 u_1 - 2\Omega u_7 + \gamma\Omega u_3 - \frac{1}{T_4}u_4 \tag{3d}$$

$$\dot{u}_5 = \omega_2 u_2 - \Omega u_3 - 2\gamma\Omega(u_8 \text{-} u_7) - \frac{1}{T_5}u_5 \tag{3e}$$

$$\dot{u}_6 = (\omega_1 + \omega_2)u_3 - \Omega u_2 + \gamma\Omega u_1 - \frac{1}{T_6}u_6 \tag{3f}$$

$$\dot{u}_7 = 2\Omega u_4 - \gamma\Omega u_5 - \frac{1}{T_7}(u_7 + 1) \tag{3g}$$

$$\dot{u}_8 = \Omega u_4 + \gamma\Omega u_5 - \frac{1}{T_8}(u_8 + 1) \tag{3h}$$

The macroscopic polarization takes the form

$$P_x = -N\langle\mu\rangle = -N\mu_{12}(u_1 + \gamma u_2) \tag{4}$$

Where N is the density of the atomic medium. In a dense medium, according to the Lorentz-Lorenz relation[14], the microscopic local electric field E_L, which couples with atomic or molecular dipole moments, is related to the external field E_x and nonlinear volume polarization P_x in the isotropic homogeneous medium.

$$E_L = E_x + P_x/3\varepsilon_0 \tag{5}$$

In order to investigate the propagation properties of an ultrashort pulse in a ladder-type atomic medium, we should simultaneously employ Maxwell Eqs. (1) and Bloch Eqs. (2). It is very difficult to solve analytically the both equations, so we use the finite-difference time-domain (FDTD) method[18] and the predictor corrector (PC) method[4] to obtain its numerical solution. The initial input fields are

$$E_x(z,t=0)=\widetilde{E}_x(z,t=0)\cos\left[\omega_p(z/c-z_0/c)\right]=E_0\sec h\left[1.76(z/c-z_0/c)/\tau_p\right]\cos\left[\omega_p(z/c-z_0/c)\right]$$

and

$$H_y(t=0,z)=(\varepsilon_0/\mu_0)^{1/2}E_x(t=0,z).$$

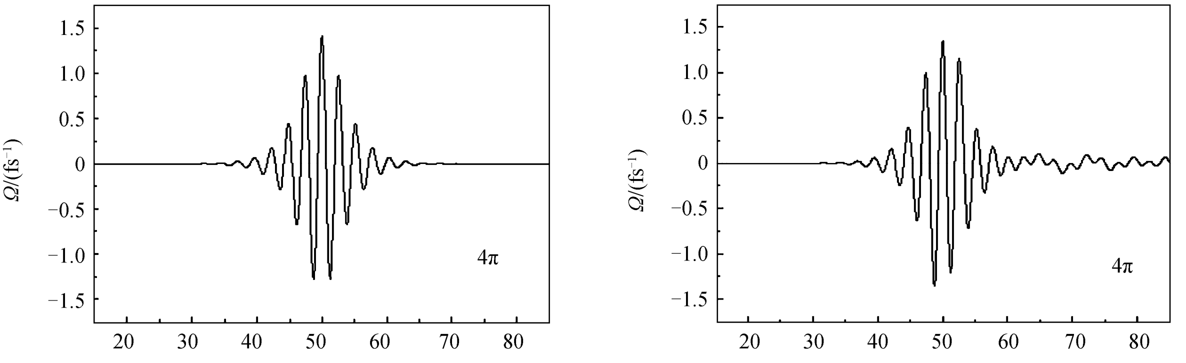
Where E_0 is the maximal electric field amplitude, τ_p is the full wide at half-maximum (FWHM) of the pulse intensity envelope, and ω_p is the pulse central frequency. The input envelope area can be gotten by $A=\mu_{12}E_0\tau_p\pi/1.76\hbar$. The choice of z_0 ensures that the pulse penetrates negligibly into the medium at $t=0$. In the following numerical calculations, we take values of the pulse and material parameters as: $\omega_1=2.41477\ \mathrm{fs}^{-1}$, $\omega_2=2.42815\ \mathrm{fs}^{-1}$, $\omega_p=2.41477\ \mathrm{fs}^{-1}$, $\tau_p=5$ fs, $\mu_{12}=4.976\times10^{-29}$ Asm and $\gamma=0.3$. The relaxation times corresponding to the decay of the real state vector components are set to the uniform value

$$T_1=\cdots=T_8=1\times10^{-9}\,\mathrm{s}.$$

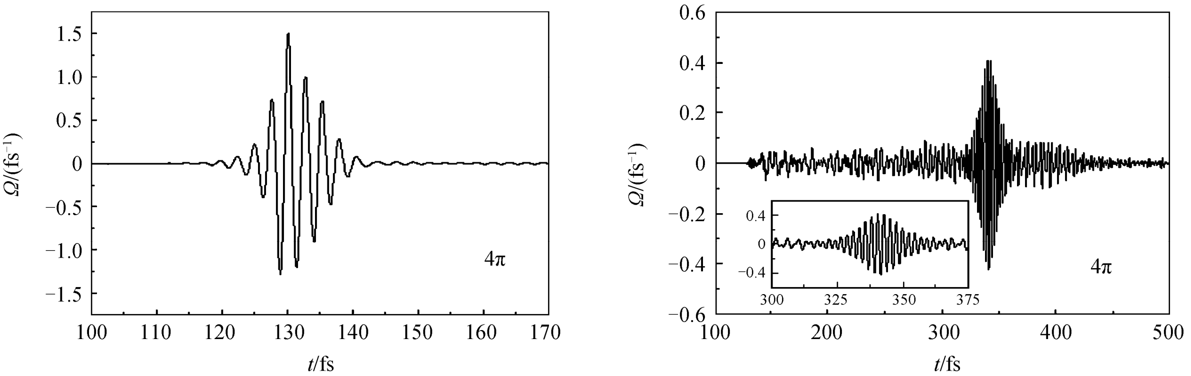
2 Numerical results and analysis

In this section, we use numerical solutions of the coupled Maxwell-Bloch Eqs (1) and (2) to analyze time evolution of the femtosecond few-cycle laser pulse at different propagation distances in the dense medium and compare it with the case in the corresponding dilute medium. The numerical result shows that, the time evolution in the dense medium is different from that in the dilute medium, and the difference will become more evident with increasing of the initial area of the pulse. In the following, we only take the initial pulse area being 4π and 8π as example to show this.

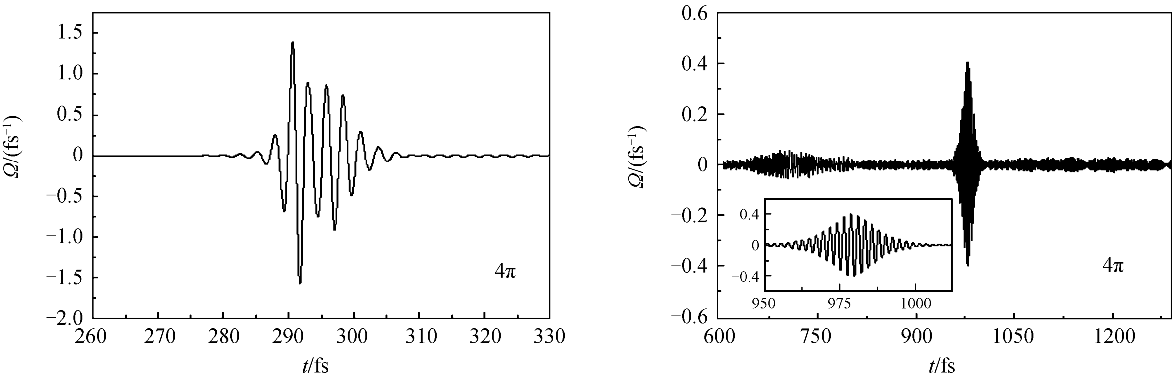
Fig. 2 depicts the time evolution of the 4π pulse at different distances in the dilute ($N=1\times10^{24}\ \mathrm{m}^{-3}$) and dense ($N=1\times10^{26}\ \mathrm{m}^{-3}$) mediums. From Fig. 2 (a),(c),(e), we can see that, in the dilute medium, at $z=0$ μm, the pulse presents a normal Rabi oscillation; but with propagation distance increasing, the pulse trailing edge becomes thinner and pulse leading edge becomes thicker, i. e. incomplete Rabi flop occurs. This phenomenon is caused by electric field time derivative[4]. Fig. 2 (b),(d),(f) show that the pulse evolution in a dense medium is different from that in the dilute medium. At $z=0$ μm, only small irregular oscillations appear at pulse leading edge. With propagation distance increasing, evident oscillation with longer duration appears at pulse leading and trailing edges, the pulse amplitude becomes smaller, but the pulse splitting doesn′t arise. And the differences between dilute and dense cases become more obvious when propagation



(a) z=0μm,in the dilute medium (b) z=0μm,in the dense medium with LFC



(c) z=24μm,in the dilute medium (d) z=24μm,in the dense medium with LFC



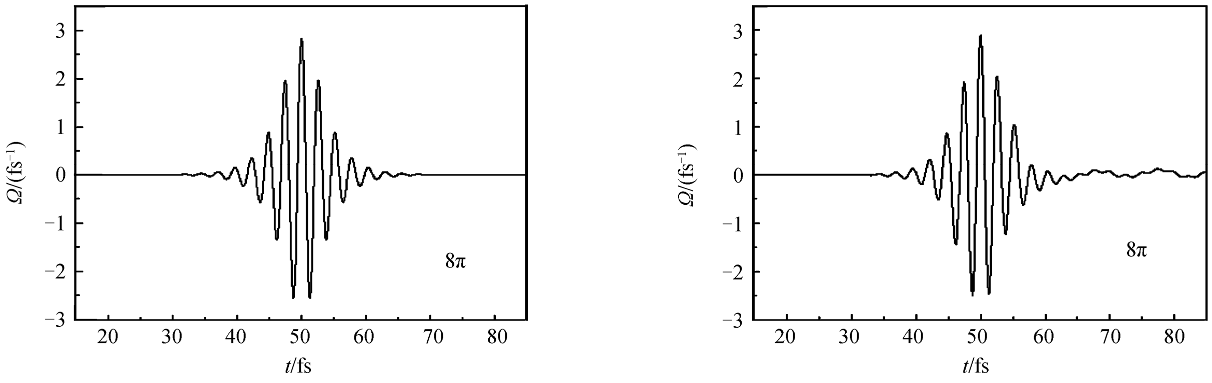
(e) z=72μm,in the dilute medium

(f) z=72μm,in the dense medium with LFC

Fig. 2 Time evolutions of 4π pulse at the distances $Z=0$, 24, 72 μm

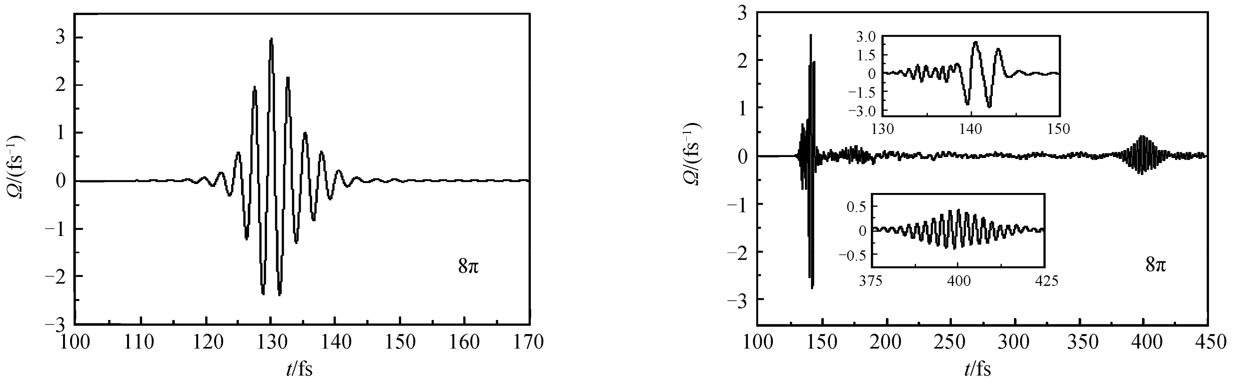
distance increases. In addition, at the same distance except the initial location (at $z=0$ μm), the time that the pulse appears in the dense medium is later than that in the dilute medium and this phenomenon is more and more remarkable with propagation distance increasing. This shows that the group velocity of the pulse becomes smaller in the dense medium. It is because that the pulse amplitude in the dense medium is smaller than that in the dilute medium and the group velocity is proportional to the square of the pulse amplitude[19].

Fig. 3 illustrates the time evolution of 8π pulse in the dilute ($N=1\times10^{24}$ m^{-3}) and dense ($N=1\times10^{26}$ m^{-3}) mediums. From Fig. 3 (a),(c).(e), we can see that, in the dilute medium, the time evolution of 8π pulse is similar to that of 4π pulse except that pulse amplitude increases. In the dense medium, the pulse evolution is irregular and pulse splitting occurs except the initial position ($z=0$ μm). And this is quite different from the 4π pulse case. At $z=24$ μm and $z=72$ μm, the pulse respectively splits into two and three sub-pulses with different shapes, and these pulse shapes are different from those in the dilute medium.



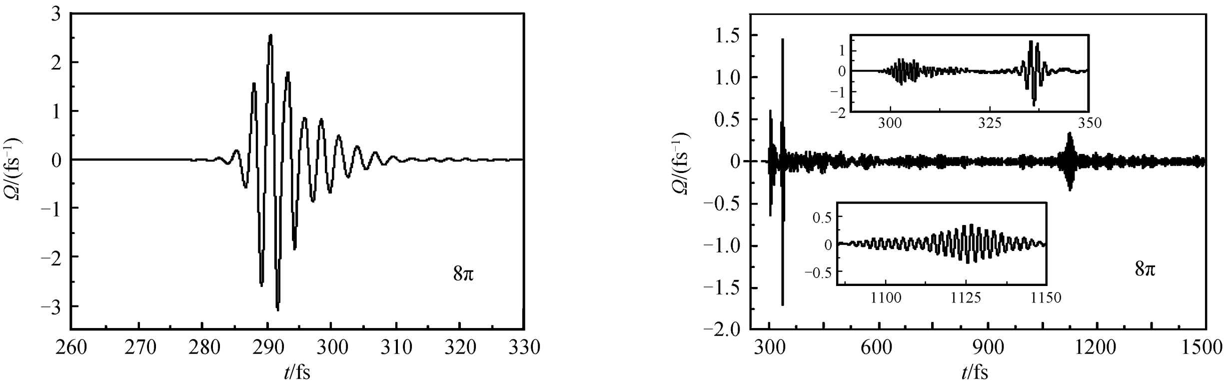
(a) z=0μm,in the dilute medium

(b) z=0μm,in the dense medium with LFC



(c) z=24μm,in the dilute medium

(d) z=24μm,in the dense medium with LFC



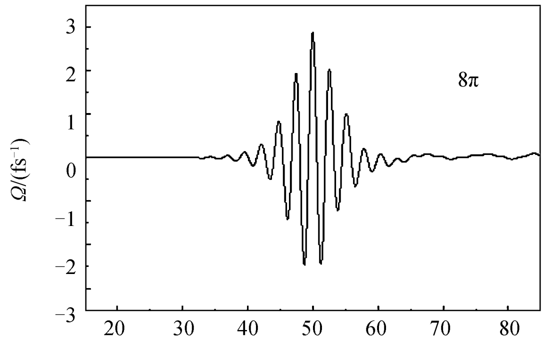
(e) z=72μm, in the dilute medium

(f) z=72μm, in the dense medium with LFC

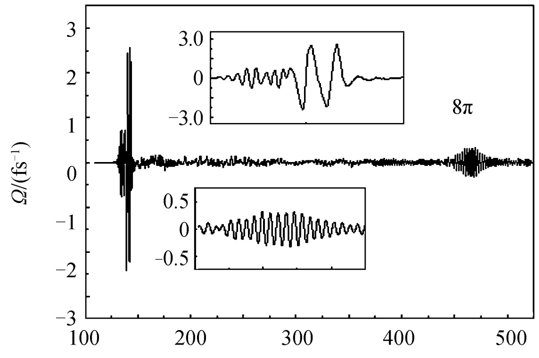
Fig. 3 Time evolutions of 8π pulse at the distances $Z=0$, 24, 72 μm

In the dense medium, with propagation distance increasing, the main pulse amplitude becomes smaller. The group velocity in the dense medium is still smaller than that in the dilute medium. In the propagation process, time evolutions of the pulse shape in the dense medium are much different from those in the dilute medium. The reason producing above difference is from two effects in the dense medium: the effect of LFC from NDD interaction and the effect of stronger polarization field than that in the dilute medium. And both effects enhance the light-matter interaction. In order to compare relative size of the two effects, we give a brief discussion by taking the case of 8π pulse as an example.

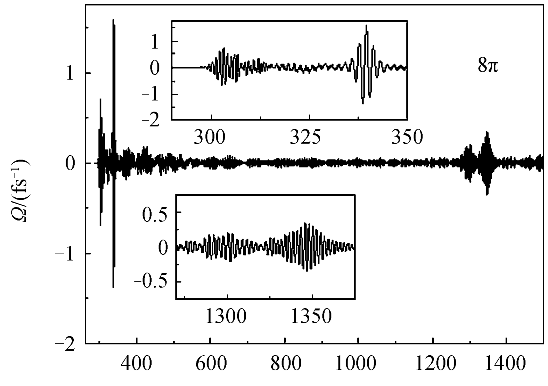
Fig. 4 shows the time evolution of 8π pulse shape at $z=0$, 24, 72 μm in the dense ($N=1\times 10^{26}$ m^{-3}) medium when LFC is not considered. Comparing Fig. 4 (a)～(c) with Fig. 3 (a), (c), (e), we can see that time evolutions of the pulse in the dense medium when LFC is absent is much different from that in the dilute medium. Comparing Fig. 4 (a)～(c) with Fig. 3(b), (d), (f) shows that time evolution of the pulse when LFC is absent also has some obvious differences from that with LFC except the initial position ($z=0$ μm). 1) At the distance $z=24$ μm, the maximum amplitude of two sub-pulses when LFC is absent is little smaller than that in the case with LFC, the moment at which the second sub-pulse appears when LFC is absent is evidently latter than that in the case with LFC. 2) At the distance $z=72$ μm, the pulse splits into four sub-pulses in the case without LFC but the pulse splits into three sub-pulses in the case with LFC, and the maximum amplitude of two sub-pulses when LFC is absent is little smaller than that in the case with LFC, the moment at which the third sub-pulse appears when LFC is absent is much later than that in the case with LFC.



(a) z=0μm, in the dense medium without LFC



(b) z=24μm, in the dense medium without LFC



(c) z=72μm, in the dense medium without LFC

Fig. 4 Time evolutions of 8π pulse at the distances $Z=0$, 24, 72 μm in the dense medium and LFC is not considered

From above analysis we can see that, even though the main cause producing considerable difference of time evolutions of the pulse shape in the dilute and dense mediums is due to that in the dense medium there is a stronger polarization field than that in the dilute medium, but the role of LFC also cannot be neglected, and the stronger polarization field is due to the atomic density increasing.

3 Conclusions

Using the numerical solutions of the full Maxwell-Bloch equations without SVEA and RWA, we investigated propagation behavior of few-cycle laser pulse in dense ladder-type three-level atomic medium and compared it with the case in the corresponding dilute medium. We find that, in the propagation process, time evolution of the pulse in the dense medium is different from that in the dilute medium, and the difference will become more evident with increasing of the initial pulse area. When the initial pulse area is smaller, in the propagating process, the pulse shape in the dilute medium just have a small change; the pulse shape in the dense medium has considerable variation but the pulse splitting doesn't occur. When the initial pulse area is larger enough, for the dense medium case the pulse splits into sub-pulses with different numbers and shapes at different propagation distances while for the dilute medium case variation of the pulse shape is still smaller in propagation process. The cause producing above difference is from two effects in the dense medium: the effect of the local-field correction (LFC) from the near dipole-dipole interaction and effect of stronger polarization field than that in the dilute medium. In which the effect of stronger polarization field plays a main role, but the role of LFC also cannot be neglected, and the stronger polarization field is due to the atomic density increasing.

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周期量级激光脉冲在不同密度的原子介质中传播行为的比较

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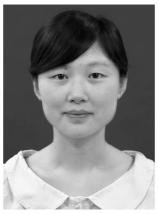
摘　要：利用不含慢变包络近似和旋波近似的全波 Maxwell-Bloch 方程组的数值解，研究了周期量级超短激光脉冲在 Ladder 型三能级原子介质中的传播行为并与在相应的稀疏介质中的情况进行了比较. 我们发现，在传播过程中，超短脉冲在稠密介质中的时间演化规律与在稀疏介质中明显不同，而且这种差别将随初始脉冲面积的增大而加大. 当初始脉冲面积较小时，在传播过程中，脉冲形状在稀疏介质中只有小的改变而在稠密介质中却有显著的变化. 当初始脉冲面积足够大时，在稠密介质中在不同的传播距离处脉冲分裂为不同数量和形状的亚脉冲；在稀疏介质中脉冲形状在传播过程中仍然只有小的改变。产生以上差别的原因在于稠密原子介质中近偶极-偶极（NDD）相互作用导致的局域场修正（LFC） 及比稀疏原子介质更强的极化电场的影响. 其中，更强的极化电场的影响起着主要的作用，但局域场修正的作用也不能忽略，而极化电场的增强是由于原子密度的增加.

关键词：周期量级激光脉冲；近偶极-偶极相互作用；稠密介质；传播行为



WANG Lei　was born in 1983. She is currently pursuing the M. S. degree, and her research interests focus on high field laser physics.

FAN Xi-jun　was born in 1947. Now he is a professor and Doctoral Supervisor, and his research interests focus on solid physics, nonlinear optics, quantum optics and high field laser physics.