第 39 卷第 2 期
2010 年 2 月

光 子 学 报
ACTA PHOTONICA SINICA

Vol. 39 No. 2
February 2010

Article ID:1004-4213(2010)02-0202-3

Study on Coupled-mode Theory of Terahertz Wire Waveguides

LIANG Hua-wei,RUAN Shuang-chen,ZHANG Min,SU Hong
(*Shenzhen Key Laboratory of laser Engineering*,*College of Electronic Science and Technology*, *Shenzhen University*,*Shenzhen*,518060,*China*)

Abstract: A coupled-mode theory for treating the coupling of two terahertz (THz) wire waveguides was studied using the coupled-mode assumption. Both coupling equations and coupling coefficients were obtained by using approximations equivalent with those in coupled-mode theory of optical waveguides. Furthermore, the solutions and coupling characteristics of coupled-mode equations for two same wire waveguides were obtained. The present theory is useful for treating problems involving energy exchange between the wire waveguides.

Key words:Coupled-mode Theory; Terahertz; Wire waveguide
CLCN:TN2 **Document Code**:A **doi**:10.3788/gzxb20103902.0202

0 Introduction

THz waveguide techniques have attracted a lot of attention over the last several years[1-3]. Especially, Wang and Mittleman demonstrated that a single metal wire which can be used as a THz waveguide with low propagation losses and negligible group velocity dispersion[4]. Subsequently, many researchers and scientists have studied wave propagation on a single metal wire in the THz frequency range[5-9]. The basic demonstration is based on a proposal that Sommerfeld made in the late nineteenth century, in which he showed that a single cylindrical conductor of finite conductivity could support a guided wave mode[10]. Wave propagation on multi metal wires has also drawn some research interest[11-12]. Shvets et al proposed a tapered multiwire array supporting subwavelength transverse electromagnetic (TEM) waves[11]. Mbonye et al investigated the coupling efficiency of a two-wire configuration by using a commercially available finite element modeling (FEM) software[12].

In this paper, coupled-mode theory is developed firstly to describe coupling between two THz wire waveguides where the couple-mode approximation is valid. Both coupling equations and coupling coefficients for THz wire waveguides are obtained by using the approximations equivalent with those in coupled-mode theory of optical waveguides[13-15]. Furthermore, the solutions and coupling characteristics of coupled-mode equations for two same wire waveguides are obtained. The results obtained agree with that of experimental measurement obtained by Kang and Mittleman[6]. The present theory, which can treat coupling between two wire waveguides, is quite useful for designing THz integrated circuit.

1 Coupled-mode theory of THz wire waveguides

Fig. 1 shows the configuration of two THz wire waveguides. When the two wire waveguides are not too closely, the magnetic field amplitude can be written as

$$H = A_1(z)\zeta_1(x,y)e^{-i\beta_1 z} + A_2(z)\zeta_2(x,y)e^{-i\beta_2 z} \quad (1)$$

$\zeta_1(x,y)$ and $\zeta_2(x,y)$, which can be obtained in literature[16], are magnetic amplitude distributions of modes on wire waveguides β_1 and β_2 without the coupling perturbation, respectively. β_1 and β_2 are complex propagation constant. $A_1(z)$ and $A_2(z)$ are complex amplitudes.

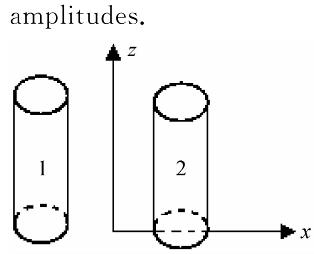

Fig. 1 Configuration of two THz wire waveguides

The distribution of complex refractive index can be written as

$$n^2(x,y) = n_s^2(x,y) + \Delta n_1^2(x,y) + \Delta n_2^2(x,y) \quad (2)$$

$$\Delta n_1^2(x,y) = n_1^2(x,y) - n_s^2(x,y) = (c^2/\omega^2)[k_1^2(x,y) - k_s^2(x,y)]$$

$$\Delta n_1^2(x,y) = n_2^2(x,y) - n_s^2(x,y) = (c^2/\omega^2)[k_2^2(x,y) - k_s^2(x,y)]$$

* Supported by the Fund Project for Shenzhen fundamental research programme (JC200903120019A) and the Fund Project for Shenzhen Key Lab R&D Programme (200806170029A)
Tel:0755- 26958265 Email:scruan@szu.edu.cn
Received date:2009- 04- 10 Revised date:2009- 07- 07

$n_s = k_s c/\omega$, $n_1 = k_1 c/\omega$, $n_2 = k_2 c/\omega$ is wave propagation constant outside the two wire, and are wave propagation constant inside wire 1 and wire 2. Both $\zeta_1(x,y)$ and $\zeta_2(x,y)$ are transverse magnetic mode, so they obey

$$\left\{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\omega^2}{c^2}[n_s^2+\Delta n_1^2(x,y)]\right\}\times \zeta_1(x,y)=\beta_1^2\zeta_1(x,y) \quad (3)$$

$$\left\{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\omega^2}{c^2}[n_s^2+\Delta n_2^2(x,y)]\right\}\times \zeta_2(x,y)=\beta_2^2\zeta_2(x,y) \quad (4)$$

In the presence of a perturbation, the total field amplitude obeys the wave equation[13-14]

$$\left\{\nabla^2+\frac{\omega^2}{c^2}[n_s^2+\Delta n_1^2(x,y)+\Delta n_2^2(x,y)]\right\}H=0 \quad (5)$$

By using equ. (1) ~ (5) and assuming "slow" variation[13], we give

$$-2\mathrm{i}\beta_1\frac{\mathrm{d}A_1(z)}{\mathrm{d}z}\zeta_1(x,y)\mathrm{e}^{-\mathrm{i}\beta_1 z}-2\mathrm{i}\beta_2\frac{\mathrm{d}A_2(z)}{\mathrm{d}z}\cdot \zeta_2(x,y)\mathrm{e}^{-\mathrm{i}\beta_1 z}=-\omega^2/c^2\Delta n_2^2(x,y)A_1(z)\cdot \zeta_1(x,y)\mathrm{e}^{-\mathrm{i}\beta_2 z}-\omega^2/c^2\Delta n_1^2(x,y)A_2(z)\cdot \zeta_2(x,y)\mathrm{e}^{-\mathrm{i}\beta_2 z} \quad (6)$$

By multiplying equ. (6) with $\zeta_1^*(x,y)\mathrm{e}^{\omega t-\beta z}$ and $\zeta_2^*(x,y)\mathrm{e}^{\omega t-\beta z}$, respectively, and scalar integrating in the whole space and then making use of the orthogonality relation, we give

$$\frac{\mathrm{d}A_1(z)}{\mathrm{d}z}=-\mathrm{i}\kappa_{12}A_2(z)\mathrm{e}^{\mathrm{i}(\beta_1-\beta_2)z}-\mathrm{i}\kappa_{11}A_1(z) \quad (7)$$

$$\frac{\mathrm{d}A_2(z)}{\mathrm{d}z}=-\mathrm{i}\kappa_{21}A_1(z)\mathrm{e}^{i(\beta_1-\beta_2)z}-\mathrm{i}\kappa_{22}A_2(z) \quad (8)$$

and

$$\kappa_{12}=\omega\varepsilon_0\int\zeta_1^*(\mathrm{x,y})\Delta n_1^2(x,y)\zeta_2(x,y)\mathrm{d}x\mathrm{d}y/4 \quad (9)$$

$$\kappa_{21}=\omega\varepsilon_0\int\zeta_2^*(x,y)\Delta n_2^2(x,y)\zeta_1(x,y)\mathrm{d}x\mathrm{d}y/4 \quad (10)$$

$$\kappa_{11}=\omega\varepsilon_0\int\zeta_1^*\Delta n_2^2(x,y)\zeta_1(x,y)\mathrm{d}x\mathrm{d}y/4 \quad (11)$$

$$\kappa_{22}=\omega\varepsilon_0\int\zeta_2^*\Delta n_1^2(x,y)\zeta_2(x,y)\mathrm{d}x\mathrm{d}y/4 \quad (12)$$

Especially, κ_{12}、κ_{21}、κ_{11}、κ_{22}、β_1、β_2 are all complex number for the two wire waveguides, and they can be written as $\kappa_{12}=\kappa_{12a}-\mathrm{i}\kappa_{12b}$, $\kappa_{21}=\kappa_{21a}-\mathrm{i}\kappa_{21b}$, $\kappa_{11}=\kappa_{11a}-\mathrm{i}\kappa_{11b}$, $\kappa_{22}=\kappa_{22a}-i\kappa_{22b}$, $\beta_1=\beta_{1a}-\mathrm{i}\beta_{1b}$, $\beta_2=\beta_{2a}-\mathrm{i}\beta_{2b}$. According to the literature[16], we can obtain the mode field distributions $\zeta_1(x,y)$ and $\zeta_2(x,y)$, and the propagation constants β_1 and β_2. Then based on equ. (9) ~ (12), we can obtain the coupling coefficients κ_{12}、κ_{21}、κ_{22} and by the numerical calculation. Subsequently, all the parameters mentioned above can be obtained.

2 Discussion of coupled-mode theory for two same THz wire waveguides

2.1 Solutions of the coupled-mode equations for two same wire waveguides

When the two wires have the same material and size, we specify $\kappa_{12a}=\kappa_{21a}=\kappa_{1a}$, $\kappa_{12b}=\kappa_{21b}=\kappa_{1b}$, $\kappa_{11a}=\kappa_{22a}=\kappa_{0a}$, $\kappa_{11b}=\kappa_{22b}=\kappa_{0b}$, $\beta_{1a}=\beta_{1b}=\beta_a$, $\beta_{1b}=\beta_{2b}=\beta_b$. As a result, equations (7) and (8) can be written as

$$\frac{\mathrm{d}A_1(z)}{\mathrm{d}z}=-\mathrm{i}\kappa_{1a}A_2(z)-\kappa_{1b}A_2(z)-\mathrm{i}\kappa_{0a}A_1(z)-\kappa_{0b}A_1(z) \quad (13)$$

$$\frac{\mathrm{d}A_2(z)}{\mathrm{d}z}=-\mathrm{i}\kappa_{1a}A_1(z)-\kappa_{1b}A_1(z)-\mathrm{i}\kappa_{0a}A_2(z)-\kappa_{0b}A_2(z) \quad (14)$$

The solutions of equations (13) and (14) are given by

$$A_1(z)=2^{-1}\mathrm{e}^{-z(\kappa_{0a}+\kappa_{0b}+\mathrm{i}\kappa_{1a}+\kappa_{1b})}\{A_1 0+A_2(0)+[A_1(0)-A_2(0)]\mathrm{e}^{2z(\mathrm{i}\kappa_{1a}+\kappa_{1b})}\} \quad (15)$$

$$A_2(z)=2^{-1}\mathrm{e}^{-z(\mathrm{i}\kappa_{0a}+\kappa_{0b}+\mathrm{i}\kappa_{1a}+\kappa_{1b})}\{A_1 0+A_2(0)+[A_2(0)-A_1(0)]\mathrm{e}^{2z(\mathrm{i}\kappa_{1a}+\kappa_{1b})}\} \quad (16)$$

so

$$A_1(z)\mathrm{e}^{-\mathrm{i}\beta_1 z}=2^{-1}\mathrm{e}^{-z(\mathrm{i}\kappa_{0a}+\kappa_{0b}+\mathrm{i}\kappa_{1a}+\kappa_{1b}+\mathrm{i}\beta_a+\beta_b)}\times\{A_1(0)+A_2(0)+[A_1(0)-A_2(0)]\cdot \mathrm{e}^{2z(\mathrm{i}\kappa_{1a}+\kappa_{1b})}\} \quad (17)$$

$$A_2(z)\mathrm{e}^{-\mathrm{i}\beta_2 z}=2^{-1}\mathrm{e}^{-z(\mathrm{i}\kappa_{0a}+\kappa_{0b}+\mathrm{i}\kappa_{1a}+\kappa_{1b}+\mathrm{i}\beta_a+\beta_b)}\times\{A_1(0)+A_2(0)+[A_2(0)-A_1(0)]\cdot \mathrm{e}^{2z(\mathrm{i}\kappa_{1a}+\kappa_{1b})}\} \quad (18)$$

The total power is expressed as

$$P=|A_1(Z)\mathrm{e}^{-\mathrm{i}\beta_1 z}|^2+|A_2(Z)\mathrm{e}^{-\mathrm{i}\beta_2 z}|^2= 2^{-1}\mathrm{e}^{-z2(\kappa_{0b}+\beta_b)}\{[A_1(0)+A_2(0)]^2\mathrm{e}^{-z2\kappa_{1b}}+ 2^{-1}[A_1(0)-A_2(0)]^2\mathrm{e}^{z2\kappa_{1b}}\} \quad (19)$$

From (17) ~ (19), we can conclude that, for the two wire waveguides, the group velocity dispersion is determined by κ_{0a}, κ_{1a} and β_a, and the attenuation due to conductivity losses is determined by κ_{0b}, κ_{1b} and β_b, respectively.

2.2 Coupling characteristics of THz wire waveguides

When $A_1(0)=1$ and $A_2(0)=0$, the total power can be written as

$$P=2^{-1}\mathrm{e}^{-z2(\kappa_{0b}+\beta_b)}(\mathrm{e}^{-z2\kappa_{1b}}+\mathrm{e}^{z2\kappa_{1b}}) \quad (20)$$

From (20), we can know that, the total power decreases gradually. The attenuation results from the conductivity losses which are usually very small[4,6].

The power on the wire waveguide 1 and 2 can be written as

$$P_1=\frac{1}{4}\mathrm{e}^{-z2(\kappa_{0b}+\beta_b)}[\mathrm{e}^{-z2\kappa_{1b}}+\mathrm{e}^{z2\kappa_{1b}}+2\cos(\kappa_{1a}z)] \quad (21)$$

$$P_2=\frac{1}{4}\mathrm{e}^{-z2(\kappa_{0b}+\beta_b)}[\mathrm{e}^{-z2\kappa_{1b}}+\mathrm{e}^{z2\kappa_{1b}}-2\cos(\kappa_{1a}z)] \quad (22)$$

If the attenuation, which is determined by κ_{0b}, κ_{1b} and β_b, can be ignored, the complete power exchange from the wire 1 to the wire 2 will happen at $z=\pi/\kappa_{1a}$. THz power variations both on the wire waveguide 1 and 2 are shown on Fig. 2. From Fig. 2, we know that when the THz wave propagates along z direction, the power on the waveguides exchanges from one to another. The similar power exchanging phenomenon in experiment is demonstrated in the literature[6].

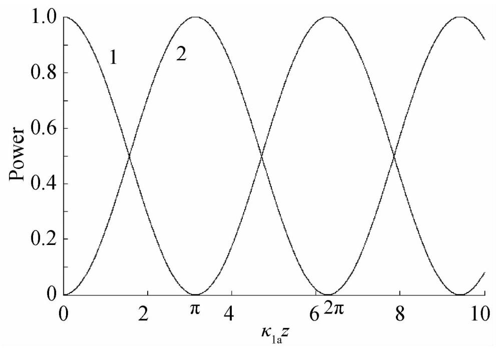

Fig. 2 THz power variations on the wire waveguide 1 and 2

3 Conclusion

A coupled-mode theory for treating the coupling of THz wire waveguides is studied. Both coupling equations and coupling coefficients are obtained by using approximations equivalent with those in coupled-mode theory of optical waveguides. Furthermore, coupling characteristics of two same wire waveguides are discussed and some important conclusions are obtained: 1) the group velocity dispersion is determined by κ_{0a}, κ_{1a} and β_a; 2) the attenuation due to conductivity losses is determined by κ_{0b}, κ_{1b} and β_b; 3) if the attenuation, which results from the conductivity losses, can be ignored, the complete power exchange between the two wires will happen.

References

[1] YEH C, SHIMABUKURO F, STANTON P, *et al*. Communication at millimeter-submillimetre wavelengths using a ceramic ribbon [J]. *Nature*, 2000, **404**(6): 584-588.

[2] MENDIS R, GRISCHKOWSKY D. Undistorted guided-wave propagation of subpicosecond terahertz pulses [J]. *Opt lett*, 2001, **26**(11): 846-848.

[3] HAN H, PARK H, CHO M, *et al*. Terahertz pulse propagation in a plastic photonic crystal fiber [J]. *Appl Phys Lett*, 2002, **80**(15): 2634-2636.

[4] WANG K, MITTLEMAN D M. Metal wires for terahertz waveguiding [J]. *Nature*, 2004, **432**(18): 376-379.

[5] JEON T, ZHANG J, GRISCHKOWSKY D. THz Sommerfeld wave propagation on a single metal wire [J]. *Appl Phys Lett*, 2005, **86**(16): 161904.

[6] WANG K and MITTLEMAN D M. Guided propagation of terahertz pulses on metal wires [J]. *JOSA B*, 2005, **22**(9): 2001-2008.

[7] WACHTER M, NAGEL M, KURZ H. Frequency-dependent characterization of THz Sommerfeld wavepropagation on single wires [J]. *Opt Express*, 2005, **13**(26): 10815-10822.

[8] WANG K and MITTLEMAN D M. Dispersion of surface plasmon polaritons on metal wires in the terahertz frequency range [J]. *Phys Rev Lett*, 2006, **96**(15): 157401.

[9] LIANG Hua-wei, RUAN Shuang-chen, ZHANG Min. Terahertz surface wave propagation and focusing on conical metal wires [J]. *Opt Express*, 2008, **16**(22): 18241-18248.

[10] CAO H, NAHATA A. Coupling of terahertz pulses onto a single metal wire waveguide using milled grooves [J]. *Opt Express*, 2005, **13**(18): 7028-7034.

[11] SHVETS G, TRENDAFILOV S, PENDRY J B, *et al*. Guiding, focusing and sensing on the subwavelength scale using metallic wire arrays [J]. *Phys Rev Lett*, 2007, **99**(5): 053903.

[12] MBONYE M K, ASTLEY V, CHAN W L, *et al*. A terahertz dual wire waveguide [C]. *Conference on Lasers and Electro-Optics (CLEO)*, 2007, 1-55752-834-9.

[13] YARIV A. Coupled-mode theory for guided-wave optics [J]. *IEEE J quantum electronics*, 1973, QE-**9**(9): 919-933.

[14] TAYLOR H F and YARIV A. Guided wave optics [J]. *IEEE*, 1974, **62**(8): 1044-1060.

[15] LIANG Hua-wei, SHI Shun-xiang, MA Lin, *et al*. Study on coupling in a multi-waveguides array [J]. *Acta Photonica Sinica*, 2008, **37**(4): 644-647.

[16] GOUBAU G. Surface waves and their application to transmission lines [J]. *J Appl Phys*, 1950, **21**(11): 1119-1128.

太赫兹金属线波导耦合模理论研究

梁华伟，阮双琛，张敏，苏红

（深圳大学 电子科学与技术学院 激光工程重点实验室，深圳 518060）

摘 要：基于耦合模假设，研究了描述两太赫兹金属线波导之间耦合特性的耦合模理论．通过采用与光波导耦合模理论等价的近似，研究得到了耦合方程和耦合系数表达式．进而，研究得到了两个相同金属线波导的耦合方程解及耦合特性．所提出的理论对于描述两金属线波导之间的能量传递具有重要的意义．

关键词：耦合模理论；太赫兹；金属线波导

LIANG Hua-wei was born in 1981. Now he has been working as a post-doctor in the Shenzhen Key Laboratory of laser Engineering of Shenzhen University. His research interests focus on the propagating and the imaging of terahertz wave.