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Measure the Phase Retardation and Birefringence of the Mica Wave Plate Using the Spectroscopic Ellipsometer*

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Abstract: In order to gain the phase retardation and birefringence of mica wave plate according to the increase of the wavelength, the phase retardation of the mica wave plate is measured continuously in the spectral region of 400～770 nm using a spectroscopic ellipsometer. After the calibration of the mica plate, the experimental data are collected by the detector and sent to the computer. From the outputted data, the retardation can be obtained according to the increase of the wavelength. With the measured phase retardation, the birefringence of the mica wave plate can be calculated. The birefringence dispersion curve and dispersion formula are also gained. The proposed method can measure a mica wave plate with arbitrary phase retardation and has the merits of convenient, quick and high accuracy.

Key words: Mica wave plate; Phase retardation; Birefringence

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0 Introduction

For a mica wave plate, the phase retardation and optic axis need to be accurately determined. At present, there are many methods to measure the phase retardation, some of which can only measure the retardation of a quarter-wave plate[1-2]. Furthermore, some methods only give the phase retardation at one or several wavelengths, but not the retardation according to the increase of the wavelength in a spectral range[3-4].

Like some of the other physical properties of mica, the dispersion relations of the refractive indices and of the birefringence may vary from one crystal to another[5-7]. Einsporn′s data give the birefringence of two mica samples favorably used by some authors[5,8]. The birefringence for one sample has an average value of −0.004 7 in the spectral range 436～707 nm and an almost fixed birefringence value of about −0.005 0 for the second sample in the same spectral range.

It is concluded from this data that at $\lambda = 589$ nm, plates of thicknesses 30.8 and 29.7 μm from the two samples respectively will act as quarter-wave plates. These thicknesses are less than typical values ranging between 32 and 36 μm for this wavelength[5]. In contrast, Shurcliff[9] considered a typical birefringence value of −0.004 0 in the visible spectrum and concluded that a plate of thickness 50 μm provides a quarter-wave phase retardation for sodium light, which is obvious an exceedingly large thickness value.

1 Measuring principle

In this paper, we develop a novel method for measuring phase retardation in a spectral range and determining optic axis of a mica wave plate using the spectroscopic ellipsometer. With the measured phase retardation, the birefringence for the mica wave plate can be accurately calculated. To demonstrate this powerful novel technique, we use a 570 nm quarter-wave plate with an arbitrary axis as an example to illustrate the measurement principles. Excellent results are obtained. This new method is particularly useful for the wave plate whose optic axis and phase retardation are not known beforehand.

The ellipsometry mainly includes the reflection ellipsometry and the transmission ellipsometry. If the sample is so transparent that the transmission wave can be measured, the transmission ellipsometry can be used. Suppose the complex amplitude transmission coefficient of the p vibration in the transmission wave is T_p,

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then $T_p=|T_p|\mathrm{e}^{\mathrm{i}\Delta_{tp}}$ (Δ_{tp} is the phase shift of the p vibration). And the complex amplitude transmission coefficient of the s vibration is $T_s=|T_s|\mathrm{e}^{\mathrm{i}\Delta_{ts}}$ (Δ_{ts} is the phase shift of the s vibration). The ratio of T_p to T_s which can be measured by the ellipsometer is shown as

$$\rho_t=T_p/T_s=\tan\psi_t\mathrm{e}^{\mathrm{i}\Delta_t} \tag{1}$$

where

$$\psi_t=\arctan|\rho_t| \tag{2}$$

$$\Delta_t=\Delta_{tp}-\Delta_{ts} \tag{3}$$

The mica wave plate is a thin slice which is made by muscovite mica. When a linearly polarized light vertically entrances a wave plate, it is decomposed into o light (s vibration) and e light (p vibration). The refractive indexes are n_o and n_e respectively. Because the two lights have different speeds in the wave plate, after passing through the wave plate with the thickness of d the phase retardation between them is:

$$\Delta=\Delta_{tp}-\Delta_{ts}=-2\pi(n_\mathrm{e}-n_\mathrm{o})d/\lambda \tag{4}$$

where ($n_\mathrm{e}-n_\mathrm{o}$) is the birefringence at the wavelength λ. The negative sign is introduced in Eq. (4) since mica is a negative crystal ($n_\mathrm{e}<n_\mathrm{o}$). From the above discussion, the transmission ellipsometry shoud be used to measure the phase retardation of the mica wave plate.

There are multiple reflections within the mica plate because of its little thickness shown in Fig. 1. We set the fast and slow axes as the x and y axes in the rectangular coordinates. The propagation direction is set as the z axis. I is the incident light and d is the width of the plate. R_1, R_2, R_3…… are the reflected light and T_1, T_2, T_3 …… are the transmission light.

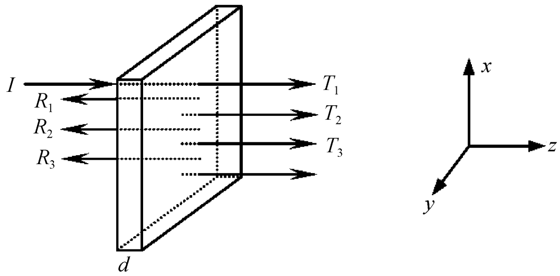


Fig. 1 Multiple reflections in mica wave plate

It is quite evident that all the reflected light and the transmitted light should be in the same line. We separate them just to show the procedure more clearly. We set T_{ix} and T_{iy} as the transmitted light at the x and y directions. To the x direction, the composed light is

$$\tau_x=T_{1x}+T_{2x}+T_{3x}+\cdots=t_x^2-t_x^2r_x^2\mathrm{e}^{-\mathrm{i}2\delta_x}+t_x^2r_x^4\mathrm{e}^{-\mathrm{i}4\delta_x}+\cdots \tag{5}$$

t_x, r_x and δ_x are given below

$$r_x=\frac{n-n_x}{n+n_x},\ t_x=\frac{2n}{n+n_x},\ \delta_x=\frac{2\pi n_x d}{\lambda} \tag{6}$$

where n is the refractive index of the incident medium, n_x is the refractive index at the x direction, λ is the incident wavelength. So the composed light at x direction is

$$\tau_x=t_x^2/(1+r_x^2\mathrm{e}^{-\mathrm{i}2\delta_x}) \tag{7}$$

For the same reason the composed light at y direction is

$$\tau_y=t_y^2/(1+r_y^2\mathrm{e}^{-\mathrm{i}2\delta_y}) \tag{8}$$

As we all know the Jones matrix is $\begin{bmatrix}\mathrm{e}^{-\mathrm{i}\delta_x} & 0\\ 0 & \mathrm{e}^{-\mathrm{i}\delta_y}\end{bmatrix}$ without the consideration of the multiple reflections within the mica plate. Considering the multiple reflections, the Jones matrix should be

$$\begin{bmatrix}\dfrac{t_x^2}{1+r_x^2\mathrm{e}^{-\mathrm{i}2\delta_x}} & 0\\ 0 & \dfrac{t_y^2}{1+r_y^2\mathrm{e}^{-\mathrm{i}2\delta_y}}\end{bmatrix}\begin{bmatrix}\mathrm{e}^{-\mathrm{i}\delta_x} & 0\\ 0 & \mathrm{e}^{-\mathrm{i}\delta_y}\end{bmatrix}=$$

$$\begin{bmatrix}\dfrac{t_x^2\mathrm{e}^{-\mathrm{i}\delta_x}}{1+r_x^2\mathrm{e}^{-\mathrm{i}2\delta_x}} & 0\\ 0 & \dfrac{t_y^2\mathrm{e}^{-\mathrm{i}\delta_y}}{1+r_y^2\mathrm{e}^{-\mathrm{i}2\delta_y}}\end{bmatrix} \tag{9}$$

If we make $\psi_x\mathrm{e}^{\mathrm{i}\Delta_x}=t_x^2\mathrm{e}^{-\mathrm{i}\delta_x}/(1+r_x^2\mathrm{e}^{-\mathrm{i}2\delta_x})$, $\psi_y\mathrm{e}^{\mathrm{i}\Delta_y}=t_y^2\mathrm{e}^{-\mathrm{i}\delta_y}/(1+r_y^2\mathrm{e}^{-\mathrm{i}2\delta_y})$, $\psi=\psi_y/\psi_x$, $\Delta=\Delta_y-\Delta_x$ then Eq. (9) can be set as

$$\psi_x\mathrm{e}^{\mathrm{i}\Delta_x}\begin{bmatrix}1 & 0\\ 0 & \psi\mathrm{e}^{\mathrm{i}\Delta}\end{bmatrix} \tag{10}$$

Eq. (10) is the Jones matrix of the mica plate considering the multiple reflections within it. ψ is the complex amplitude ratio of the transmitted light at the x and y directions and Δ is the phase retardation.

From calculation we gain

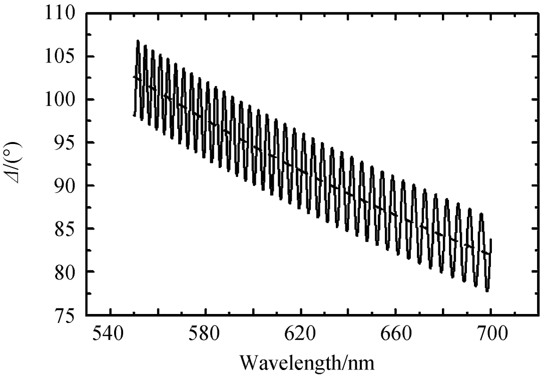
$$\begin{cases}\tan\Delta_x=\dfrac{1-r_{1x}^2}{1+r_{1x}^2}\tan\delta_x\\[2ex] \tan\Delta_y=\dfrac{1-r_{1y}^2}{1+r_{1y}^2}\tan\delta_y\end{cases} \tag{11}$$

So

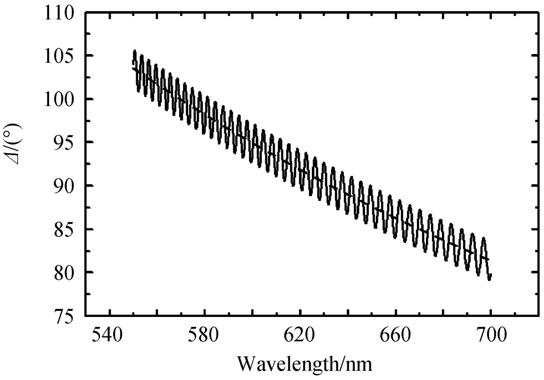
$$\tan\Delta=\frac{\tan\Delta_y-\tan\Delta_x}{1+\tan\Delta_y\tan\Delta_x}=\frac{(1-r_{1y}^2)(1+r_{1x}^2)\cdot}{(1+r_{1y}^2)(1+r_{1x}^2)+}\rightarrow$$

$$\leftarrow\frac{\tan\delta_y-(1-r_{1x}^2)(1+r_{1y}^2)\tan\delta_x}{(1-r_{1y}^2)(1-r_{1x}^2)\tan\delta_x\tan\delta_y} \tag{12}$$

Suppose a mica plate is a quarter wave plate at 633 nm. The reflection index of mica wave plate before coated is about 5.18%, from Eq. (12) we can gain the Fig. 2(a). The red imaginary line is the result from Eq. (4). If we set the reflection index as 1%, we can gain Fig. 2(b). So it is quite evident that the oscillations will be reduced a lot when reflection index becomes smaller.



(a) R=5.18%



(b) R=1%

Fig. 2 The retardation of the mica 1/4 wave plate

2 Experiment

To measure the phase retardation of the mica wave plate, broadband anti-reflective film is evaporated on it. The anti-reflective film serves two purposes: the first one is to increase the transmission and the second one is to suppress the multiple reflections within the mica plate because of its little thickness. The film coefficient is Sub/Al_2O_3 ∧ 1/WD10 ∧ 2/MgF_2 ∧ 1/Air.

The transmission curve (measured in the UV3101PC spectral photometer) of the coated sample is presented in Fig. 3. From the curve, the transmission of the o light and e light is more than 80% in the spectral range 400～770 nm. So the retardation and birefringence of the mica wave plate in this spectral range are meaningful.

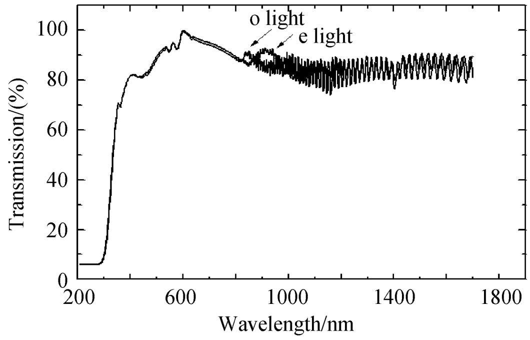


Fig. 3 Transmission curve of the coated mica wave plate

The UVISEL spectroscopic phase modulated ellipsometer (shown in Fig. 4) made by the French Jobin Yvon corporation is used in the experiment. L is the laser, P is the polarizer, S is the measured wave plate, A and A' are the analyzers, M is the modulator, M' is the monochromator, D is the detector and C is the computer. It has a continuous spectral range of 190～1 700 nm. The accuracy of the angular instrument is 0.05° which is also the accuracy of measuring other angles in the optical system.

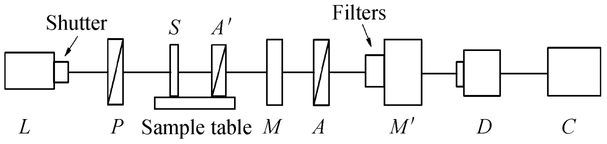


Fig. 4 UVISEL ellipsometer

The experiment procedures are as follows

1) Polarizer P and analyzer A are aligned in orientations with $P=0°$, $A=0°$. Then place another analyzer A' on the sample table. Adjust A' to make the fast axes of P and A' crossed.

2) Then place the measured plate between P and A', adjust the plate until minimum intensity is detected which means the optic axis of the measured plate is parallel or vertical to the transmission axis of the polarizer P. In the mean time the primary harmonic component $R_{\omega}=0.000\ 48$, the secondary harmonic component $R_{2\omega}=0.000\ 31$ which guarantee the optic axis of the measured plate is vertical to the incident light[10].

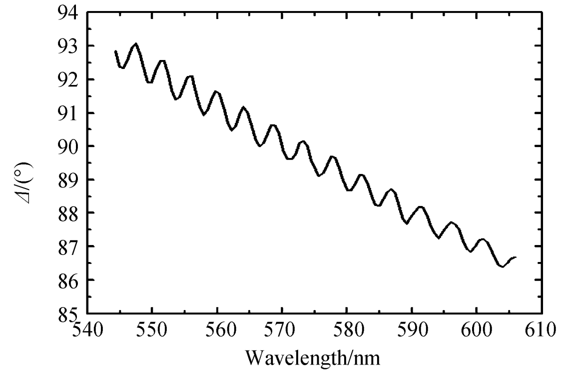
3) Take A' away and adjust $P=45°$, set the spectral range of the ellipsometer (the spectral range of this experiment is 400～770 nm), the retardation of the wave plate can be gained from the computer.

The experiment is made at the temperature (20 ± 0.2)℃. The thickness of the plate was measured at different points by a digital micrometer with accuracy 0.5 μm and the mean value of the thickness d is 32 μm. With the measured phase retardation data, the birefringence of the mica plate can be calculated from Eq. (4). The results are presented in Table 1. The wavelength interval is 10 nm which can be set a smaller value.

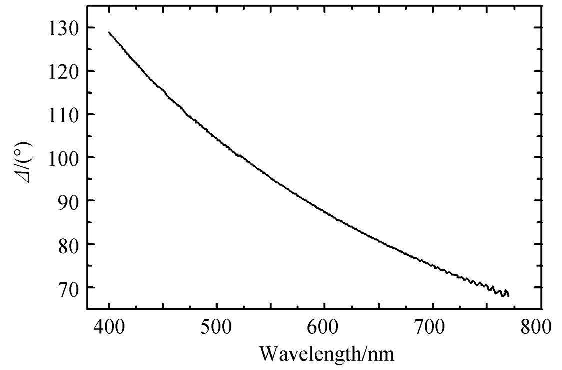
Table 1 Phase retardation and birefringence of the mica wave plate (after coated)

λ/nm	Δ/(°)	$-(n_e-n_o)\times10^{-3}$
400	126.674	4.398 40
410	123.890	4.409 28
420	120.975	4.410 55
430	118.357	4.417 84
440	115.828	4.423 99
450	113.283	4.425 12
460	111.091	4.435 93
470	108.825	4.439 91
480	106.812	4.450 50
490	104.777	4.456 66
500	102.814	4.462 41
510	100.214	4.436 56
520	98.833	4.461 21
530	96.643	4.446 25
540	95.569	4.479 80
550	94.089	4.492 10
560	92.522	4.497 60
570	90.545	4.480 09
580	88.539	4.457 69
590	87.470	4.479 80
600	86.265	4.492 97
610	84.524	4.475 66
620	83.251	4.480 52
630	82.184	4.494 44
640	80.951	4.497 28
650	79.551	4.488 56
660	77.862	4.460 84
670	76.434	4.445 38
680	75.201	4.438 95
690	73.995	4.431 99
700	73.104	4.442 08
710	73.110	4.505 91
720	73.266	4.579 13
730	71.087	4.504 65
740	68.378	4.392 34
750	70.734	4.605 08
760	66.052	4.357 60
770	68.622	4.586 71

The phase retardation of the mica wave plate is measured before and after coated as a contrast. Fig. 5 (a) shows a curve for the wavelength dependence of the retardation for the sample before coated. Fig. 5 (b) shows the curve for the same wave plate after coated. It is quite evident that the oscillations are avoided from 400 nm to 770 nm through the evaporation of broadband anti-reflective film. So the phase retardation accuracy is enhanced. From Fig. 5 (b) the retardation of the plate is 90.545° at 570 nm. The phase retardation measurement results have oscillations above 700nm because the evaporation center of the broadband anti-reflective film is 570 nm. So the oscillations occur at the longer wavelength because of the multiple reflections within the plate.



(a) Before coated



(b) After coated

Fig. 5 Phase retardation curves for the mica wave plate before and after coated

3 Results and discussion

The birefringence curve is shown in Fig. 6. In Fig. 6, the birefringence for the mica wave plate oscillates a little according to the increase of the wavelength from 400 nm to 700 nm. The measurement results also have larger oscillations above 700 nm for the same reason mentioned above. But the maximum amplitude is 0.247×10^{-3} which is in accordance with the first group of data in Ref. [5] with the maximum amplitude of

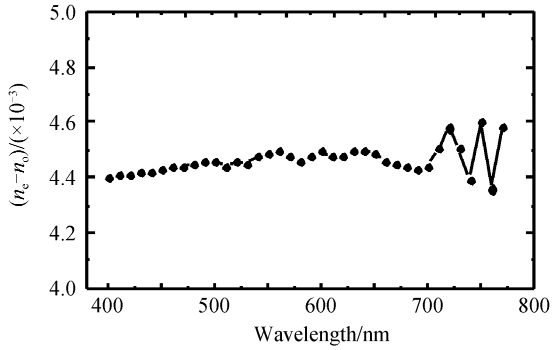


Fig. 6 Birefringence curve for the mica wave plate sample

0.275×10^{-3}. In Ref. [9] the oscillations in the birefringence curve are not so evident because the method they adopt can only gain birefringence at the wavelengths when wave plates act as quarter-wave or half-wave phase retarders. Compared with Ref. [5， 9]， our data are richer so our birefringence curve is more accurate. Expressing birefringence as a power series of the form[11]

$$(n_e-n_o)=c_0+c_1\lambda+c_2\lambda^2+c_3\lambda^3+\cdots \qquad (13)$$

(the unit of λ is 10^{-12} m) and neglecting higher order terms， a least-squares fit to our data gives： $c_0=-2.078\,65\times10^{-3}$， $c_1=-11.867\,10\times10^{-3}$， $c_2=19.543\,84\times10^{-3}$， $c_3=-10.694\,54\times10^{-3}$.

Sources of error in this work arise from the angular instrument in the spectroscopic ellipsometer and error in the thickness measurement of the plate. The former error from the angular instrument is 0.05°. The latter arise from the digital micrometer with accuracy 0.5 μm.

From Eq. (4) the error $-\delta(n_e-n_o)=\lambda V/2\pi(d\pm\delta d)=34.4\ \mathrm{m}^{-1}\times\lambda\ (\delta d=0.5\ \mu\mathrm{m})$. In the spectral range 400 ~ 770 nm the maximum error $-\delta(n_e-n_o)=34.4\times770\times10^{-9}=2.6\times10^{-5}$. Considering the two factors the estimated maximum uncertainty in the values of birefringence is then 3×10^{-5}.

4 Conclusions

We have demonstrated a new method to measure the phase retardation and the optic axis of mica plate by using a spectroscopic ellipsometer. The method has the following advantages. First， the phase retardation is directly obtained without further calculations or analysis of the results in a large spectral range rather than at one or several wavelengths. The second advantage is that the optic axis of the measured plate can be accurately determined. Finally， the birefringence of the mica wave plate can be calculated from the retardation data. The described method is simple， quick and accurate. This new method is particularly useful for the wave plate whose optic axis and phase retardation are not known beforehand.

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椭偏法测量云母波片相位延迟量及双折射率

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摘　要：为了得到云母波片的相位延迟量和双折射率随波长的变化关系，利用椭偏光谱仪连续测量了云母波片在400～770 nm光谱范围内的延迟量．在对云母波片进行校准后，测量的数据被光电探测器收集并输送到计算机，根据输出的数据可以得到云母波片的相位延迟量随波长的变化．利用测得的延迟量计算出了云母波片在一定光谱范围内的双折射率，得到了云母波片的双折射率色散曲线，并通过拟合得到了双折射率色散公式．该方法能测量任意波片的相位延迟量，并且具有测量方便、周期短、精度高等特点．

关键词：云母波片；相位延迟量；双折射率

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勘　误

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