第 39 卷第 11 期
2010 年 11 月

光 子 学 报
ACTA PHOTONICA SINICA

Vol. 39 No. 11
November 2010

**Article ID**: 1004-4213(2010)11-2073-5

# Economic and Simple Controlled Teleportation of an Arbitrary Two-qubit State Using Five-qubit Cluster State*

LI Yuan-hua[1], LIU Jun-chang[1], NIE Yi-you[1,2,†]

(1 *Department of Physics, Jiangxi Normal University, Nanchang* 330022, *China*)

(2 *Key Laboratory of Optoelectronic & Telecommunication of Jiangxi Province, Nanchang* 330022, *China*)

**Abstract**: A new application of the five-qubit cluster state is investigated for economic and simple controlled teleportation of an arbitrary two-qubit state. In this scheme, a five-qubit cluster state is shared by a sender (Alice), a controller (Charlie) and a receiver (Bob), and the sender only needs to perform Bell-state measurements and the controller performs a single-qubit projective measurement. The receiver can reconstruct the arbitrary two-qubit state by performing some appropriate unitary transformations on his qubits after he knows the measured results of both the sender and the controller. This controlled teleportation scheme is deterministic, and the probability of successful controlled teleportation is 100%. In comparison with the controlled teleportation scheme using the same quantum channel, the proposed scheme do not need to make multi-particle joint measurement that makes this scheme simpler.

**Key words**: Controlled teleportation; Cluster state; Bell-State Measurements(BSM)

**CLCN**: O431. 2; TN918　　**Document Code**: A　　**doi**: 10. 3788/gzxb20103911. 2073

## 0 Introduction

Quantum teleportation is an important ingredient in distributed quantum networks, which exploits entanglement for transferring a quantum state between two or more parties. Since Bennett et al. presented the protocol of quantum teleportation in 1993[1], it has become one of the most important fields of quantum information. Several authors have devised protocols for the teleportation by using quantum entangled channels[2-8]. The first controlled teleportation protocol was proposed in 1998 using GHZ state[9]. The basic idea of controlled teleportation is to transport an unknown quantum state with a controller, and it has been under the extensive investigations[10-19]. In these schemes the controlled teleportation of an unknown qubit state is through a three-qubit GHZ state or W state as the quantum channel along with classical communication. However, the entanglement in multi-qubit case is more complicated than in three-qubit case. Briegel and Raussendorf[20] have shown that cluster states have some particular characters in the case of $N>3$. For instance, the cluster state is maximally connected and its persistency is better than one of GHZ-class state. In other words, the cluster state has the properties of both the GHZ-class and the W-class entangled states, and is harder to destroy by local operations than GHZ-class states[21-22]. In recent years, many teleportation protocols are proposed using the multi-particle cluster state as quantum channel. Nie et al. proposed a scheme of non-maximally entangled controlled teleportation using a four-particle cluster state[23]. Zhang and Liu proposed an economic and deterministic quantum teleportation of an arbitrary bipartite state using a four particles cluster state as quantum channel[24]. It has been shown that the four and five particles cluster states are important resources for teleportation of an arbitrary two particles state[25].

In this paper, we proposed a new scheme for the controlled teleportation of an arbitrary two-qubit state using a five-qubit cluster state as the quantum channel. In this scheme, a five-qubit cluster state is prepared and shared by a sender, a controller and a receiver. The sender performs Bell-state measurements (BSMs) on her qubit pairs, respectively. Then the controller performs a single qubit projective measurement (SPM) on his

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* Supported by the National Natural Science Foundation of China (60807014), the Natural Science Foundation of Jiangxi Province, China (2009GZW0005), the Research Foundation of State Key Laboratory of Advanced Optical Communication Systems and Networks, and the Research Foundation of the Education Department of Jiangxi Province, China (GJJ09153)

† Tel: 0791- 8120376　　Email: nieyiyou@163. com

Received date: 2010- 04- 09　　Revised date: 2010- 06- 02

qubit. Finally the receiver performs some appropriate unitary transformations on his qubits according to the measure results of the sender and the controller. Thus the task of controlled teleportation of an arbitrary two-qubit state is completed. In section 1，we investigate the controlled teleportation of an arbitrary two-qubit state.

## 1 Controlled teleportation of an arbitrary two-qubit state

Controlled teleportation of an arbitrary two-qubit state was frst proposed by Deng et al.[26-27] using four Bell pairs among two controllers and then generalized to M agents. Later，the same purpose is achieved by Muralidharan and Panigrahi[25] using the five-qubit cluster state as the quantum channel. In Ref. [25]，the sender Alice first combined the original state with the five-qubit cluster state and made a four-qubit measurement. In order to recover the original state，the receiver Bob can ask the controller Charlie to perform a single-qubit projective measurement (SPM) on his qubit. Having known the outcomes of both their measurements，the receiver Bob can do appropriate unitary transformations to get back the original state. In this section，we propose a new protocol for the controlled teleportation of an arbitrary two-qubit state using the five-qubit cluster state as the quantum channel with BSMs. We will see that this protocol requires a lesser number of qubits than Deng′s scheme[26] and is simpler than Muralidharan′s scheme[25].

This controlled teleportation scheme can be described as follows. Suppose the sender Alice has an unknown arbitrary two-qubit state

$$|\psi\rangle_{ab}=\alpha|00\rangle+\mu|10\rangle+\gamma|01\rangle+\beta|11\rangle \tag{1}$$

where $|\alpha|^2+|\mu|^2+|\gamma|^2+|\beta|^2=1$. She prepares a cluster state with five qubits 1,2,3,4 and 5[20]

$$|C_5\rangle_{12345}=\frac{1}{2}(|00000\rangle+|00111\rangle+|11101\rangle+|11010\rangle)_{12345} \tag{2}$$

Firstly，Alice sends the qubit 4 to the controller Charlie，and the qubits 2 and 3 to the receiver Bob，respectively. Then，the combined state of the seven qubits can be expressed as

$$|\Psi\rangle_{ab12345}=|\psi\rangle_{ab}\otimes|C_5\rangle_{12345} \tag{3}$$

To achieve the purpose of controlled teleportation，Alice performs BSMs on her qubit pairs $(a,1)$ and $(b，5)$，respectively，and Alice has 16 kinds of possible measure results with equal probability 1/16. There are also 16 kinds of corresponding collapse states $|\varphi^i\rangle_{234}(i=1,2,\cdots,16)$ of qubits 2，3 and 4 after the measurement by Alice. The outcome of the measurement performed by Alice and the corresponding collapse state of Bob-Charlie′s system are shown in Table 1.

**Table 1 The outcome of the measurements performed by Alice and the corresponding states obtained by Bob and Charlie，where the normalization factors have been omitted for convenience**

| Alice′s measure result | State obtained by Bob and Charlie |
|---|---|
| $\|\Phi^+\rangle_{a1}\|\Phi^+\rangle_{b5}$ | $\|\varphi^1\rangle_{234}=\alpha\|000\rangle+\mu\|101\rangle+\gamma\|011\rangle+\beta\|110\rangle$ |
| $\|\Phi^+\rangle_{a1}\|\Phi^-\rangle_{b5}$ | $\|\varphi^2\rangle_{234}=\alpha\|000\rangle+\mu\|101\rangle-\gamma\|011\rangle-\beta\|110\rangle$ |
| $\|\Phi^-\rangle_{a1}\|\Phi^+\rangle_{b5}$ | $\|\varphi^3\rangle_{234}=\alpha\|000\rangle-\mu\|101\rangle+\gamma\|011\rangle-\beta\|110\rangle$ |
| $\|\Phi^-\rangle_{a1}\|\Phi^-\rangle_{b5}$ | $\|\varphi^4\rangle_{234}=\alpha\|000\rangle-\mu\|101\rangle-\gamma\|011\rangle+\beta\|110\rangle$ |
| $\|\Phi^+\rangle_{a1}\|\Psi^+\rangle_{b5}$ | $\|\varphi^5\rangle_{234}=\alpha\|011\rangle+\mu\|110\rangle+\gamma\|000\rangle+\beta\|101\rangle$ |
| $\|\Phi^+\rangle_{a1}\|\Psi^-\rangle_{b5}$ | $\|\varphi^6\rangle_{234}=\alpha\|011\rangle+\mu\|110\rangle-\gamma\|000\rangle-\beta\|101\rangle$ |
| $\|\Phi^-\rangle_{a1}\|\Psi^+\rangle_{b5}$ | $\|\varphi^7\rangle_{234}=\alpha\|011\rangle-\mu\|110\rangle+\gamma\|000\rangle-\beta\|101\rangle$ |
| $\|\Phi^-\rangle_{a1}\|\Psi^-\rangle_{b5}$ | $\|\varphi^8\rangle_{234}=\alpha\|011\rangle-\mu\|110\rangle-\gamma\|000\rangle+\beta\|101\rangle$ |
| $\|\Psi^+\rangle_{a1}\|\Phi^+\rangle_{b5}$ | $\|\varphi^9\rangle_{234}=\alpha\|101\rangle+\mu\|000\rangle+\gamma\|110\rangle+\beta\|011\rangle$ |
| $\|\Psi^+\rangle_{a1}\|\Phi^-\rangle_{b5}$ | $\|\varphi^{10}\rangle_{234}=\alpha\|101\rangle+\mu\|000\rangle-\gamma\|110\rangle-\beta\|011\rangle$ |
| $\|\Psi^-\rangle_{a1}\|\Phi^+\rangle_{b5}$ | $\|\varphi^{11}\rangle_{234}=\alpha\|101\rangle-\mu\|000\rangle+\gamma\|110\rangle-\beta\|011\rangle$ |
| $\|\Psi^-\rangle_{a1}\|\Phi^-\rangle_{b5}$ | $\|\varphi^{12}\rangle_{234}=\alpha\|101\rangle-\mu\|000\rangle-\gamma\|110\rangle+\beta\|011\rangle$ |
| $\|\Psi^+\rangle_{a1}\|\Psi^+\rangle_{b5}$ | $\|\varphi^{13}\rangle_{234}=\alpha\|110\rangle+\mu\|011\rangle+\gamma\|101\rangle+\beta\|000\rangle$ |
| $\|\Psi^+\rangle_{a1}\|\Psi^-\rangle_{b5}$ | $\|\varphi^{14}\rangle_{234}=\alpha\|110\rangle+\mu\|011\rangle-\gamma\|101\rangle-\beta\|000\rangle$ |
| $\|\Psi^-\rangle_{a1}\|\Psi^+\rangle_{b5}$ | $\|\varphi^{15}\rangle_{234}=\alpha\|110\rangle-\mu\|011\rangle+\gamma\|101\rangle-\beta\|000\rangle$ |
| $\|\Psi^-\rangle_{a1}\|\Psi^-\rangle_{b5}$ | $\|\varphi^{16}\rangle_{234}=\alpha\|110\rangle-\mu\|011\rangle-\gamma\|101\rangle+\beta\|000\rangle$ |

where $|\Phi^\pm\rangle=(|00\rangle\pm|11\rangle)/\sqrt{2}$ and $|\Psi^\pm\rangle=(|01\rangle\pm|10\rangle)/\sqrt{2}$ are four Bell states. Then Alice tells the result of her measurement to Bob and Charlie via a classical channel. If the controller Charlie allows Bob to get the initial state that Alice wants to send to Bob，then Charlie carries out a SPM on qubit 4 under the basis $|\pm\rangle=(|0\rangle\pm|1\rangle)/\sqrt{2}$. The outcome of the measurement performed by Charlie and the corresponding state obtained by Bob are shown in Table 2.

Then Charlie tells Bob about his result via a classical channel. Finally，Bob needs to perform an appropriate unitary transformation on his qubits 2 and 3 according to the results of Alice and Charlie so as to obtain the initial state that Alice wants to send to him. These unitary transformations are shown in the Table 3. The security of this scheme against eavesdropping and cheating can be assured by using the same check and proof methods proposed in Refs. [8,25,28]，so this scheme can be made to be secure.

**Table 2 The outcome of the measurements performed by Charlie and the corresponding state obtained by Bob, where the normalization factors have been omitted for convenience**

| Charlie′s result | $\lvert\varphi^i\rangle_{23}^-$ the states obtained by Bob | Charlie′s result | $\lvert\varphi^i\rangle_{23}^+$ the states obtained by Bob |
|---|---|---|---|
| $\lvert -\rangle_4^1$ | $\lvert\varphi^1\rangle_{23}^-=\alpha\lvert 00\rangle-\mu\lvert 10\rangle-\gamma\lvert 01\rangle+\beta\lvert 11\rangle$ | $\lvert +\rangle_4^1$ | $\lvert\varphi^1\rangle_{23}^+=\alpha\lvert 00\rangle+\mu\lvert 10\rangle+\gamma\lvert 01\rangle+\beta\lvert 11\rangle$ |
| $\lvert -\rangle_4^2$ | $\lvert\varphi^2\rangle_{23}^-=\alpha\lvert 00\rangle-\mu\lvert 10\rangle+\gamma\lvert 01\rangle-\beta\lvert 11\rangle$ | $\lvert +\rangle_4^2$ | $\lvert\varphi^2\rangle_{23}^+=\alpha\lvert 00\rangle+\mu\lvert 10\rangle-\gamma\lvert 01\rangle-\beta\lvert 11\rangle$ |
| $\lvert -\rangle_4^3$ | $\lvert\varphi^3\rangle_{23}^-=\alpha\lvert 00\rangle+\mu\lvert 10\rangle-\gamma\lvert 01\rangle-\beta\lvert 11\rangle$ | $\lvert +\rangle_4^3$ | $\lvert\varphi^3\rangle_{23}^+=\alpha\lvert 00\rangle-\mu\lvert 10\rangle+\gamma\lvert 01\rangle-\beta\lvert 11\rangle$ |
| $\lvert -\rangle_4^4$ | $\lvert\varphi^4\rangle_{23}^-=\alpha\lvert 00\rangle+\mu\lvert 10\rangle+\gamma\lvert 01\rangle+\beta\lvert 11\rangle$ | $\lvert +\rangle_4^4$ | $\lvert\varphi^4\rangle_{23}^+=\alpha\lvert 00\rangle-\mu\lvert 10\rangle-\gamma\lvert 01\rangle+\beta\lvert 11\rangle$ |
| $\lvert -\rangle_4^5$ | $\lvert\varphi^5\rangle_{23}^-=-\alpha\lvert 01\rangle+\mu\lvert 11\rangle+\gamma\lvert 00\rangle-\beta\lvert 10\rangle$ | $\lvert +\rangle_4^5$ | $\lvert\varphi^5\rangle_{23}^+=\alpha\lvert 01\rangle+\mu\lvert 11\rangle+\gamma\lvert 00\rangle+\beta\lvert 10\rangle$ |
| $\lvert -\rangle_4^6$ | $\lvert\varphi^6\rangle_{23}^-=-\alpha\lvert 01\rangle+\mu\lvert 11\rangle-\gamma\lvert 00\rangle+\beta\lvert 10\rangle$ | $\lvert +\rangle_4^6$ | $\lvert\varphi^6\rangle_{23}^+=\alpha\lvert 01\rangle+\mu\lvert 11\rangle-\gamma\lvert 00\rangle-\beta\lvert 10\rangle$ |
| $\lvert -\rangle_4^7$ | $\lvert\varphi^7\rangle_{23}^-=-\alpha\lvert 01\rangle-\mu\lvert 11\rangle+\gamma\lvert 00\rangle+\beta\lvert 10\rangle$ | $\lvert +\rangle_4^7$ | $\lvert\varphi^7\rangle_{23}^+=\alpha\lvert 01\rangle-\mu\lvert 11\rangle+\gamma\lvert 00\rangle-\beta\lvert 10\rangle$ |
| $\lvert -\rangle_4^8$ | $\lvert\varphi^8\rangle_{23}^-=-\alpha\lvert 01\rangle-\mu\lvert 11\rangle-\gamma\lvert 00\rangle-\beta\lvert 10\rangle$ | $\lvert +\rangle_4^8$ | $\lvert\varphi^8\rangle_{23}^+=\alpha\lvert 01\rangle-\mu\lvert 11\rangle-\gamma\lvert 00\rangle+\beta\lvert 10\rangle$ |
| $\lvert -\rangle_4^9$ | $\lvert\varphi^9\rangle_{23}^-=-\alpha\lvert 10\rangle+\mu\lvert 00\rangle+\gamma\lvert 11\rangle-\beta\lvert 01\rangle$ | $\lvert +\rangle_4^9$ | $\lvert\varphi^9\rangle_{23}^+=\alpha\lvert 10\rangle+\mu\lvert 00\rangle+\gamma\lvert 11\rangle+\beta\lvert 01\rangle$ |
| $\lvert -\rangle_4^{10}$ | $\lvert\varphi^{10}\rangle_{23}^-=-\alpha\lvert 10\rangle+\mu\lvert 00\rangle-\gamma\lvert 11\rangle+\beta\lvert 01\rangle$ | $\lvert +\rangle_4^{10}$ | $\lvert\varphi^{10}\rangle_{23}^+=\alpha\lvert 10\rangle+\mu\lvert 00\rangle-\gamma\lvert 11\rangle-\beta\lvert 01\rangle$ |
| $\lvert -\rangle_4^{11}$ | $\lvert\varphi^{11}\rangle_{23}^-=-\alpha\lvert 10\rangle-\mu\lvert 00\rangle+\gamma\lvert 11\rangle+\beta\lvert 01\rangle$ | $\lvert +\rangle_4^{11}$ | $\lvert\varphi^{11}\rangle_{23}^+=\alpha\lvert 10\rangle-\mu\lvert 00\rangle+\gamma\lvert 11\rangle-\beta\lvert 01\rangle$ |
| $\lvert -\rangle_4^{12}$ | $\lvert\varphi^{12}\rangle_{23}^-=-\alpha\lvert 10\rangle-\mu\lvert 00\rangle-\gamma\lvert 11\rangle-\beta\lvert 01\rangle$ | $\lvert +\rangle_4^{12}$ | $\lvert\varphi^{12}\rangle_{23}^+=\alpha\lvert 10\rangle-\mu\lvert 00\rangle-\gamma\lvert 11\rangle+\beta\lvert 01\rangle$ |
| $\lvert -\rangle_4^{13}$ | $\lvert\varphi^{13}\rangle_{23}^-=\alpha\lvert 11\rangle-\mu\lvert 01\rangle-\gamma\lvert 10\rangle+\beta\lvert 00\rangle$ | $\lvert +\rangle_4^{13}$ | $\lvert\varphi^{13}\rangle_{23}^+=\alpha\lvert 11\rangle+\mu\lvert 01\rangle+\gamma\lvert 10\rangle+\beta\lvert 00\rangle$ |
| $\lvert -\rangle_4^{14}$ | $\lvert\varphi^{14}\rangle_{23}^-=\alpha\lvert 11\rangle-\mu\lvert 01\rangle+\gamma\lvert 10\rangle-\beta\lvert 00\rangle$ | $\lvert +\rangle_4^{14}$ | $\lvert\varphi^{14}\rangle_{23}^+=\alpha\lvert 11\rangle+\mu\lvert 01\rangle-\gamma\lvert 10\rangle-\beta\lvert 00\rangle$ |
| $\lvert -\rangle_4^{15}$ | $\lvert\varphi^{15}\rangle_{23}^-=\alpha\lvert 11\rangle+\mu\lvert 01\rangle-\gamma\lvert 10\rangle-\beta\lvert 00\rangle$ | $\lvert +\rangle_4^{15}$ | $\lvert\varphi^{15}\rangle_{23}^+=\alpha\lvert 11\rangle-\mu\lvert 01\rangle+\gamma\lvert 10\rangle-\beta\lvert 00\rangle$ |
| $\lvert -\rangle_4^{16}$ | $\lvert\varphi^{16}\rangle_{23}^-=\alpha\lvert 11\rangle+\mu\lvert 01\rangle+\gamma\lvert 10\rangle+\beta\lvert 00\rangle$ | $\lvert +\rangle_4^{16}$ | $\lvert\varphi^{16}\rangle_{23}^+=\alpha\lvert 11\rangle-\mu\lvert 01\rangle-\gamma\lvert 10\rangle+\beta\lvert 00\rangle$ |

**Table 3 The correspondingly unitary transformation performed by Bob according to the state obtained by Bob**

| $\lvert\psi^i\rangle_{23}^-$ | $U_{23}^{i-}$ | $\lvert\psi^i\rangle_{23}^-$ | $U_{23}^{i-}$ | $\lvert\psi^i\rangle_{23}^+$ | $U_{23}^{i+}$ | $\lvert\psi^i\rangle_{23}^+$ | $U_{23}^{i+}$ |
|---|---|---|---|---|---|---|---|
| $\lvert\varphi^1\rangle_{23}^-$ | $\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}$ | $\lvert\varphi^9\rangle_{23}^-$ | $\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}$ | $\lvert\varphi^1\rangle_{23}^+$ | $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$ | $\lvert\varphi^9\rangle_{23}^+$ | $\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$ |
| $\lvert\varphi^2\rangle_{23}^-$ | $\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}$ | $\lvert\varphi^{10}\rangle_{23}^-$ | $\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}$ | $\lvert\varphi^2\rangle_{23}^+$ | $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$ | $\lvert\varphi^{10}\rangle_{23}^+$ | $\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix}$ |
| $\lvert\varphi^3\rangle_{23}^-$ | $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$ | $\lvert\varphi^{11}\rangle_{23}^-$ | $\begin{pmatrix}0&-1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$ | $\lvert\varphi^3\rangle_{23}^+$ | $\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}$ | $\lvert\varphi^{11}\rangle_{23}^+$ | $\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}$ |
| $\lvert\varphi^4\rangle_{23}^-$ | $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$ | $\lvert\varphi^{12}\rangle_{23}^-$ | $\begin{pmatrix}0&-1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix}$ | $\lvert\varphi^4\rangle_{23}^+$ | $\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}$ | $\lvert\varphi^{12}\rangle_{23}^+$ | $\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}$ |
| $\lvert\varphi^5\rangle_{23}^-$ | $\begin{pmatrix}0&0&-1&0\\0&0&0&1\\1&0&0&0\\0&-1&0&0\end{pmatrix}$ | $\lvert\varphi^{13}\rangle_{23}^-$ | $\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}$ | $\lvert\varphi^5\rangle_{23}^+$ | $\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}$ | $\lvert\varphi^{13}\rangle_{23}^+$ | $\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}$ |
| $\lvert\varphi^6\rangle_{23}^-$ | $\begin{pmatrix}0&0&-1&0\\0&0&0&1\\-1&0&0&0\\0&1&0&0\end{pmatrix}$ | $\lvert\varphi^{14}\rangle_{23}^-$ | $\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&1&0&0\\-1&0&0&0\end{pmatrix}$ | $\lvert\varphi^6\rangle_{23}^+$ | $\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix}$ | $\lvert\varphi^{14}\rangle_{23}^+$ | $\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}$ |
| $\lvert\varphi^7\rangle_{23}^-$ | $\begin{pmatrix}0&0&-1&0\\0&0&0&-1\\1&0&0&0\\0&1&0&0\end{pmatrix}$ | $\lvert\varphi^{15}\rangle_{23}^-$ | $\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}$ | $\lvert\varphi^7\rangle_{23}^+$ | $\begin{pmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{pmatrix}$ | $\lvert\varphi^{15}\rangle_{23}^+$ | $\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&1&0&0\\-1&0&0&0\end{pmatrix}$ |
| $\lvert\varphi^8\rangle_{23}^-$ | $\begin{pmatrix}0&0&-1&0\\0&0&0&-1\\-1&0&0&0\\0&-1&0&0\end{pmatrix}$ | $\lvert\varphi^{16}\rangle_{23}^-$ | $\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}$ | $\lvert\varphi^8\rangle_{23}^+$ | $\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}$ | $\lvert\varphi^{16}\rangle_{23}^+$ | $\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}$ |

## 2 Conclusion

In this paper, we proposed a new scheme for controlled teleportation of an arbitrary two-qubit state by using five-qubit cluster state. In this scheme, a five-particle cluster state is prepared and shared by a sender, a controller and a receiver. The sender needs to perform BSMs on her qubit pairs, respectively, then the controller performs SPM on his qubit under the basis $\{|+\rangle, |-\rangle\}$. Finally the receiver performs some appropriately unitary transformation on his qubits according to the measure results from the sender and the controller. Thus the task of controlled teleportation of an arbitrary two-qubit state is completed. This scheme is secure against eavesdropping attacks, and the probability of successful controlled teleportation is 100%. Without the cooperation of controller, the receiver cannot reconstruct the initial arbitrary two-qubit state by himself.

Compared with Muralidharan and Panigrahi's scheme, our scheme is much simpler than that, because in our scheme only need to make Bell-state measurements, and do not need to make multi-particle joint measurements. In this scheme, the measurement means are simple and easy to fulfill in the experiment. Nowadays, it has been shown that the five-qubit cluster state is important resource for teleportation of an arbitrary two-qubit state. This property of the scheme can be utilized to construct a controlled quantum channel, which may be useful in the future quantum computation.

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# 基于五粒子团簇态实现经济和简单的二粒子任意态的可控隐形传态

李渊华[1],刘俊昌[1],聂义友[1, 2]

(1 江西师范大学 物理与通信电子学院,南昌 330022)

(2 江西省光电子与通信重点实验室,南昌 330022)

**摘　要**:通过对五粒子团簇态新应用的研究,提出了一个经济和简单的二粒子任意态的可控隐形传态方案.在这个方案中,发送者(Alice)、控制者(Charlie)和接收者(Bob)共享一个五粒子团簇态,发送者只需要执行Bell基测量,而控制者也仅需要执行单粒子投影测量.接受者根据发送者和控制者的测量结果,对自己拥有的粒子做适当的幺正变换,就可以重建发送者的二粒子任意态.这个可控隐形传态方案是决定性的,成功的概率为100%.与使用相同的量子信道进行二粒子任意态的可控隐形传送方案相比,不需要执行多粒子的联合测量,从而使得这个方案更加简单.

**关键词**:可控量子隐形传态;团簇态;Bell基测量

**LI Yuan-hua** was born in 1984. Now he is a M. S. degree candidate at College of Physics and Communication Electronics, Jiangxi Normal University. His research interests focus on quantum optics and quantum information.

**NIE Yi-you** was born in 1963. Now he is a professor at College of Physics and Communication Electronics, Jiangxi Normal University, and his research interests focus on quantum optics, quantum computation and quantum information.