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Pancharatnam Phase in a Tavis-Cummings Model with Nonlinearity*

LIAO Hao-xiang, WANG Fa-qiang†, LIANG Rui-sheng

(*Laboratory of Photonic Information Technology, School of Information and Optoelectronic Science and Engineering, South China Normal University, Guangzhou* 510006, *China*)

Abstract: The Pancharatnam phase of a Tavis-Cummings model with the coherent state in Kerr medium is investigated theoretically and numerically. An exact expression of the Pancharatnam phase under this general system is obtained. The numerical results show that the dipole-dipole coupling strength between two atoms can change the quasi-period of the Pancharatnam phase evolution, and it is also proved that the Pancharatnam phase contains the information of atoms and the field. In the nonlinear interaction regime, the nonlinearity can cause the chaotic behavior of the Pancharatnam phase.

Key words: Quantum optics; Pancharatnam phase; Tavis-Cummings; Kerr medium

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0 Introduction

In recent years, the quantum phase such as the Pancharatnam phase[1-6] and geometric phase[7-8] plays an important role in the field of physics, quantum computation, quantum communication and quantum cryptography. The concept of the phase in this paper was proposed in 1956 by Pancharatnam in his studies of interference effect of polarized light waves[1]. After that, many generalizations of Pancharatnam phase have been used in all kinds of physical systems. For example, M. Abdel-At et al. have studied the Pancharatnam phase of a two-mode optical field with Kerr nonlinearity[4]. And the Pancharatnam phases of two two-level atoms interacting with a single mode field in both the coherent state and Fock state are also analyzed[5-6].

It is known that nonlinear interaction plays an important role in the quantum optics. Lately, the properties and application in nonlinear quantum optics models have been studied extensively. Abdel. Shafy et al. researched the non-classical effects in a three-level atom one-mode system with arbitrary forms of nonlinearities[9]. Squeezing and entanglement in circuit-QED is efficiently generated[10-11], using a nanomechanical resonator as a cavity and superconducting charge qubit as a nonlinear media. Experimental efforts are underway[12]. M. M. He, et al. reported the Berry phase in the Tavis-Coummings model[8] and the two-atom Jaynes-Cumimings model with Kerr medium[13].

In this paper, we extend the Jaynes-Cumimings model[14] of Ref. [13] to the Tavis-Coummings model[15] where the dipole-dipole coupling between two atoms is included. Instead of Berry phase, we investigate the time-dependent evolution of the Pancharatnam phase of this more general case. In the present work, we firstly get the exact time-dependent expressions for the final state of the system by solving the Schrodinger equation. Then, we obtain the expressions of the Pancharatnam phase evolution. At last, we analyze the influence of some parameters on the Pancharatnam phase of the system by numerical calculation.

1 Formulation of the problem

The system considered here contains two two-level atoms and single-mode quantized cavity which is filled with nonlinear Kerr medium. We consider the non-degenerate case, which means pairs of photons are created or absorbed by the quantized radiation field. The Hamiltonian of this system[16] in the rotating wave approximation is given by ($\hbar = c = 1$)

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{in}} \tag{1}$$

where

$$\hat{H}_0 = \omega \hat{a}^+ a + \frac{\omega_0}{2}(\sigma_1^z + \sigma_2^z) \tag{2}$$

$$\hat{H}_{\text{in}} = \lambda_1 \hat{a} e^{i\Delta t}(\sigma_1^+ + \sigma_2^+) + \lambda_2 \hat{a}^+ e^{-i\Delta t}(\sigma_1^- + \sigma_2^-) + \Omega\sigma_1^+\sigma_2^- + \Omega\sigma_2^+\sigma_1^- + \chi \hat{a}^+\hat{a}\hat{a}^+\hat{a} \tag{3}$$

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† Tel: 020-39310311 Email: fqwang@scnu.edu.cn

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with the detuning parameters $\Delta=\omega-\omega_0$, ω_0 is the atomic transition, ω is the field mode frequency, $\hat{a}(\hat{a}^+)$ is the annihilation(creation) operator of the field mode, $\sigma_i^+(\sigma_i^-)$ and σ_i^z are the atomic raising (lowering) and inversion operators of two atoms, Ω is the dipole-dipole coupling strength between two atoms, λ_i represents atom-field coupling constant. We denote by χ the dispersive part of the third-order nonlinearity of Kerr medium.

For simplicity, we consider the resonance case i. e. $\Delta=0$ and the case of identical atoms i. e. $\lambda_1=\lambda_2=\lambda$. We assume that the cavity field is prepared initially in a coherent state

$$|\alpha\rangle=\sum_{n=0}^{\infty}e^{-|\alpha|^2/2}\alpha^n/\sqrt{n!}\,|n\rangle$$

where $\alpha=\sqrt{\bar{n}}e^{i\xi}$, $\bar{n}$ and ξ represent the average photon number and the direction angle of the excitation for the field mode, and we set $\xi=0$ for simplicity. Two atoms are initially in an entangled state[17-18] i. e. $|\psi(0)\rangle_{\text{atoms}}=\cos\theta\,|e_1g_2\rangle+\sin\theta\,|g_1e_2\rangle$. Then the initial state of the system is given by

$$|\psi(0)\rangle=\sum_{n=0}^{\infty}F_n(\cos\theta\,|e_1g_2\rangle+\sin\theta\,|g_1e_2\rangle)\otimes|n\rangle$$

$$F_n=e^{-|\alpha|^2/2}\alpha^n/\sqrt{n!} \tag{4}$$

Finally, we get the time-dependent analytical solution of the Schrodinger equation in the interaction picture with

$$i\hbar\frac{d}{dt}|\psi(t)\rangle=\hat{H}_{\text{in}}|\psi(t)\rangle \tag{5}$$

for the state vector $|\psi(t)\rangle$ at any time $t>0$

$$|\psi(t)\rangle=\sum_{n=0}^{\infty}F_n(a_j(t)|e_1e_2\rangle|n\rangle+b_j(t)|e_1g_2\rangle|n+1\rangle+c_j|g_1e_2\rangle|n+1\rangle+d_j(t)|g_1g_2\rangle|n+2\rangle) \tag{6}$$

The coefficients are given by

$$a_j(t)=A(e^{-iat}-e^{-ibt})(\cos\theta+\sin\theta)$$

$$b_j(t)=B\{[2e^{-i(f-\Omega)t}+e^{-iat}+e^{-ibt}]\cos\theta+[-2e^{-i(f-\Omega)t}+e^{-iat}+e^{-ibt}]\sin\theta\}+C(e^{-iat}-e^{-ibt})\Delta(\cos\theta+\sin\theta)$$

$$c_j(t)=B\{[-2e^{-i(f-\Omega)t}+e^{-iat}+e^{-ibt}]\cos\theta+[2e^{-i(f-\Omega)t}+e^{-iat}+e^{-ibt}]\sin\theta\}+C(e^{-iat}-e^{-ibt})\Delta(\cos\theta+\sin\theta)$$

$$d_j(t)=D(e^{-iat}-e^{-ibt})(\cos\theta+\sin\theta)$$

where

$$A=\frac{\Omega^2\beta^2\gamma-\beta^4\gamma+\Omega^2\gamma^3-2\beta^2\gamma^3-\gamma^5}{R}$$

$$B=\frac{\frac{1}{2}\Omega^4\mu^2+7\Omega^2\beta^4+7\Omega^2\gamma^4+14\Omega^2\beta^2\gamma^2}{R}-\frac{12\beta^2\gamma^2\mu-8\beta^6-8\gamma^6}{R}$$

$$C=\frac{\frac{1}{2}\Omega^3\mu-\Omega\beta^4-2\Omega\beta^2\gamma^2-\Omega\gamma^4}{R}$$

$$D=\frac{\Omega^2\beta^3-\beta^5+\Omega^2\beta\gamma^2-2\beta^2\gamma^2-\beta\gamma^4}{R}$$

with

$$\gamma=\lambda\sqrt{n+1},\beta=\lambda\sqrt{n+2},\mu=2(\gamma_n^2+\beta_n^2),f=\chi n^2$$

$$\Delta=\sqrt{\Omega^2+8\beta^2+8\gamma^2},a=\frac{2f+\Omega+\Delta}{2},b=\frac{2f+\Omega-\Delta}{2}$$

$$R=(\Omega^3+8\Omega\beta^2+8\Omega\gamma^2)^2-(\Omega^2\Delta+2\beta^2\Delta+2\gamma^2\Delta)^2$$

2 The pancharatnam phase

For a quantum system evolving from an initial state to a final state in an arbitrary way, the Pancharatnam phase $\varphi_t(t)$ between the vector $|\psi(o)\rangle$ and $|\psi(t)\rangle$ is given by

$$\varphi_t(t)=\arg\langle\psi(o)|\psi(t)\rangle \tag{7}$$

We use the wave function equation(6), and definition(7) to calculate the Pancharatnam phase $\varphi_t(t)$. After some algebraic manipulations the Pancharatnam phase is read as

$$\varphi_t(t)=-\arcsin\left(\frac{Y(t)}{\sqrt{X^2(t)+Y^2(t)}}\right) \tag{8}$$

$$Y(t)=-\sum_{m=1}^{\infty}F_m\sum_{n=0}^{\infty}2F_n\{B(\cos\theta+\sin\theta)\cdot[\cos(at)+\cos(bt)]+C\Delta(\cos\theta+\sin\theta)\cdot[\cos(at)-\cos(bt)]\}$$

$$X(t)=\sum_{m=1}^{\infty}F_m\sum_{n=0}^{\infty}2F_n\{B(\cos\theta+\sin\theta)\cdot[\sin(at)+\sin(bt)]+C\Delta(\cos\theta+\sin\theta)\cdot[\sin(at)-\sin(bt)]\}$$

Then we can make numerical calculation to discuss the behavior of the Pancharatnam phase on the dynamics of the system.

3 Results of calculations

First of all, it should be noted that we have invoked reasonable truncation criteria for all infinite series of computing. Some typical results of the time-dependent evolution of the Pancharatnam phase are displayed thereinafter.

In Fig. 1, we have plot the Pancharatnam phase $\varphi_t(t)$ as a function of the scaled time λt for different values of the dipole-dipole coupling strength Ω between two atoms when there is no Kerr medium in the cavity, i. e. ,($\chi=0$). We can find that the phase initially oscillates around zero and collapses to zero in a short time, then it appears the continuous collapse-revival phenomenon for a range of interaction times, and later it exhibits another obvious oscillation again (see Fig. 1(a) and(b)), which is similar to the case of consideration of the cavity field in a Fock

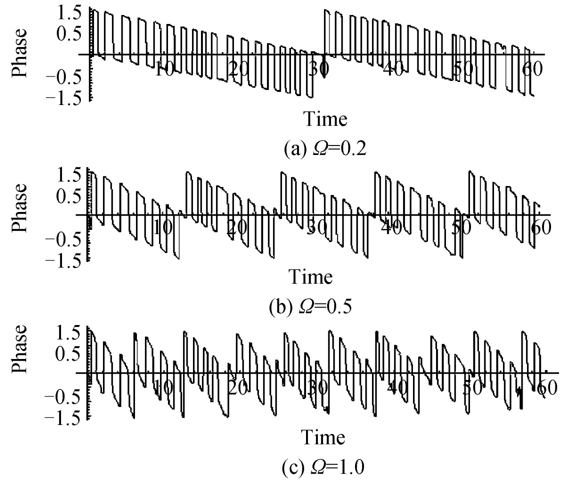

Fig. 1　The Pancharntnam phase $\varphi_t(t)$ as a function of scaled time (λt) (assume that $\bar{n}=2, \theta=\pi/4$)

state[5]. After enhancing the coupling strength, it is remarked that the phase has yet the similar behavior. However, the period becomes smaller (see Fig. 1(b) and(c)), which can be understood to be the information of phase exchanges more frequently.

In Fig. 2, it is presented that the results of the Pancharatnam phase for different initial entangled states of two atoms. The graph for $\theta=\pi/6$ shows that there exits still the quasi-period collapse-revival phenomenon, however, it exhibits some anomalies in the period comparing to the case of $\theta=\pi/4$(see Fig. 1(b) and 2(a)). An interesting case, in which $\theta=\pi/2$, the periodicity is shown more clearly, and some sharp peaks can be observed distinctly(see Fig. 2(b)), which can be understood the interference in phase space [5]. This means that the initial state of two atoms can affect the behavior of the Pancharatnam phase.

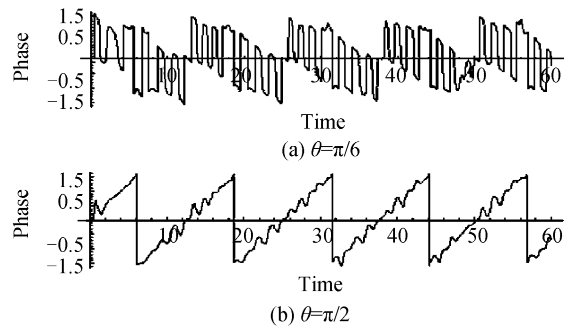

Fig. 2　The Pancharntnam phase $\varphi_t(t)$ as a function of scaled time (λt)(assume that $\bar{n}=2, \Omega=0.5, \chi=0$)

To research the influence of the Kerr medium in the Pancharatnam phase $\varphi_t(t)$, we set different values of χ. The result is shown in Fig. 3. In Fig. 3 (a) and(b), we see that there is still a periodic collapse-revival phenomenon when the nonlinearity is small. And with the increase of nonlinearity, the periodicity becomes bigger and bigger, even is changed markedly (see Fig. 3 (c)). Once the nonlinearity is great enough, the periodicity in Fig. 2(a) and(b) is no longer present, and chaotic behavior begins(see Fig. 3(d)).

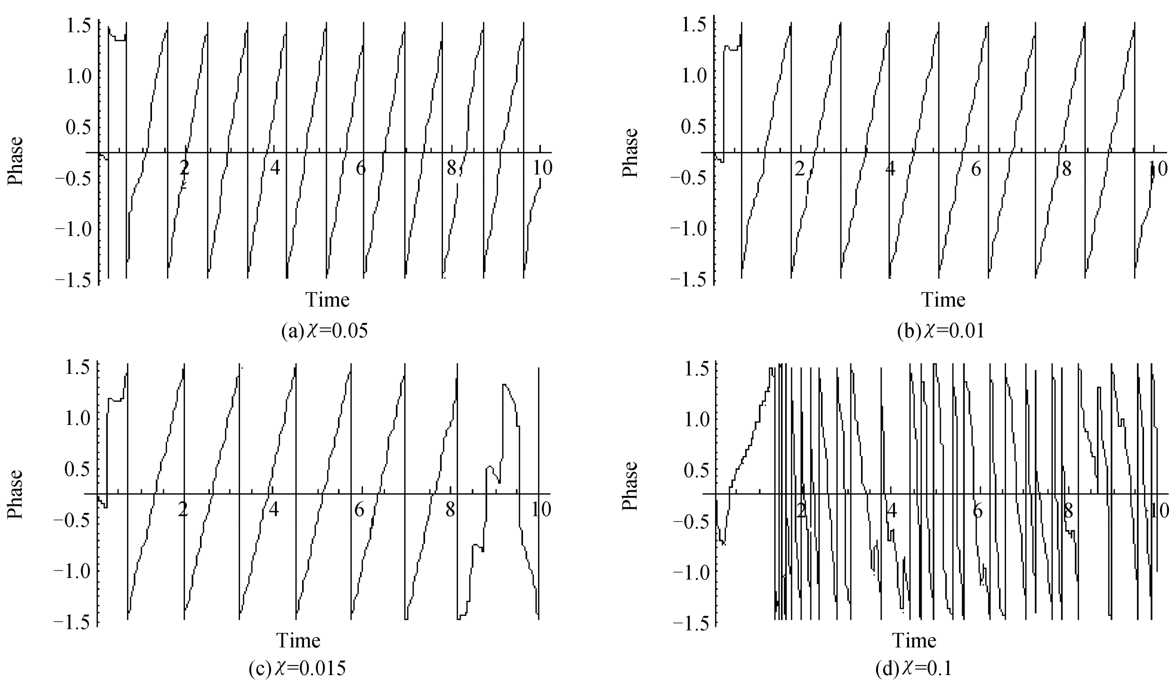

Fig. 3　The Pancharntnam phase $\varphi_t(t)$ as a function of scaled time (λt)(assume that $\bar{n}=2, \Omega=0.5, \theta=\pi/4$)

The phases studied here are physically observable through a "structured" approach or can be estimated by other techniques of phase estimation.

4 Conclusions

In this paper we studied the Pancharatnam phase of Tavis-Cummings model with Kerr medium theoretically and numerically. An exact expression is obtained. The effect of both Kerr nonlinearity and the dipole-dipole coupling strength between two atoms is analyzed. It shows that the Pancharatnam phase is sensitive to the Kerr nonlinearity, also the evolvement of the phase can be controlled by the initial state of the atoms and the dipole-dipole coupling strength, which appears great importance to the application such as quantum computation and nanotechnology.

References

[1] PANCHARATNAM S. Generalized theory of interference and its applications[J]. *Proe Indian Acad Sci Scct A*, 1956, **44**(6): 247-417.

[2] GANG Z Y, ZANG L B. Quantum phases of Pancharatnam type for a general spin in a time-dependent magnetic field[J]. *Chin Phys Lett*, 1997, **14**(11): 801-804.

[3] LAWANDE Q V, LAWANDE S V, JOSHI A. Pancharatnam phase for a system of a two-level atom interacting with a quantized field in a cavity[J]. *Phys Lett A*, 1999, **251**(3): 164-168.

[4] ATY M A, KHALEK S A, OBADA. F Pancharatnam phase of two-mode optical fields with kerr nonlinearity[J]. *Opt Rev*, 2000, **7**(6):499-504.

[5] ATY M A. The pancharatnam phase of a two-level atom in the presence of another two-level atom in a cavity[J]. *J Opt B: Quant Semiclass Opt*, 2003, **5**(4): 349-354.

[6] METWALLY N. Entanglement and pancharatnam phase for two two-level atoms interacting with a single mode field[J]. *Int J Theor Phys*, 2008, **47**(3): 623-633.

[7] YI X X, L WANG C, ZHENG T Y. Berry phase in a composite system[J]. *Phys Rev Lett*, 2004, **92**(15): 150406.

[8] HE M M, CHEN G, LIANG J Q. Berry phase in Tavis-Cummings model[J]. *Eur J Phys D*, 2007, **44**(3): 581-583.

[9] SHAFY A, OBADA F, ATY M A. Nonclassical effects in a three-level atom one mode system with arbitrary forms of nonlinearities[J]. *Physica A*, 2003, **329**(1): 53-67.

[10] HUO W Y, LONG G L. Entanglement and squeezing in solid state circuit[J]. *New J Phys*, 2008, **10**(1): 013026.

[11] HUO W Y, LONG G L. Generation of squeezed states of nanomechanical resonator using three-wave mixing[J]. *Appl Phys Lett*, 2008, **92**(3): 133102.

[12] ETAKI S, POOT M, MAHBOOB I, *et al*. Motion detection of a micromechanical resonator embedded in a d. c. SQUID [J]. *Nature Phys*, 2008, **4**(8): 785-788.

[13] BU S P, ZHANG G F, LIU J, *et al*. Berry phase in a two-atom Jaynes - Cummings model with Kerr medium[J]. *Phys Scr*, 2008, **78**(6): 065008.

[14] LIU Wang-yun, AN Yu-ying, YANG Zhi-yong. Influence of frequency detuning on evolution of quantum field entropy in multi mode nondegenerate multi-photon Jaynes-Cummings Model[J]. *Acta Photonica Sinica*, 2008, **37**(5): 1057-1062.

[15] TAVIS M, CUMMINGS F W. Exact solution for an N molecule radiation field Hamiltonian[J]. *Phys Rev*, 1968, **170**(2): 379-384.

[16] HE M M, XU C T, CHEN N, *et al*. Quantum phase transition of entanglement in Heisenberg spin model with quantized field[J]. *Eur J Phys D*, 2006, **39**(2):313-320.

[17] CHEN Mei-feng, MA Song-she. Probabilistic remote preparation of two atom entangled state[J]. *Acta Photonica Sinica*, 2008, **37**(1): 188-191.

[18] CAI Xin-hua, NIE Jian-jun, GUO Jie-rong. Entanglement translation and quantum teleportation of the single-photon entangled state[J]. *Acta Photonica Sinica*, 2006, **35**(5): 776-779.

非线性 Tavis-Cummings 模型的 Pancharatnam 相

廖浩祥,王发强,梁瑞生

(华南师范大学 信息光电子科技学院 光子信息技术高校重点实验室,广州 510006)

摘　要:对处于相干态光场的非线性 Tavis-Cummings 模型中 Pancharatnam 相进行了理论上和数值上的研究,并获得了这个量子系统中 Pancharatnam 相的精确表达式.分析表明:两原子之间的偶极相互作用强度能改变 Pancharatnam 相演化的周期,并证明了 Pancharatnam 相包含原子和场的信息,同时,非线性的出现会导致 Pancharatnam 相的演化出现混沌行为.

关键词:量子光学; Pancharatnam 相; Tavis-Cummings; 克尔介质

LIAO Hao-xiang was born in 1984. Now he is a M. S. degree candidate at Laboratory of Photon Information Technology, South China Normal University. His research interests focus on quantum optics and quantum information.

WANG Fa-qiang is a professor, and his research interests focus on quantum optics and quantum information.