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Entanglement Concentration of Single-photon Entangled States*

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Abstract: By the analysis of the action characteristic of the beam-splitter, presenting a method of generating the single-photon entangled states utilizing optics beam-splitter and single-photon source. A scheme on entanglement concentration of the single-photon entanglement states is designed. In this scheme, the two single-photon partially entangled states are utilized as the quantum channel. The process of the entanglement concentration is achieved by using the 50/50 symmetric beam splitters, the photon detectors and with the help of classical information. Results show that to distill some maximally entangled states from partially entangled pure states is possible by using this method.

Key words: Entanglement concentration; Single-photon entanglement states; Optics beam-splitter; Detection of photon

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0 Introduction

(Quantum entanglement is one of the essential features of quantum mechanics. In recent years, however, entanglement has generated much interest in the quantum information processing such as quantum teleportation[1], superdense coding[2], quantum key distribution[3], telecoloning[4] and quantum positioning and clock synchronization[5] et al.

In an efficient realization of quantum information processing, highly entangled states play a key role. When an entangled state is initially prepared in a non-maximally entangled state, it needs to be distilled to a highly entangled state before using it for quantum information processing. To obtain highly entangled states from less entangled states, researchers have proposed the idea of entanglement concentration and purification, and proved that it is possible to convert two copies of a less entangled state into one copy of a more entangled state by using only local operations and classical communication[6]. Also, some entanglement concentration and purification protocols have been investigated[7-9]. Here we show that single-photon entanglement concentration can be realized in a very simple way using only linear optical elements and photon detectors.

1 Single-photon entanglement

There are different types of entanglement for the light fields. In the Caltech teleportation experiment[10], a unknown coherent state was transmitted from a sender to a receiver, the state used for realizing the teleportation was a two-mode squeezed state. In the proposal of teleportation presented by S. J. van Enk et al[11], the transported state was an arbitrary coherent superposition state of two coherent states and the quantum channel was an entangled coherent state. In the previous papers[12], we have researched using the entangled squeezed vacuum states for realizing quantum information processing.

Recent developments in experimental techniques for generating and manipulating the single photon[13] have made quantum information processing utilizing single particle entanglement feasible, here, single particle entanglement refers to the entanglement between the single photon state and the vacuum state.

We consider single-photon entangled state of the form

$$|\varphi\rangle_{12}=\cos\eta|1>_1|0>_2-\sin\eta|0>_1|1\rangle_2 \qquad (1)$$

where $|1\rangle$ is a single-photon state, $|0\rangle$ is a vacuum state, the subscripts 1 and 2 refer to two modes of the light field, respectively. This is a bipartite entangled pure state, its reduced density matrices is[14]

$$\boldsymbol{\rho}^{(1)}=\boldsymbol{\rho}^{(2)}=\begin{bmatrix}\cos^2\eta & 0\\ 0 & \sin^2\eta\end{bmatrix} \qquad (2)$$

and its Von Neumann entropy is

$$S(\boldsymbol{\rho})=S(\boldsymbol{\rho}^{(1)})=S(\boldsymbol{\rho}^{(2)})=H(\sin^2\eta) \qquad (3)$$

where H is the Shannon entropy, which the probability distribution is $(\sin^2\eta, \cos^2\eta)$. When $\theta=\pm\pi/4$ the reduced density matrices of $|\varphi>_{12}$ is

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$$\boldsymbol{\rho}^{(1)}=\boldsymbol{\rho}^{(2)}=\begin{bmatrix}1/2 & 0\\ 0 & 1/2\end{bmatrix} \tag{4}$$

and it has the maximally partial entropy

$$S(\boldsymbol{\rho}^{(1)})=S(\boldsymbol{\rho}^{(2)})=-\mathrm{tr}(\boldsymbol{\rho}^{(1)}\,\mathbf{log}\,\boldsymbol{\rho}^{(1)})=1 \tag{5}$$

Actually, $|\phi\rangle_{12}$ has became a Bell state at this time

$$|\phi\rangle_{12}=\frac{1}{\sqrt{2}}(|1\rangle_1|0\rangle_2-|0\rangle_1|1\rangle_2) \tag{6}$$

it is a maximally entangled state.

Now we consider how to generate the single-photon entangled state. The experimental setup consists of a single photon source (S) and a lossless 50/50 beam-splitter BS, which can generate a single photon entangled state.

Let $\hat{a}_1$ and $\hat{a}_2$ denote the bosonic annihilation operators of the two light beams entering the two input ports of the beam-splitter and $\hat{b}_1$ and $\hat{b}_2$ denote the bosonic annihilation operators of the two light beams leaving the two output ports of the 50/50 beam-splitter, respectively. The boundary conditions at the surface of the beam-splitter lead to the mode transformation[15]

$$\begin{pmatrix}\hat{b}_1\\ \hat{b}_2\end{pmatrix}=\begin{pmatrix}\sqrt{R} & \sqrt{1-R}\\ -\sqrt{1-R} & \sqrt{R}\end{pmatrix}\begin{pmatrix}\hat{a}_1\\ \hat{a}_2\end{pmatrix} \tag{7}$$

where R is the reflectively of the beam-splitter. If setting $R=1/2$, the beam-splitter is of 50/50. Then we obtain

$$\begin{aligned}\hat{b}_1&=\hat{a}_1/\sqrt{2}+\hat{a}_2/\sqrt{2}\\ \hat{b}_2&=-\hat{a}_1/\sqrt{2}+\hat{a}_2/\sqrt{2}\end{aligned} \tag{8}$$

We assume that the two input light beams of the beam-splitter BS are in a single photon state and a vacuum state

$$|\Psi\rangle_{12}=|1\rangle_1|0\rangle_2=\hat{a}_1^+|0\rangle_1|0\rangle_2 \tag{9}$$

After interaction with the beam-splitter, the output state emerging from the beam-splitter BS is given by

$$\begin{aligned}|\Psi'\rangle_{12}&=\frac{1}{\sqrt{2}}(\hat{b}_1^+-\hat{b}_2^+)|0\rangle_1|0\rangle_2=\\ &(|1\rangle_1|0\rangle_2-|0\rangle_1|1\rangle_2)/\sqrt{2}\end{aligned} \tag{10}$$

This is a single-photon entangled state, we have proven its entanglement entropy is 1.

2 Entanglement concentration

In quantum information, researchers need maximally entangled states for an efficient realization of quantum information processing. If the initially entangled state is in a pure but not maximally entangled state, this state may be distilled to a maximally entangled state before using it for quantum information processing through entanglement concentration. Here we show that for a single-photon entangled state the entanglement concentration may be realized by using a beam-splitter and photo-detectors.

Suppose two people, Alice and Bob, share a partially entangled state

$$|\phi>_{13}=\cos\eta|1\rangle_1|0\rangle_3-\sin\eta|0\rangle_1|1\rangle_3 \tag{11}$$

Here, the real phase factor η, $0<\eta<\pi/2$, determines the degree of entanglement for state (11), from which we want to distill the maximally entangled state.

After sharing the quantum channel with Bob, first Alice prepares a pair of particles which are in the same entangled state as the quantum channel

$$|\phi\rangle_{24}=\cos\eta|1\rangle_2|0\rangle_4-\sin\eta|0\rangle_2|1\rangle_4 \tag{12}$$

The initial state of the whole system consisting of the four subsystems is then given by

$$|\Psi\rangle_{1234}=|\phi\rangle_{13}\otimes|\phi\rangle_{24} \tag{13}$$

For convenience, now we assume mode 3 on Bob′s side and modes 1, 2, and 4 on Alice′s side, then state $|\Psi\rangle_{1234}$ can be explicitly written as

$$\begin{aligned}|\Psi\rangle_{1234}=&\cos^2\eta\,\hat{a}_1^+\hat{a}_2^+|0\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4-\cos\eta\cdot\\ &\sin\eta\,\hat{a}_1^+|0\rangle_1|0\rangle_2|0\rangle_3|1\rangle_4-\sin\eta\cos\eta\,\hat{a}_2^+\cdot\\ &|0\rangle_1|0\rangle_2|1\rangle_3|0\rangle_4+\sin^2\eta|0\rangle_1|0\rangle_2|1\rangle_3|1\rangle_4\end{aligned} \tag{14}$$

Secondly Alice lets modes 1 and 2 enter the input ports of the beam-splitter. After interacting with the beam-splitter, the stat of the whole system becomes

$$\begin{aligned}|\Psi'\rangle_{1234}=&\frac{1}{2}\cos^2\eta(\hat{b}_1^+-\hat{b}_2^+)(\hat{b}_1^++\hat{b}_2^+)|0\rangle_1\cdot\\ &|0\rangle_2|0\rangle_3|0\rangle_4-\frac{1}{\sqrt{2}}\cos\eta\sin\eta(\hat{b}_1^+-\hat{b}_2^+)|0\rangle_1\cdot\\ &|0\rangle_2|0\rangle_3|1\rangle_4-\frac{1}{\sqrt{2}}\sin\eta\cos\eta(\hat{b}_1^++\hat{b}_2^+)|0\rangle_1\cdot\\ &|0\rangle_2|1\rangle_3|0\rangle_4+\sin^2\eta|0\rangle_1|0\rangle_2|1\rangle_3|1\rangle_4\end{aligned} \tag{15}$$

Its expansion in terms of Fork states is

$$\begin{aligned}|\Psi'\rangle_{1234}=&\frac{1}{\sqrt{2}}\cos^2\eta(|2\rangle_1|0\rangle_2-|0\rangle_1|2\rangle_2)|0\rangle_3\cdot\\ &|0\rangle_4)-\frac{1}{\sqrt{2}}\cos\eta\sin\eta|1\rangle_1|0\rangle_2(|0\rangle_3|1\rangle_4+\\ &|1\rangle_3|0\rangle_4)-\frac{1}{\sqrt{2}}\sin\eta\cos\eta|0\rangle_1|1\rangle_2(|0\rangle_3|1\rangle_4-\\ &|1\rangle_3|0\rangle_4)+\sin^2\eta|0\rangle_1|0\rangle_2|1\rangle_1|1\rangle_4\end{aligned} \tag{16}$$

Thirdly Alice performs a two-mode photon number measurement on mode 1 and 2 on her side. When one photon is detected at the mode 1 and zero photon is detected at the mode 2 simultaneously, the state of whole system collapses into

$$|\Psi''\rangle_{34}\rightarrow\sin\eta\cos\eta(|0\rangle_3|1\rangle_4+|1\rangle_3|0\rangle_4)/\sqrt{2} \tag{17}$$

When zero photon is detected at the mode 1 and one photon is detected at the mode 2 simultaneously, the state of whole system collapses into

$$|\Psi''\rangle_{34}\rightarrow\sin\eta\cos\eta(|0\rangle_3|1\rangle_4-|1\rangle_3|0\rangle_4)/\sqrt{2} \tag{18}$$

After normalization again, we obtain two maximally entangled states

$$|\Psi''\rangle_{34}=(|0\rangle_3|1\rangle_4\pm|1\rangle_3|0\rangle_4)/\sqrt{2} \tag{19}$$

we have proven that their amount of entanglement is exactly one ebit.

From Eq. (16) we can obtain the probability of finding one photon at the mode 1 and zero photon at the mode 2 simultaneously to be

$$P_{10}=|{}_1\langle 1|{}_2\langle 0|\Psi'\rangle_{1234}|^2=(\sin^2\eta\cos^2\eta)/2 \tag{20}$$

and finding zero photon at the mode 1 and one photon at the mode 2 simultaneously to be

$$P_{01}=|{}_1\langle 0|{}_2\langle 1|\Psi'\rangle_{1234}|^2=(\sin^2\eta\cos^2\eta)/2 \tag{21}$$

Then we obtain the probability of distilling maximally entangled states from non-maximally entangled states to be

$$P=P_{10}+P_{01}=\sin^2\eta\cos^2\eta \tag{22}$$

This implies that, no matter how small the initial entanglement is, it is possible to distill some maximally entangled states from partially entangled pure states.

3 Conclusion

To summarize, on the basis of the function of the beam-splitter and single-photon detecting technology, we described a method to prepare the single-photon entangled states. A scheme for distilling maximally entangled states from among partially entangled pure states by using a 50/50 beam splitter and photo detectors is proposed. We have proven that to distill some maximally entangled states from the initial partially entangled pure states is possible. Owing to recent developments in experimental techniques for generating and manipulating the single-photon, these schemes should be feasible in theory and practice.

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单光子纠缠态的纠缠浓缩

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摘　要:通过分析光学分束器对单光子态的作用特点,描述了一个利用单光子态和真空态制备纠缠单光子态的方法;提出了一个实现单光子纠缠态纠缠浓缩的方案.在这个方案中,两个单光子部分纠缠态被用来作为量子信道,通过利用光学分束器作用和单光子探测器探测完成了这个纠缠浓缩的过程.结果表明,对于单光子纠缠态,利用这种方法总有一定的几率可以从部分纠缠纯态中提纯出最大纠缠态.

关键词:纠缠浓缩;单光子纠缠态;光束分离器;光子探测

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